

An Estimation of Critical Electron Density at Just Starting Breakdown in Gases

Mase. H

Professor Emeritus of Ibaraki University

Electric Field Threshold E^* for Electron Impact Ionization of Neutral Atoms

$$E^* = \frac{\phi_i}{\lambda_i} \quad \leftarrow \quad \frac{1}{2} m v_e^2 = e E^* \lambda_i = e \phi_i$$

ϕ_i Ionization Potential

$\lambda_i = (\sigma_i n)^{-1}$ m.f.p. for electron impact ionization

$E > E^*$, $\frac{dn_e}{dt} > 0$ for $\lambda_i < \text{gap length} \rightarrow$ spark or breakdown,

here any particle loss and energy loss except for ionization are out of account.

cf $E^* = 30 \text{ kV/cm}$ for dielectric strength in atmospheric air

If $\phi_i = 15 \text{ V}$, λ_i becomes $5 \times 10^{-4} \text{ cm}$ \leftarrow reasonable

$$V_{BD} = E^* d = \left(\frac{\phi_i}{\lambda_{i0}} \right) \cdot p d \quad \leftarrow \text{Paschen's law}$$

Temporal and Spatial Growth of Electron Density

$$\frac{dn_e}{dt} = R_g n_e = \left(\frac{\nu}{\lambda_i} \right) n_e > 0$$

$$n_e = n_0 e^{R_g t}$$

$$\frac{dn_e}{dx} = \left(\frac{R_g}{\nu} \right) n_e = \left(\frac{1}{\lambda_i} \right) n_e > 0$$

$$n_e = n_0 e^{x/\lambda_i}$$

here, $E \geq E^*$ and any particle loss is out of account.

energy loss for excitation is also out of account.

$$\text{energy balance } \frac{d\varepsilon}{dt} = \sigma E^2 - e R_g n_e \phi_i \geq 0$$

Structure of Electron Avalanche

$$\frac{dn_e}{dx} = \frac{1}{\lambda_i} n_e, \quad n_e = n_{e0} e^{x/\lambda_i}$$

where n_{e0} is initial electron density

Length x' of electron avalanche

$$n_e \approx n'_e \quad x' \approx \lambda_i \ln \left[\frac{n'_e}{n_{e0}} \right]$$

cf ions are at rest

initial electrons;

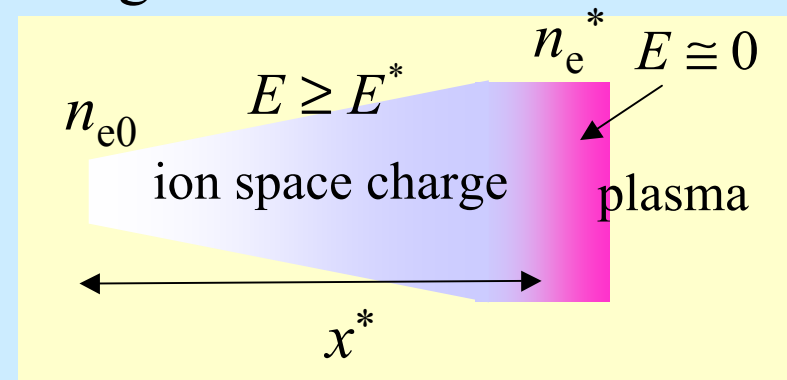
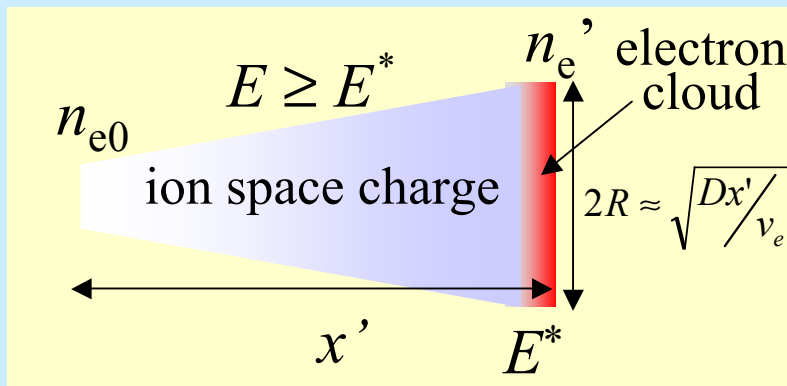
1.spontaneously generated

