Reason why the electric field varies linearly in the normal glow cathode fall region

-Formation of ion sheath composed of uniform space charge layer-.

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Many experimental results of the electric field in the cathode fall region demonstrated that the field decreases linearly with distance from the cathode in the normal glow discharge. However, the explanation for these observed evidence have been not satisfied with the conventional theories that are based on the collisional ionization sheath model. The conventional theories still remain the serious problems to be solved, although they are applied in lot of textbooks on the gaseous discharges.

About 25 years ago, we proposed the new conceptual model of cathode sheath, which is used the collisionless sheath model and the plasma balance equations. This model could clearly explain the reason why the electric field varies linearly in the normal glow cathode fall region. Here, we describe once more the outline of the explanation for the linearly varying field in the normal glow cathode fall.

Conventional models of ion sheathes

At first we consider the ion space charge layer (ion sheath) around the negatively biased electrode in the plasma. The studies on the ion sheath have been developed since 1929 (Langmuir).

The structure of ion sheath is determined by the motion of ions, the continuity of current and the bias voltage. As well known, the ion sheathes are categorized by the motion of ions into following three types. The the mean velocity $v(r)$ of ions is expressed as a fucnction of the acceleration ($\phi(r) = \int E(r)dr$ and/or $E(r)$).

1. collisionless sheath model. $d << \lambda_i$

$$v(r) \propto \sqrt{\phi(r)}$$

2. collisional seath model (I) , $d >> \lambda_i$

$$v(r) \propto \mu E(r)$$

3. collisional seath model (II) , $d > \lambda_I$

$$v(r) \propto \mu' \sqrt{E(r)}$$

where d, λ_i μ and μ’ are sheath thickness, ion m.f.p. and two types of ion mobility, respectively.

The equation of continuity in the steady state flow is given by

$$J = en(r)v(r) = \text{constant}$$

where J and $n(r)$ are ion current density and number density of ions, respectively. The continuity of ion current show that the space charge density $\rho(r) \propto 1/v(r)$ decreases with the distance r from the sheath edge, because the ions are accelerated toward the negatively biased electrode and then the velocity of ions increase with r.

One-dimensional Poison's equation indicates that the linearly varying space charge field occurs in a case of the uniform space charge layer.

$$\frac{dE}{dr} \propto \rho(r) = \text{constant}$$

In above sheath models, the uniform space charge layer never occurs, and then they could not explain the linearly varying field in the ion sheath.

$$\frac{dE}{dr} \propto \rho(r) \neq \text{constant} \quad .$$

New sheath model

Now we consider such an ion sheath in which the ion-neutral atom collisions occur at extremely rare intervals in the sheath. The interactions between ions and their parent gas atoms frequently happens with resonant charge exchange process,

$$A^+_{fast} + A_{slow} \rightarrow A_{fast} + A^+_{slow} \quad ,$$

The generated slow ions might be staying in the ion sheath for relatively long time compared with the fast ions. Though the ion sheath is in the category of above type I model, the sheath is composed with tow kinds of ion groups, that is, the fast ion group and the slow ion group (this fact had been confirmed experimentally). The mean velocity of ions is no longer allowed to be written by simple expression, $v = (2e\phi /M)^{1/2}$. We proposed a new model for the collision less sheath regime. New sheath model is based on the ion flows with having the tow-component velocity distribution of fast and slow ions. The fast ion group carries the ion current and the slow ion group forms the space charge layer. In this model, the space charge layer is discussed independently of the continuity equation of the current..

$$J = en_f v_f + en_s v_s = J_B$$

$$\varepsilon_o \frac{d^2\phi}{dx^2} = -e(n_f + n_s) \approx en_s$$

where J_B is Bohm current.

Assuming the formation of uniform space charge layer due to the slow ions group, the potential structure of ion sheath that is biased up to ϕ_s is derived as follows.

The slow ions is generated by the charge exchange collision between the fast ion and the neutral atoms,

$$\frac{dn_s}{dt} = \frac{n}{\tau_c} - \frac{n_s}{\tau_{ts}}$$

where $\tau_c \sim \lambda_i/\langle v_f \rangle$ and $\tau_{ts} \sim d_s/\langle v_s \rangle$ are charge exchange collision time of fast ions and slow ions' transit time across the sheath, respectively. If $dn_s/dt > 0$, the slow ions would accumulate up to required density in the sheath region. The density n_s of slow ions is estimated to be

. $$n_s \approx \frac{J_B}{ec_S} = n$$

because the acoustic speed c_S seems to be in the same order of average speed $\langle v_s \rangle$ of slow ions.

