

Conflict between Personal Freedom and Democratic Decision-making.

Shojiro Yasui

Graduate School of Decision Science and Technology,
Tokyo Institute of Technology
2-12-1, Ookayama, Meguro-ku, Tokyo 152-8552, Japan

ABSTRACT

As one of the justification of public debate, it is said there is a common understanding such that a decision based on democratic process such as collective agreement has itself justification. Unfortunately, in collective choice, democratic decision process does not guarantee personal freedom. Many study such as “Arrow’s general possibility theorem” (1963) has revealed that basic democratic conditions such as “unrestricted domain” (each individuals has un-restricted alternatives.), “independence from irrelevant alternative” and “Pareto optimality” cannot satisfy the condition of “non-dictatorship”. If we interpret collective choice as multiple criteria decision of single person, Arrow’s theorem also affirms impossibility of implementation of multi-criteria decision, although some physiological studies show opposite results. To resolve this issue, this paper proposes the decision model which employs cardinal criteria and relaxes the condition of “unrestricted domain”, and the social welfare function (SWF) to be used for the model. The main result of this paper is to reveal that the proposed SWF can meet all aforementioned conditions except “unrestricted domain”. Since we can reasonably assume the cardinal criterion in practice, Arrow’s theorem is not a real obstacle to conduct multi-criteria decision making. From the perspective of collective choice, “absolute freedom (unrestricted domain)” of choice cannot be maintained in democratic decision process, however, other Conditions I, P and D can be achieved by introducing a reasonable restriction of individual preference by utilizing cardinally measurable preferences.

1. Introduction

When you have to choose a hotel you stay, how do you select it? Someone focuses on charges; the other person focuses ambiances or else. Most of you decide the hotel by utilizing multiple criteria, such as charges, location, ambiances, services, etc. In this situation, what do you think if you are allowed to employ only a single particular criterion for your decision and cannot utilize other numerous indicators? That is what J.K. Arrow

proved in 1950, and it is called as “Arrow’s general impossibility theorem” (Arrow 1950, 1951, 1963).

Arrow’s social welfare function is a particular type of collective choice rule that specifies orderings for the society as “outcome” by utilizing the information about multiple individuals’ ordering of preference as “input”. The individuals’ ordering is assumed to be reflexive, transitive and complete. *1

We should note that the collective choice by Arrow’s social welfare function specifies “orderings” for the society, not only the “best alternative”. Each and every binary relationship has to be determined by the function.

1.1 Arrow’s frame work

In typical Arrow’s frame work, X is the set of all alternatives or outcomes. It has at least two members. We let $P(X)$ denote the set of orderings on X . The set N of individuals whose preferences are to be consulted is the finite set $\{1, 2, \dots, n\}$ with n is more than 1. Then $P(X)^N$ is the set of ordering on X for the set of N . A domain P is some non-empty subset of $P(X)^N$, a member p of $P(X)^N$ is called a profile, and it assigns the ordering $p(i)$ to individual $i \in N$, where where $p(i)$ is interpreted as i ’s preference ordering at profile p . A social welfare function for outcome set X and domain P is a function f from P into the set of complete binary relations on X .

The “Arrow’s framework” has five characteristics as follows. (Campbell and Kelly 2002)

- (1) The set X of alternatives is unstructured.
- (2) There is a finite set N of “individuals”, which is fixed. Typically, the members of N are different people, who have preference over the alternatives in X . However, May (1954) and Arrow and Raynaud (1986) regard N as decision criteria. For example, X is a set of hotels and the set N is the set of scheme of preference. For each attribute, the ranking reflects the tastes of a single individual. In this interpretation, we can analyze the single person’s decision making which is based on more than 2 criteria. This interpretation of N has been used in many papers employing “multi-criteria decision making”. (van Delft and Nijkamp, 1977)
- (3) Social choice is sensitive only to the ordinal property of individual preference. Specifically, the informational base for a social choice procedure is a family of profiles, where a profile is an assignment of a preference over X to each individual N . The input to the social choice procedure is a preference over X to each individual N .
- (4) The output of a social choice procedure is an ordering of the alternatives in X , called a social ordering, rather than the selection of one or more members of X .
- (5) Preference revelation is non-strategic. We do not consider what happens when “the ideals of the just society meet with the play of self interest” (Arrow 1997).