2.externally generated
(pre-ionization)

3.photo-ionization due to light
emission from front of
avalanche (feedback)

4.2ry emission by ion bom-
bardment (γ process, feedback)

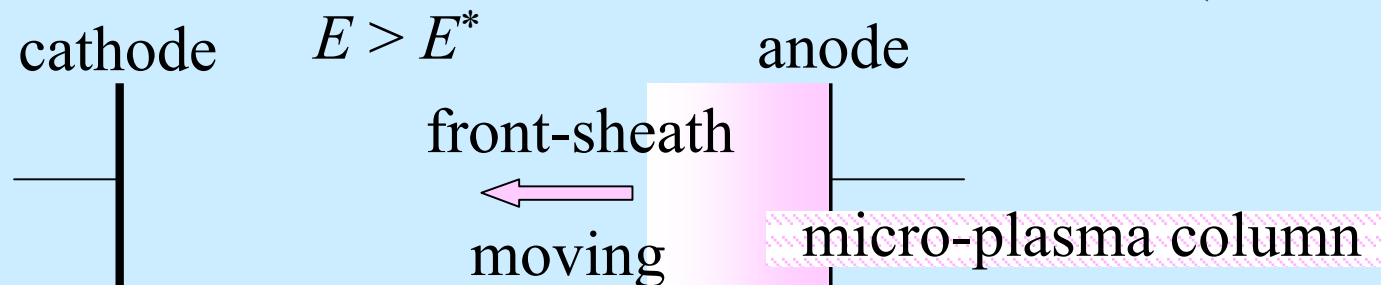
before and after starting breakdown



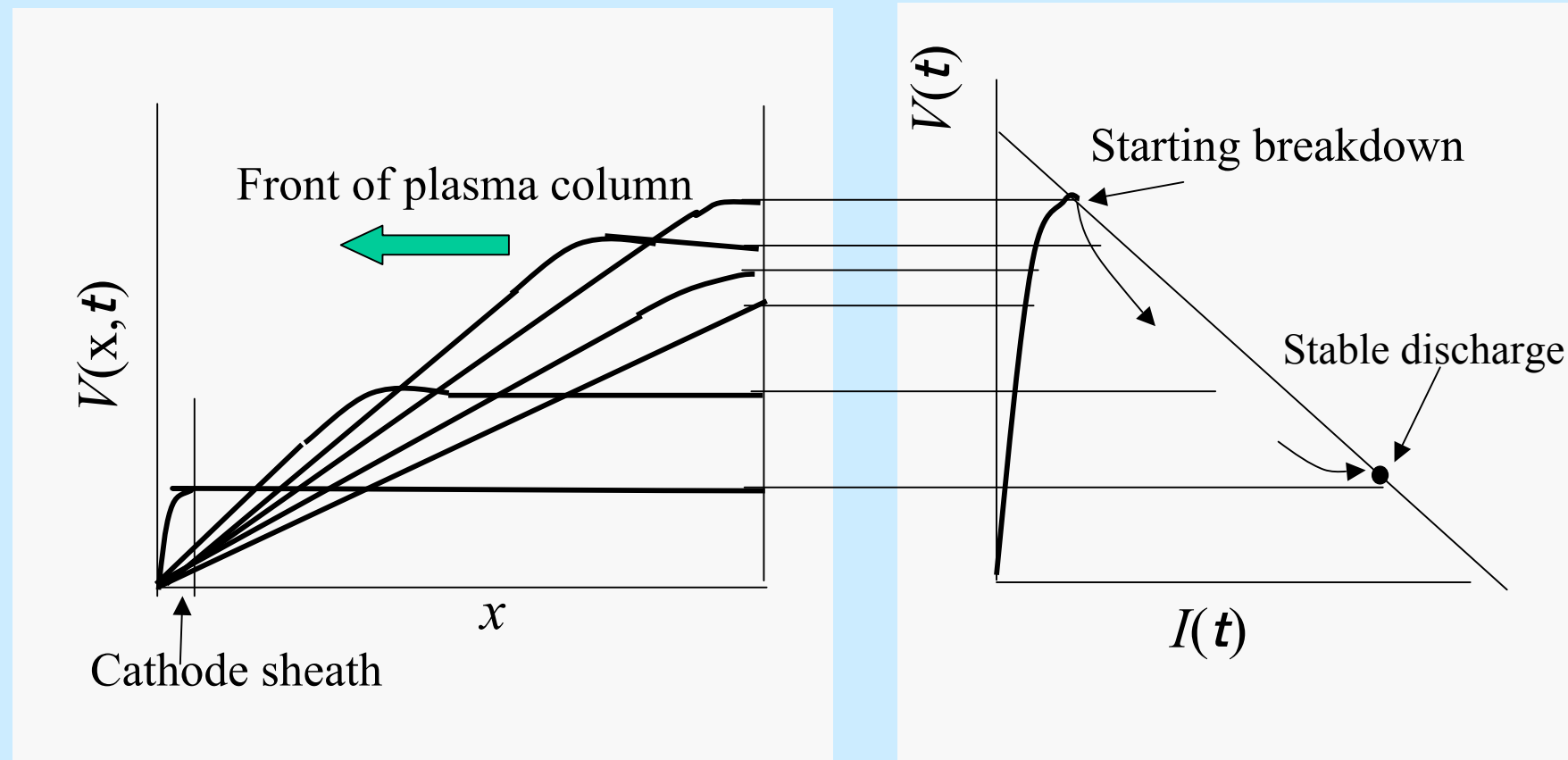
Scenario of Starting Electric Breakdown in Gases

