Models of Cathode Sheath in Low-Pressure Glows

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Abstract: A brief review of cathode sheath models and some defects of them are described. The conventional theories of the cathode sheath in the glow mode are based on collisional sheath in which the electron multiplication is essential process and hence they contain some principal problems to be solved, when we will derive the self-consistent solution. We propose a new concept for comprehensive treatment of the cathode sheath in the cold- and hot- cathode discharges. The new concept is based on the collisionless sheath model and plasma balance equations. In addition, tow-component velocity distribution of ions is introduced for the explanation of linearly varying field in the normal glow mode. The new concept makes it possible to clear the structure of cathode sheath in both the cold- and the hot- cathode discharges.
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1. Introduction

The cathode sheath, a plasma boundary layer between the cathode and the plasma, always occurs in the diode discharges, even if the cathode is cold and/or hot. The cathode sheath plays very important role to sustain the discharge plasma, because most part of the applied voltage across the electrodes appears in the cathode sheath. That explains why many experimental and theoretical studies on the cathode sheath, especially in the cold-cathode discharge, have been made continuously since 1920's [1].

The object of any theory is to obtain relation between the potential drop V_c , the current J and sheath thickness d_c under the conditions of known constant of the gas, pressure and cathode material (e.g. ionization cross-section of the gas and γ for the cathode). In normal glows, the characteristic values, V_{cn}, J_n and d_{cn} are in existence and the electric field in the cathode sheath decreases linearly with the distance from the cathode. These facts confirmed experimentally must be explained.

In this paper, a brief review of these theories is described and some questions about them are discussed. These theories usually need some assumptions to agree with the experimental results and they still involve some principal problems to be solved, when we will derive the self-consistent solution. We also discussed a new concept of the cathode sheath and comprehensive treatment including the externally heated hot-cathode discharge. The concept based on the plasma balance equations and the collisionless sheath model without electron multiplication. In addition, the motion of ions in the intermediate range between mobility limit and free fall limit is introduced for explanation of the characteristics of cathode sheath in the normal glow mode.

2. Brief review of theories

The theories of cathode sheath are classified into two groups, (1) the treatment as collisionless ion sheath in which any ionization does not occur and (2) the models based on collisional sheath in which the electron multiplication is essential process.

2.1 Collisionless sheath model

In some early theories, the cathode sheath was regarded as a collisionless ion sheath where the field is zero at the edge of the plasma (the source of ions). The relationship between V_c , J_i and d_c is given by Child-Langmuir Law,

$$\boldsymbol{J_i} = \frac{4\varepsilon_0}{9}\sqrt{\frac{2\boldsymbol{e}}{\boldsymbol{M}}}\frac{\boldsymbol{V_c}^{3/2}}{\boldsymbol{d_c}^{2}}, \qquad (1)$$

where J_i is the ion current ($J_i = J/(1+\gamma)$)
This equation was applied to the cathode sheath by Guntherschulze and Ryde, and agreed remarkably with the experimental results in the abnormal glow discharge. Moreover, it is possible to build up a simple picture of the cathode sheath of both cold- and hot- cathode discharges.

However, these theories were abandoned by the reason why the linear variation of the field, the ionization, and the collisions with molecules were not explained [1]. An essentially weak point of the theories would be such that V_c, J and d_c , especially these values in the normal glow mode, could not obtain independently.

2.2 Collisional sheath model

Most conventional theories in the cold-cathode discharge belong to this group. The collisional sheath theories have been developed after the pioneering study by Von Engel and Steenbeck [2]. These theories are based on the collisional sheath in which the electron multiplication,

$$M = \exp[\int \alpha \, dx], \quad \alpha/p = f(E/p) \qquad (2)$$

and the self-sustaining condition,

$$\gamma(M-1) = 1 \qquad (3)$$

are essential [1-4]. The situation in the cathode sheath is complicated and then it is very difficult to calculate $J = f(V_c, d_c)$.

These theories usually need some assumptions to agree with the experimental results and they still involve some principal problems to be solved, when we will derive the self-consistent solution [5-8]. One of the questions in the theories is the application of ionization coefficient $\alpha / p = f(E/p)$ to the cathode sheath. The expression of ionization coefficient as a function of E/p is valid only when $\Lambda >> d_c >> \lambda_e$, where Λ is the characteristic length of spatial variation of electric field and λ_e is electron mean free path. In the cathode sheath, the electric field varies strongly and then Λ is nearly equal to d_c [4,9,10]. Another defect in the theories is incompleteness of argument about stability of the cathode sheath. If equation (3) were used, the cathode sheath would be destroyed [10] (details are discussed in next section). Moreover, it is almost impossible to apply them to the cathode sheath in the hot-cathode discharges,

3. Necessity for stable cathode sheath

In order to exist the stable cathode sheath, it is necessary to form the positive space charge layer in the large part of cathode sheath in contact region with plasma. In addition, the cathode sheath must satisfy the condition for sustaining the discharge plasma.