From one-dimensional Poisson's equation, the linearly varying field is easily derived,

$$E(x) \approx -\left(\frac{end_s}{\varepsilon_0}\right)\left(\frac{x}{d_s}\right) = -\left(\frac{\phi_s}{2d_s}\right)\left(\frac{x}{d_s}\right) \quad (1)$$

The potential structure becomes parabolic profile.

$$\phi(x) \approx -\phi_s\left(\frac{x}{d_s}\right)^2 \quad (2)$$

The current-voltage characteristics and sheath thickness are as follows.

$$J = 2\varepsilon_0\sqrt{\frac{KT_e}{M}}\left(\frac{\phi_s}{d_s^2}\right)$$
$$d_s = \sqrt{\frac{2e\phi_s}{KT}}\lambda_D \quad (3)$$

where T_e is the temperature of plasma electrons and λ_D is Debye length.

In the conventional collisionless sheath model, Child-Langmuir Equation is

$$J = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{M}}\frac{\phi_s^{\frac{3}{2}}}{d_s^2}$$

$$d_s = \frac{\sqrt{2}}{3}\left(\frac{2e\phi_s}{KT}\right)^{\frac{3}{4}}\lambda_D$$

The sheath thickness obtained from Child-Langmuir Equation is larger than that of equation (3). This means that the averaged space charge density ($J/\langle v_f\rangle$) in the former is dilute compared with that (J/c_s) of the later.

Linearly varying field in normal glow cathode fall region

We return to the main subject. From above discussion we can understand that the linear varying field in the cathode fall region caused by the formation of uniform space charge layer. In the normal glow mode, the magnitude of cathode fall and the discharge current density take the minimum values, that is, the plasma density is most dilute. Therefore the thickness of cathode sheath becomes the largest thickness $d_{cn} \sim \lambda^*$ where any ionization does not occurs ($d_{cn} < \lambda^*$ mean ionization length for electron impact ionization, in the stable ion sheath any ionization should never occurs). The type of normal glow cathode sheath seems to be the category of new sheath model that is mentioned above.

The magnitude V_{cn} of normal glow cathode fall is obtained from the plasma balance equations and the condition for self-sustaining discharge. Considering the simplest case, we can derive V_{cn}

$$\frac{dn_i}{dt}L \approx \frac{n_e v_e L}{\lambda *} - n_i v_i = 0$$

$$\frac{d\varepsilon}{dt}L \approx V_{cn}J_e - \frac{n_e v_e L}{\lambda *}(e\phi_i) = V_{cn}J_e - \phi_i J_i = 0 \qquad (4)$$

$$J_e = \gamma J_i, \qquad J = J_e + J_i = (\gamma + 1)J_i \approx j_i$$

where L is the electrode separation (or length of plasma column) and γ is the rate of γ process. Using these equations, V_{cn} is given by the simple expression

$$V_{cn} = \frac{\phi_i}{\gamma} \qquad (5)$$

The linearly varying field E is easily obtained as follows.

$$E = \left(\frac{V_{cn}}{2d_{cn}}\right)\left(\frac{x}{d_{cn}}\right) = \left(\frac{V_{cn}}{2\lambda^*}\right)\left(\frac{x}{\lambda^*}\right) \qquad d_{cn} \approx \lambda^* = \frac{\lambda_0^*}{p} \qquad (6)$$

The current density J_n in normal glow mode is written by

$$\frac{J_n}{p^2} = 2\varepsilon_0\sqrt{\frac{KT_e}{M}}\left(\frac{V_{cn}}{(d_{cn}p)^2}\right) = 2\varepsilon_0\sqrt{\frac{KT_e}{M}}\left(\frac{V_{cn}}{(\lambda_0^*)^2}\right) = \text{constant} \qquad (7)$$

The calculated values of V_{cn}, d_{cn} and J_n/p^2 are in generally good agreement with the experimental results. The appearance of the linearly varying field in normal glow

cathode fall would be explained clearly by the new sheath model.
For the cathode fall in abnormal glow mode, the conventional collisionless sheath model could be applied, because the sheath thickness becomes much smaller due to the increase of charge density and any ion-neutral atom collision does not occurs.
The more detailed discussion is described in Proc.ISPC-8 Tokyo 1987(*) and the unpublished document “Models of Cathode Sheath in Low-Pressure Glow” in STPIG web site(**)
(*) http://134.147.148.178/ispcdocs/ispc8/content/8/08-0273.pdf
(**) http://www.ne.jp/asahi/iupl/sunsan-tuee/doc/Models-CS.pdf