1. $E > E^*$ is applied field across the electrodes. n_{e0} occurs at any time.
2. $dn_e/dt > 0$ in front of electron avalanche. E is extremely distorted by the space charge.
3. $n_e = n^* = n_i$ n^* is the critical electron density at just starting breakdown.
 >> appearance of micro-plasma in front of electron avalanche or near the anode.
4. E is increasing in the space between the cathode and front of the plasma column
 >> enhanced ionization >> expansion of plasma column.
5. Front of the plasma column (that is **virtual anode**) is moving toward the cathode.
 go back # 4. < positive feedback process > (* More detailed discussion is required to introduce the energy minimum principle.)
6. When the front-sheath comes at the cathode, the discharge gap makes it to flashover .
7. If the front-sheath satisfies the self-sustaining condition of discharge , the glow discharge or the arc discharge occurs.

In the case, the front-sheath means so-called “**cathode sheath**”. (*Appendix II*)



Evolution of Potential Structure before and after Starting Breakdown



A Hypothesis for the Appearance of Micro-Plasma at Anode Near Region

$$E^* = \frac{\phi_i}{\lambda_i} = \frac{T_{eV} / e}{\lambda_D}$$

T_{eV} electron temperature in eV

λ_D Debye shielding length

$E \approx 0$ in the plasma column

E^* would be shielded electrically by the polarization of plasma particles. It is best that T_{eV} should be confirmed theoretically by the plasma balance equation. Usually T_{eV} is in the range of several eV.

Critical Electron Density n_e^* at Just Starting Breakdown in Gases

$$\frac{\phi_i}{\lambda_i} = \frac{T_{eV}/e}{\lambda_D}, \quad \frac{1}{\lambda_D} = \left(\frac{n_e^* e^2}{\epsilon_0 T_{eV}} \right)^{1/2}$$

$$n_e^* = \frac{\epsilon_0}{e} \left(\frac{e\phi_i}{T_{eV}} \right) \frac{\phi_i}{\lambda_{i0}^2} p^2$$

$$\frac{e\phi_i}{T_{eV}} \approx 10, \quad \phi_i = 10 \text{ V}, \quad \lambda_i = 5 \times 10^{-4} / P [\text{atm}] \text{ cm}$$

$$n_e^* \approx 2 \times 10^{14} p^2 \text{ cm}^{-3} \quad p \text{ in atm}$$

Relationship between critical electron density and space charge density in front-sheath