1.2 Arrow's general possibility theorem

Arrow proved general possibility theorem in 1950, which consists of imposing certain conditions on a social welfare function f , and showing that these conditions are mutually incompatible. In other words, rule f which can satisfy condition U, P and I must be dictatorial. These conditions are as follows ;(Campbell and Kelly 2002)

Condition U (unrestricted domain): The domain of the rule f must include all logically possible combinations of individual orderings.

Condition P (weak Pareto principle): the rule f must satisfy the Pareto principle in the weak form, i.e. if everyone prefers x to y , then society must also prefer x to y .

Condition I (independence of irrelevant alternative) : Social choice over a set of alternatives must depend on the orderings of the individuals only over those alternatives, not on anything else, e.g., on rankings of "irrelevant" alternatives that are not involved in this choice.

Condition D (non-dictatorship): Rule f should not be dictatorial. In other words, there should be no individual such that whenever he prefers x to y , society must prefer x to y , irresponsibly of the preference of everyone else.

1.3 Application of Arrow's Theorem to Multi-criteria Decision Making

If we regard N as the set of schemes of preference of single person, the interpretation of Arrow's theorem tells us that any person has to make his/her decision depending on only a single particular criterion. In other words, multi-criteria decision making cannot be done. For example, if you want to decide which hotel to go, you cannot utilize combined information of charges and ambience, but have to be based on only charges. This conclusion is not acceptable instinctively, and also contradicts some physiological studies. (Miller, 1956)

To resolve this issue, we formulate the decision-making models which can examine multi-criteria decision making, and in line with the model, we would like to evaluate the applicability of conditions to the model. Given a condition that cannot be applicable to the model, we can be freed from the constraints of Arrow's theorem.

2. The Model

In the model we use in this paper, we denote X as a set of hotels $X=\{A,B,C\}$, and the set $N =\{1,2\}$ as the set of scheme of preference denoting charges and ambiances respectively.

Criterion 1 orders the hotels according to the charges; criterion 2 orders them by ambiances as shown in the table below. A single person has social welfare function f such that determines the ordering of hotels based on the preferences of each criterion.

Table 1

1: hotel charges	2: ambiances
Hotel A (\$100)	Hotel B
Hotel B (\$200)	Hotel C
Hotel C (\$ 300)	Hotel A

You are assumed to be staying in Hotel B, and want to decide whether you should move to Hotel A or Hotel C, or keep staying Hotel B.

2.1 Cardinality and unrestricted domain

Let us focus on the criterion 1. This criterion is consisted of “cardinal utility” which is measurable by using an objective scale, and comparable interpersonally. In cardinal criterion, even if the unit of the indicators (e.g. \$, Yen, Euro, etc) are transformed, the ordering itself does not change. I.e., charges of Hotel A are more expensive than those of Hotel B, whatever the customers use USD, JPY, GBP or EUR. Therefore, the order of the criterion 1 cannot prefer B to A whoever the person employs this criterion, unless the customers like better expensive than inexpensive in general. This constraint breaches the Condition U, therefore, the decision making by utilizing this physiological model are not unavoidably dictatorial according to Arrow’s theorem.

On the other hand, the criterion 2 is consisted of “ordinal utility” which cannot be measured using an objective scale, but we are capable of ranking alternatives.

The other question arises. If one of the criteria are cardinal and does not fulfil the Condition U, is there any social welfare function complying with the Condition I, P, and D? The arrow’s theorem just say there is no social welfare function which satisfy all conditions of U, I, P and D at the same time, but does not say there are social welfare functions that satisfy the conditions of I, P and D.

2.2 Proposed Social Welfare Function

Let me propose the social welfare function as follows.

- (I) If everyone prefers x to y , then society must also prefer x to y . (Condition P (weak Pareto principle))
- (II) If there is a conflict between criteria (individuals), i.e., criterion 1 prefer x to y , and criterion 2 prefer y to x , and if x is a certain social state (the “status quo”) which will

be the outcome if the two individuals fail to strike a bargain, the society must prefer x to y .

- (III) If there is a conflict between criteria (individuals), and if both x and y are not “status quo”, and if the cardinal criterion prefers x to y , the society must prefer x to y .

As for rule (I), if at least one individual strictly prefers x to y , and every individual regards x to be at least as good as y , then the society should prefer x to y . This criterion has an obvious appeal.

