

Sustaining Voltage for DC Diode Discharges

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Comprehensive discussion of the sustaining voltage for dc diode discharges with both *cold*- and *hot*- cathodes using the same conceptual model, which is based on the collisionless cathode sheath model and the plasma balance equations.

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V_C	cathode fall potential
ΔV	potential drop across plasma column
V_{AC}	voltage across electrodes
d_C	cathode sheath thickness
d_A	electrode separation
R_{gen}, R_{loss}	generation and loss rates of plasma particles
ϕ_i	ionization potential
η	efficiency of ionization by 1 ry electron beam
J_{eb}	1 ry electron beam current density
J_i	ion current density
J	discharge current density $= J_{eb} + J_i$
γ	coefficient of γ process $= \left(\frac{J_{eb}}{J_i} \right)$

Table of symbols 2/2

λ_i	mean ionization length
J_{th}	thermoionic emission current density
m	electron mass
M	ion mass
n_i	ion density
n	plasma density $=n_i=n_e$
c_s	sound speed
σ	plasma conductivity
E	electric field in plasma column
r_p	radius of plasma column
α_r	coefficient of volume recombination
Ω	volume of plasma column

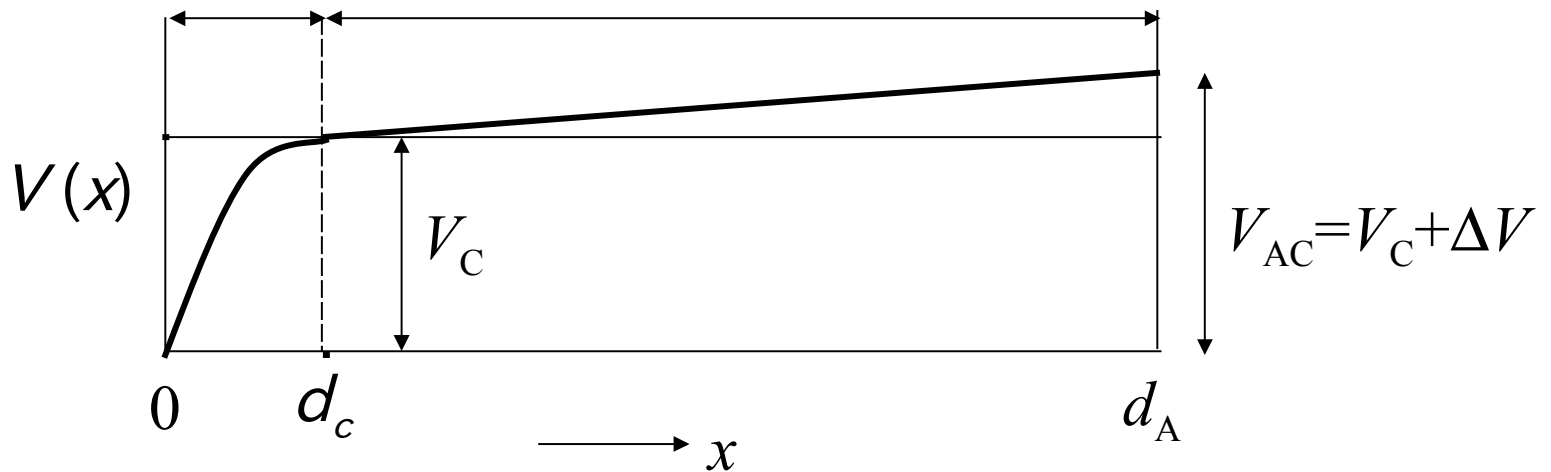
Cathode (cold)

Anode



Cathode Sheath

Plasma Column



Here detailed structure is neglected

Usually $d_c \ll d_A$

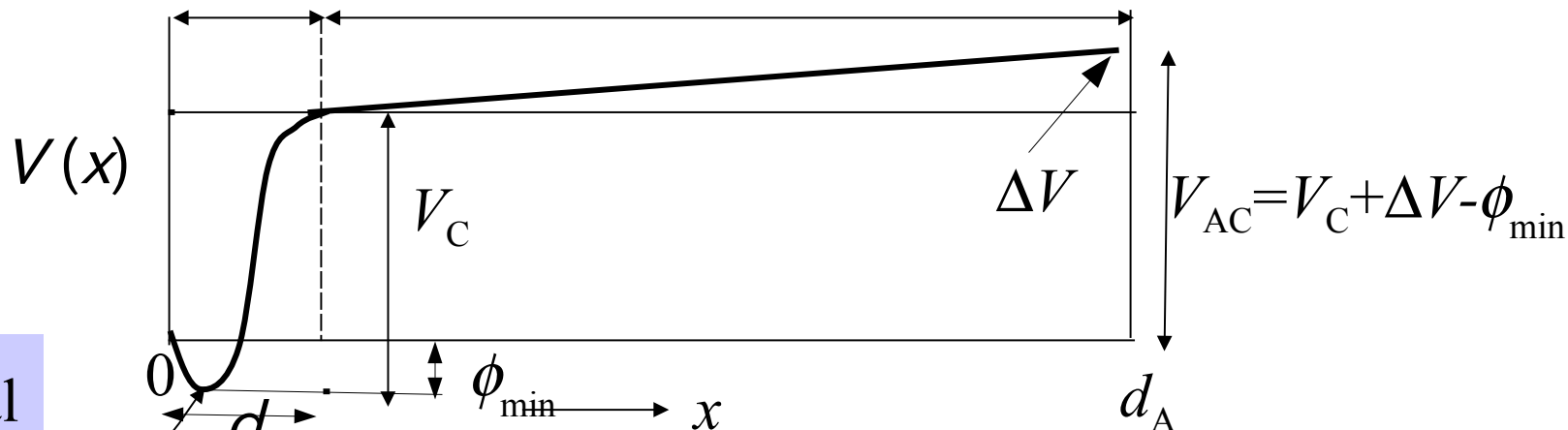
Outline of Potential Structure in DC Diode Discharges with Hot- Cathode