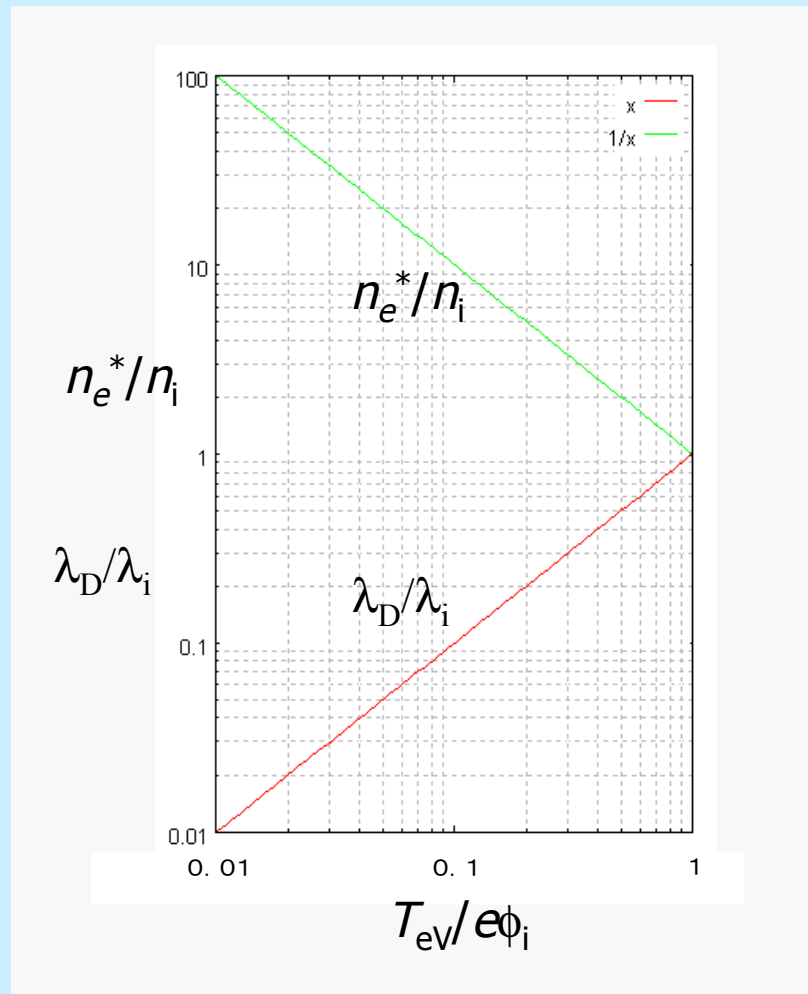
$$n_e^* = \left(\frac{e\phi_i}{T_{eV}} \right) \frac{\epsilon_0}{e} \frac{\phi_i}{\lambda_i^2},$$

$$\frac{\epsilon_0}{e} \nabla^2 \phi \approx \frac{\epsilon_0}{e} \frac{\phi_i}{\lambda_i^2} \approx n_i \quad \text{in front - sheath}$$

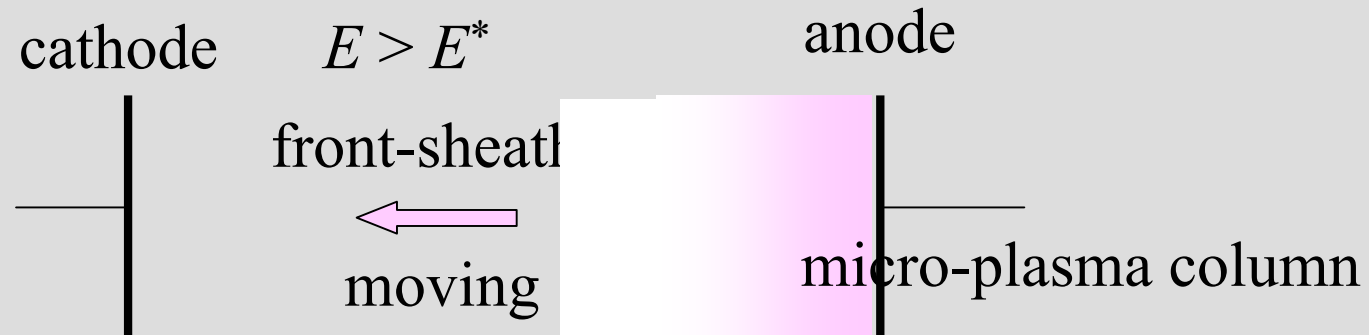
$$n_e^* \approx \left(\frac{e\phi_i}{T_{eV}} \right) n_i$$