As for rule (II), in predicting the actual outcome of a conflict of the preference, the status quo is clearly relevant, for it defines what will happen in the absence of the parties agreeing to a cooperative solution, as stated in Nash’s “bargaining problem” (Nash 1950).

As for rule (III), ordering in the cardinal criterion does not change even if the unit of the utility (i.e. \$, Yen, Euro, etc) is linearly transformed. Moreover, if the function of liner transformation is specified by “exchange rate”, interpersonal comparison of utility can be carried out. Because of such advantages, it is reasonable that the cardinal criterion precede other criteria.

2.3 The property of reflexivity, transitivity and completeness

As we employ the binary relation “at least as good as” in input ordering, the binary relations of outcome are also reflexive. The outcome of the proposed SWF has the property of completeness, because the function can determine the preference between all binary combinations of alternatives.

As for the property of transitivity, we need deeper consideration and examine the following proposition.

Proposition: If the set of alternative is $X = \{A, B, C\}$, and the set of criteria is $N = \{1, 2\}$, and if one of the criterion is cardinal, the orderings determined by the proposed social welfare function always satisfy the property of transitivity.

(Proof)

Let me assume that the criterion 1 is cardinal one. The binary relations which does not satisfy the condition of weak Pareto and comprises B (status quo) are the relations which the criterion 2 prefer B to A, or C to B only. In these cases, based on the rule (II), the social ordering must prefer B to A, or B to C respectively.

The binary relation to which the rule (II) does not apply is the one which the criterion 2 prefer C to A only. In this case, the rule (III) stipulates that the social choice must prefer A to C.

The above analysis reveals that;

- If and only if the criterion 2 (non-cardinal) prefers B to A, the social ordering must prefer B to A.
- In the other cases including if the criterion 2 prefer C to B, or C to A, the social ordering must be consistent with the ordering of criterion 1 (cardinal) which is preferring B to C, or A to C. Therefore, the social ordering conforms to the ordering of criterion 1.

As results, if and only if the criterion 2 prefers B to A, the social ordering becomes $B \succ A \succ C$. Otherwise, the social ordering adhere to the cardinal criterion's ordering. Therefore, all social orderings are transitive.
(End of proof)

2.4. Adaptation to Aforementioned Conditions

The proposed social welfare function meets the Condition P clearly. Also, The function also satisfies the Condition I, because rule (II) only affect the binary relation between A and B or B and C, and does not have any impact on the other binary relation, including the relation between A and C.

To examine the adaptation of the Condition D, let me show the application of the rule to the model shown in Table 1. As the first step, binary relations to meet weak Pareto principle are identified. B (status quo) is preferred to C in both criteria, then the society must prefer B (status quo) to C. As for the relation between A and B, there is no consensus among criteria (individuals). Therefore, rule (II) is applied, you must prefer B to A. Lastly, two criteria have conflict over the binary relation between A and C, rule (III) stipulates you to prefer A to C. Through these steps, the social ordering appears as $B \succ A \succ C$. This result clearly remarks there is no dictator in this decision-making. Also, this relation of three alternatives complies with reflexivity, transitivity and completeness.

3. Conclusion

The analysis above reveals that it is possible to formulate the social welfare function which can fulfil the Condition P, I and D, in case one of the alternative is the state of status quo, and one of the criteria is cardinal criterion, in the use of the model which comprises three alternatives and two criteria.

In practice, we can reasonably assume the multi-criteria decision making which incorporates the cardinal criterion and the state of status quo. Therefore, we do not have to overreact to Arrow's theorem in the application of multiple-criteria decision.

The results of the analysis can be interpreted that “absolute freedom (unrestricted domain)” cannot be maintained in democratic collective choice, however, other Conditions I, P and D can be achieved by introducing a reasonable restriction of individual preferences by utilization of cardinal measurable preferences.

Since this paper examines a limited situation, the study to seek further generalization is desired.

*1: “An ordering is a ranking of all alternatives vis-à-vis each other. Consider the relation “at least as good as.” First, it must be “transitive,” i.e., if x is at least as good as y, and y is at least as good as z, then x should be at least as good as z. Second, the relation must be “reflexive,” i.e., every alternative x must be thought to be at least as good as itself. Third, the relation must be “complete,” i.e., for any pair of alternatives x and y, either x is at least good as y, or y is at least as good as x (or possibly both).” (Sen, 1970))

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