Hot Cathode

Anode



Cathode Sheath Plasma Column



Potential
minmum

Here detailed structure is neglected

Usually $d_c \ll d_A$

Derivation of Sustaining Voltage

Plasma Balance Equations;
Continuity of number density and energy

$$\frac{dn_i}{dt} d_{AC} = R_{gen} - R_{loss} = 0$$

$$\frac{d\varepsilon}{dt} d_{AC} = \eta V_C J_{eb} + \sigma E^2 (d_{AC} - d_C) - R_{gen} e \phi_i = 0$$

ϕ_i is ionization potential

J_{eb} is 1ry electron beam current density

$\eta \leq 1$ is ionization efficiency of 1ry electron beam

Derivation of Sustaining Voltage

Glow Discharge

In the case of relatively short discharge tubes

Potential drop along the plasma column is neglected

$R_{\text{loss}} = n_i v_i$, surface and volume losses are neglected

$$R_{\text{gen}} - R_{\text{loss}} = R_{\text{gen}} - n_i v_i = 0$$

$$\eta V_C J_{eb} - R_{\text{gen}} e \varphi_i = \eta V_C J_{eb} - e \varphi_i n_i v_i = 0$$

$$V_C = \frac{e \varphi_i n_i v_i}{\eta J_{eb}} = \frac{\varphi_i J_i}{\eta J_{eb}} = \left(\frac{1}{\eta} \right) \frac{\varphi_i}{\gamma}, \quad \frac{J_{eb}}{J_i} = \gamma < 1$$

Self-sustaining condition

$$\eta = 1, \text{ ie } \frac{\lambda^i}{\gamma} < d_{AC}$$

λ^i is mean ionization length



most low impedance
glow discharge

$$V_C = \frac{\varphi_i}{\gamma}$$

Derivation of Sustaining Voltage

Externally Heating Arc Discharge

In the case of relatively short discharge tubes

Potential drop along the plasma column is neglected

$R_{\text{loss}} = n_i v_i$, surface and volume losses are neglected

$$V_C = \frac{e \phi_i n_i v_i}{\eta J_{eb}} = \frac{\phi_i J_i}{\eta J_{eb}}, \quad \frac{J_i}{\eta J_{eb}} \geq 1$$

$$J \approx J_{eb} < J_{th}, \quad \frac{J_i}{J_{eb}} = \sqrt{\frac{m}{M}} \ll 1, \quad \text{double layer: Langmuir mode}$$

$$J \approx J_{eb} = J_{th}, \quad \frac{J_i}{J_{eb}} \leq 1, \quad \text{temperature limited mode}$$

$$\eta \frac{J_{eb}}{J_i} = 1, \quad V_C = \phi_i \quad \text{most low impedance arc discharge}$$

Derivation of Sustaining Voltage

Potential Drop along Plasma Column

$$\frac{dn_i}{dt} \Omega = R_{gen} - R_{loss} = 0$$

$$\frac{d\varepsilon}{dt} \Omega = \sigma E^2 \Omega - R_{gen} e \varphi_i = 0$$

$$R_{gen} = R_{loss} \approx n c_s (2 \pi r_p d_{AC}) \left(1 + \frac{\alpha_r n}{n c_s} \cdot \frac{r_p}{2} \right)$$

$$\Delta V = E d_{AC} = \frac{R_{loss} e \varphi_i}{\sigma E (\pi r_p)}$$

r_p is the radius of plasma column,

$$\approx \left(\frac{e n c_s}{J} \right) \left(1 + \frac{\alpha_r n^2}{n c_s} \cdot \frac{r_p}{2} \right) \cdot \left(\frac{2 d_{AC}}{r_p} \right) \cdot \varphi_t, \quad \Delta V \text{ depends on the ratio } (d_{AC}/r_p)$$

R_{loss} is surface and volume losses of plasma particles

$$E = \frac{\Delta V}{d_{AC}} \propto \left(\frac{e n c_s}{J} \right) \cdot \left(\frac{\varphi_i}{r_p} \right), \quad \text{same as Schottky's diffusion theory}$$

Sustaining Voltage for DC Diode Discharges

$$V_{AC} = V_C + \Delta V - \phi_{\min},$$

$$\phi_{\min} = 0 \quad \text{for glow mode}$$

$$V_C = \left(\frac{1}{\eta} \right) \cdot \frac{\phi_i}{\gamma} \geq \frac{\phi_i}{\gamma}, \quad \text{for glow mode}$$

$$V_C = \left(\frac{1}{\eta} \cdot \frac{J_i}{J_{eb}} \right) \cdot \phi_i \geq \phi_i, \quad \text{for arc mode}$$

$$\Delta V = \frac{R_{loss} e \phi_i}{\sigma E} \approx \left(\frac{e n c_s}{J} \right) \left(1 + \frac{\alpha_r n^2}{n c_s} \cdot \frac{r_p}{2} \right) \left(\frac{2 d_{AC}}{r_p} \right) \cdot \phi_i,$$

Sustaining Voltage as a function of J_{eb}/J_i