$$n_e^* \lambda_D \approx n_i \lambda_i$$

n_e^*/n_i and λ_D/λ_i as a function of $T_{ev}/e\phi_i$



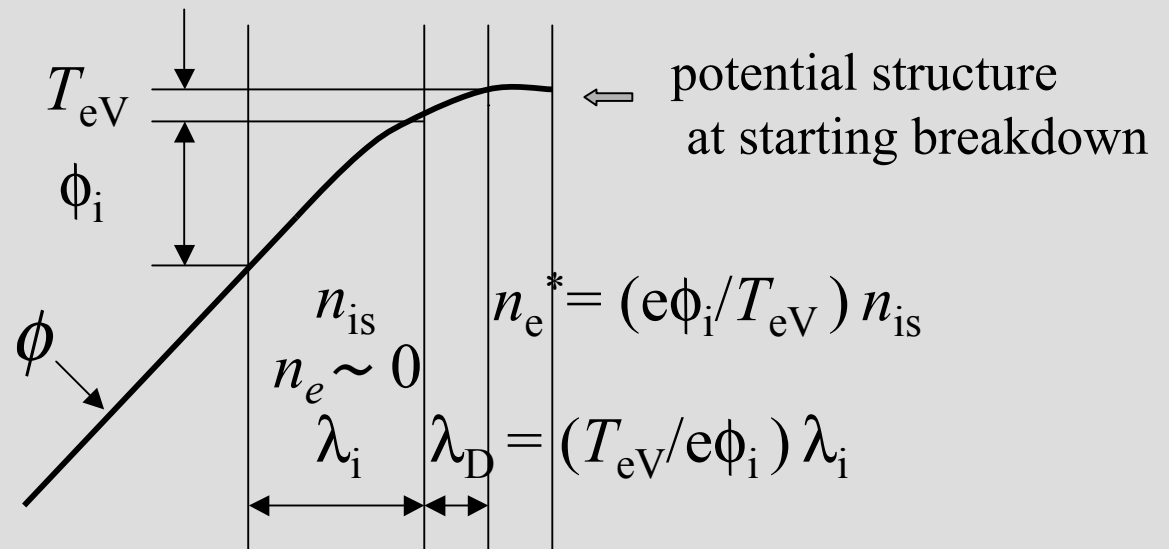
Relationship between Micro-Plasma Column and its Front-Sheath at Starting Breakdown



space charge density
in front-sheath

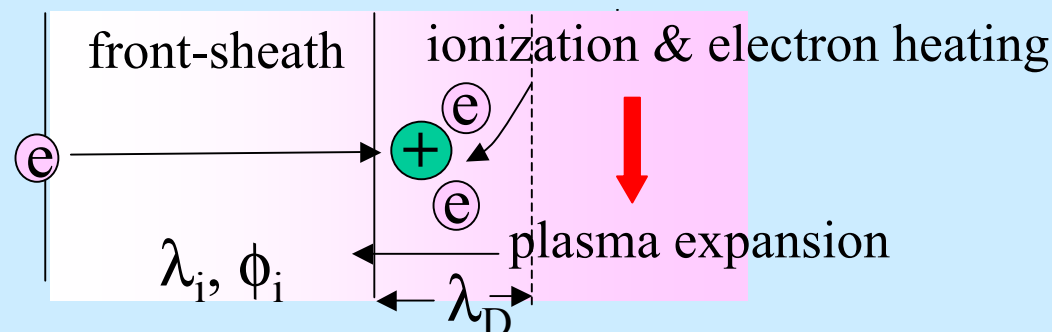
$$n_{is} = \frac{\epsilon_0}{e} \nabla^2 \phi$$

$$\approx \frac{\epsilon_0}{e} \cdot \frac{\phi_i}{\lambda_i^2} = \left(\frac{T_{eV}}{e\phi_i} \right) n_e^*$$



Velocity v_{fs} of Moving Front-Sheath (Streamer).

