(a) $S_y = -S\sin\theta = -S\dfrac{y_n - y_{n-1}}{h} = \underset{\sim\sim\sim}{-\dfrac{S}{h}(y_n - y_{n-1})}$

(b) 同様にして、 $S_y{}' = \underset{\sim\sim\sim}{-\dfrac{S}{h}(y_n - y_{n+1})}$

これより、n 番目の粒子の運動方程式は、

$$Ma_n = S + S' = \frac{S}{h}(y_{n-1} - 2y_n + y_{n+1}) \cdots\cdots ①$$

(c)

$$\begin{aligned} y_{n-1} + y_{n+1} &= A\sin\left\{\omega t - \frac{2\pi}{\lambda}h(n-1)\right\} + A\sin\left\{\omega t - \frac{2\pi}{\lambda}h(n+1)\right\} \\ &= 2A\sin\left(\omega t - \frac{2\pi}{\lambda}nh\right)\cdot\cos\left(\frac{2\pi}{\lambda}h\right) \\ &= \underset{\sim\sim\sim}{2\cos\frac{2\pi}{\lambda}h}\cdot y_n \cdots\cdots ② \end{aligned}$$

(d) $a_n = \dfrac{d^2y_n}{dt^2} = -A\omega^2\sin\left(\omega t - \dfrac{2\pi}{\lambda}nh\right) = \underset{\sim\sim}{-\omega^2}\cdot y_n \cdots\cdots ③$

(e) ① 式に ②、③ 式を代入して、

$$\begin{aligned} -M\omega^2 y_n &= -\frac{S}{h}\left(2y_n - 2\cos\frac{2\pi}{\lambda}h\cdot y_n\right) \\ &= -\frac{2S}{h}\left(1 - \cos\frac{2\pi}{\lambda}h\right)y_n \\ \therefore \omega^2 &= \frac{2S}{Mh}\left(1 - \cos\frac{2\pi}{\lambda}h\right) = \frac{4S}{Mh}\sin^2\frac{\pi}{\lambda}h \\ \therefore \omega &= \underset{\sim\sim\sim}{2\sqrt{\frac{S}{Mh}}}\cdot\sin\frac{\pi}{\lambda}h \end{aligned}$$

(f) $\rho = \underset{\sim}{\dfrac{M}{h}}$

(g) $\omega = 2\sqrt{\dfrac{S}{Mh}} \cdot \sin\dfrac{\pi}{\lambda}h \fallingdotseq 2\sqrt{\dfrac{S}{Mh}}\dfrac{\pi h}{\lambda} = 2\pi\sqrt{\dfrac{S}{\dfrac{M}{h}}}\dfrac{1}{\lambda} = 2\pi\sqrt{\dfrac{S}{\rho}}\dfrac{1}{\lambda}$

$$\therefore T = \frac{2\pi}{\omega} = \sqrt{\frac{\rho}{S}}\lambda \qquad \therefore v = \frac{\lambda}{T} = \underset{\sim\sim\sim}{\sqrt{\frac{S}{\rho}}}$$

(h) $\lambda_m = \dfrac{2L}{m} \qquad \therefore f_m = \dfrac{v}{\lambda_m} = \underset{\sim\sim\sim}{\dfrac{m}{2L}\sqrt{\dfrac{S}{\rho}}}$

(i) 数値を代入して、

$$f_1 = \frac{1}{2 \times 1.0}\sqrt{\frac{0.50 \times 9.8}{1.0 \times 10^{-3}}} = \underset{\sim\sim\sim}{35\mathrm{Hz}}$$

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