The growth of plasma volume is caused by the ionization due to fast electrons having their energy of $(1/2)mv^2 > e\phi_i$.



$$\frac{n_e^* \lambda_D}{t} = \frac{n_{se} v_e \lambda_D}{\lambda_i} \approx \left(\frac{\lambda_D}{\lambda_i} \right) \left(\frac{T_{eV}}{e\phi_i} \right) n_e^* \sqrt{\frac{2e\phi_i}{m}}$$

$$v_{fs} = \frac{\lambda_D}{t} \approx \sqrt{\frac{2e\phi_i}{m}} = 5.9 \times 10^7 \sqrt{\phi_i} \text{ cm/S}$$

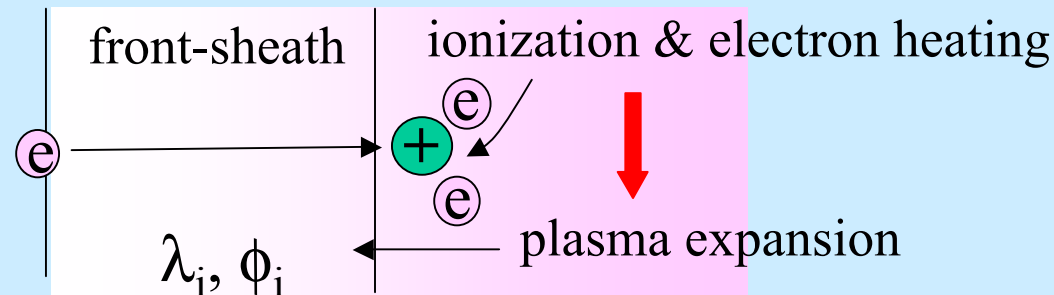
cf : expansion due to electron heating is neglected here
recent observation of $v_{fs} \sim 10^8 \text{ cm/s}$

Velocity v_{fs} of Moving Front-Sheath (Streamer).

$$v_{fs} \geq v_e = \sqrt{\frac{2e\phi_i}{m}} = 5.9 \times 10^7 \sqrt{\phi_i} \text{ cm/S}$$

Expansion of micro-plasma is caused by the ionization due to fast electrons having their energy of $(1/2)mv^2 > e\phi_i$. Velocity of moving front-sheath may be determined by the motion of fast electrons which are passing through the front-sheath without any ionization collision. So, v_{fs} could be estimated to be the same velocity of electrons accelerated up to $e\phi_i$ eV

(* More detailed discussion is required to introduce the energy minimum principle.)



cf : recent observation of $v_{fs} \sim 10^8$ cm/s

Critical current density at starting breakdown 1/2

Velocity and density of fast electrons which are passing through the front-sheath

$$v_{se} \geq \sqrt{2e\phi_i / m}$$
$$n_{se} \approx n_i = \left(\frac{T_{eV}}{e\phi_i} \right) n_e^*$$

Critical current density

$$j^* = en_{se}v_{se} \approx e \left(\frac{T_{eV}}{e\phi_i} \right) n_e^* \sqrt{\frac{2e\phi_i}{m}} = en_e^* \sqrt{\frac{2T_{eV}}{m}} \sqrt{\frac{T_{eV}}{e\phi_i}}$$

Critical current density at starting breakdown 2/2

electron thermal velocity

$$\frac{j^*}{p^2} \approx e \frac{n_e^*}{p^2} \sqrt{\frac{2T_{eV}}{m}} \left(\sqrt{\frac{T_{eV}}{e\phi_i}} \right)$$
$$\approx 1.6 \times 10^{-19} [\text{C}] \times 2 \times 10^{14} [\text{cm}^{-3}] \times 6 \times 10^7 [\text{cm/s}] \times \left(\sqrt{\frac{1}{10}} \right)$$
$$\approx 0.6 \text{ kA cm}^{-2} \text{ atm}^{-2}$$

Cross-section of micro-discharge in atmospheric pressure

$$1 \text{ mm}^2 \rightarrow I^* \sim 60 \text{ mA}$$

$$0.01 \text{ mm}^2 \rightarrow I^* \sim 0.6 \text{ mA}$$

Summary

Following hypothesis for the appearance of micro-plasma at anode near region, seems to be correct. Breakdown is just starting when the micro-plasma appears at anode near region.

$$E^* = \frac{\phi_i}{\lambda_i} = \frac{T_{eV} / e}{\lambda_D}$$

$E \approx 0$ in the plasma column

E^* is shielded electrically by the polarization of plasma particles

We can estimate the critical electron density at just starting breakdown by using the above formula.

$$n_e^* = \left(\frac{e\phi_i}{T_{eV}} \right) \frac{\epsilon_0}{e} \frac{\phi_i}{\lambda_i^2} = \left(\frac{e\phi_i}{T_{eV}} \right) \frac{\epsilon_0}{e} \frac{\phi_i}{\lambda_{i0}^2} \cdot p^2$$
$$n_e^* \approx \left(\frac{e\phi_i}{T_{eV}} \right) n_{is} \quad \frac{\epsilon_0}{e} \nabla^2 \phi \approx \frac{\epsilon_0}{e} \frac{\phi_i}{\lambda_i^2} \approx n_{is} \quad \text{in the front - sheath}$$

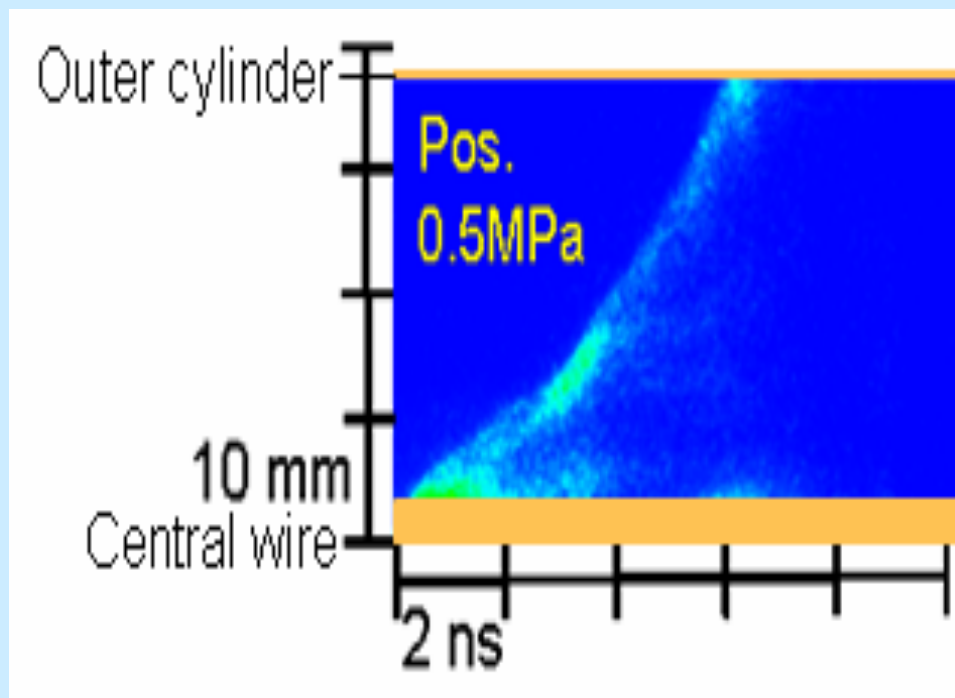
Summary *continue*

We also discuss the propagation of plasma column head toward the cathode (streamer). The propagating speed of plasma column head might be determined by the motion of fast electrons which are passing through the front-sheath without any ionization loss and then v_{fs} could be estimated to be the same velocity of electrons accelerated up to $e\phi_i$ eV (* More detailed discussion is required to introduce the *energy minimum principle*.)

$$v_{fs} \geq v_e = \sqrt{2e\phi_i/m} = 5.9 \times 10^7 \sqrt{\phi_i} \text{ cm/S}$$

Appendix I

Recent Observation of Streamer *-Kumamoto Univ.-*



$$V_s = 7.1 \text{ mm/ns}$$

Fig. 7 Streak photograph

Appendix II

Cathode Sheath in Steady State Glows

plasma balance Eqs. (*) $\frac{dn_i}{dt} \cdot L \approx \frac{n_e v_e L}{\lambda_i} - n_i v_i = 0,$

$$\frac{d\mathcal{E}}{dt} \cdot L \approx j_e V_{cs} - \frac{n_e v_e L}{\lambda_i} e \phi_i = j_e V_{cs} - j_i \phi_i = 0,$$

self - sustaining condition $j_e = \gamma \cdot j_i,$

$$V_{cs} \approx \frac{\phi_i}{\gamma},$$

$$d_{cs} \leq \lambda_i \text{ (any ionization does not occur in the sheath)}$$

* Here we consider only the most essential processes