In the case of planar sheath, the sheath potential $V(x)$ is required to satisfy the following inequality,

$$\frac{d^2V(x)}{dx^2} = -\frac{e}{\varepsilon_0}(n_i - n_e) \le 0 \qquad (4)$$

Now we consider the collisionless sheath model. At the sheath edge, the space charge effect due to primary electrons is very small compared with plasma electrons ($J_e/(eV_c/M)^{1/2} << en \exp[-e(V_p-V_c)/kT_e]$, where n, T_e and V_p are plasma density, electron temperature and plasma potential, respectively). Therefore, the Bohm sheath criterion can be applied to the formation of stable cathode sheath same as the usual ion sheath. The condition for sustaining discharge plasma is given by

$$J_e = \gamma J_i = \gamma J_B$$
$$J_B = \frac{en}{2}\sqrt{\frac{kT_e}{M}} \qquad (5)$$

where J_B is Bohm current. It means that the primary electrons must be accelerated sufficiently for generating the plasma of n and T_e. [10]. In the early collisionless model, equation (5) was not used.

On the other hand, the equation (1) in the case of collisional model is rewritten as follows,

$$\frac{d^2V(x)}{dx^2} = -\frac{J}{\varepsilon_0 v_i(x)}\left[1 - \left(1 + \frac{v_i(x)}{v_e(x)}\right)\left(\frac{\gamma}{1+\gamma}\right)\frac{J_e(x)}{J_e(0)}\right] \le 0 \qquad (4')$$

where $v_i(x)>0$ and $v_e(x)>0$ are the drift velocities of ions and electrons. This equation is derived by the assumption that no ions enter the cathode sheath from the plasma. The positive space charge layer appears in the region

$$\left.\begin{aligned} & 0 < x < d_c = \frac{1}{\overline{\alpha}} \ln\left(1+\frac{1}{\gamma}\right)\left(1+\frac{v_i(x)}{v_e(x)}\right)^{-1} \\ & \overline{\alpha} = \frac{1}{d_c}\int_0^{d_c} \alpha(x)dx \end{aligned}\right\} \qquad (6)$$

Usually the cathode sheath is treated as a gas diode with the electrode separation of d_{va}
which is given by the self-sustaining condition,

$$\overline{\alpha} d_{va} = \ln\left(1+\frac{1}{\gamma}\right) \qquad (7)$$

where the suffix va denotes the virtual anode (edge of plasma column). Difference between d_{va} and d_c is

$$d_{va} - d_c = \frac{1}{\overline{\alpha}} \ln\left(1+\frac{v_i(x)}{v_e(x)}\right)^{-1} > 0$$

This result shows that the negative space charge appears in the region from d_c to d_{va}. The excess electrons would be diffusing quickly and then the plasma would be appearing there. The equation (4) suggests the condition that the virtual anode is just moving toward the cathode and the cathode sheath is destroyed. Therefore, the stable cathode sheath could not exist if the self-sustaining condition using equations (2) and (3) are applied in this region. It seems that any electron multiplication could not occur in the stable cathode sheath. [10]

4. New concept of cathode sheath

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4.1 *Modified collisionless sheath model*

In order to correct the defect of early collisionless sheath theory, a new concept of the cathode sheath formation was proposed [9,10]. The concept consists of collisionless sheath model and the plasma balance equations. If there is no field in the plasma column (no additional heating σE^2 of plasma electrons), the plasma is kept steady-state (n, T_e = *cosnt*) by the energy input of primary electrons accelerated up to V_c. In the plane geometry (see Fig.1), the plasma balance equations are easily expressed as follows,,

$$R_i = \left(\frac{\Gamma_i}{L}\right)\left(1-\frac{R_r}{R_i}\right)^{-1} = R_{loss} \qquad (8)$$

$$e\Gamma_{el}V_c\left(1-\frac{\Gamma'_{el}}{\Gamma_{el}}\right) = e\phi_i R_i\left(1+\frac{R_\varepsilon \varepsilon}{R_i e\phi_i}\right)L \qquad (9)$$

where R_i and R_r are the rates of ionization and recombination, and $R_\varepsilon\varepsilon$ is the rate associated with energy losses except energy loss $R_i(e\phi_i)$ for ionization (e.g. excitation,. heating of plasma electrons). The flux Γ_{el}' is the flux of primary electrons passing through the plasma column without any energy loss

Using equations.(8) and (9), the sustaining condition of discharge plasma can be written by

$$\frac{\Gamma_{el}}{\Gamma_i} = \gamma = \frac{\phi_i}{V_c} \frac{\left(1 + \frac{R_\varepsilon \varepsilon}{R_i e \phi_i}\right)}{\left(1 - \frac{R_r}{R_i}\right)\left(1 - \frac{\Gamma'_{el}}{\Gamma_{el}}\right)}$$

and then the potential drop V_c is obtained as follows,

$$\left.\begin{aligned} V_c &= \eta\left(\frac{\phi_i}{\gamma}\right) F(n_{et}) \\ \eta &= \left(\frac{1 + \frac{R_\varepsilon \varepsilon}{R_i e \phi_i}}{1 - \frac{R_r}{R_i}}\right) \geq 1 \\ F(n_{et}) &= \frac{1}{1 - \frac{\Gamma'_{el}}{\Gamma_{el}}} \geq 1 \end{aligned}\right\} \qquad (10)$$

The potential drop of cathode sheath is determined by (ϕ_i/γ) and other discharge conditions (η and $F(n_{et})$). The factor η depends on energy loss rate and particle loss rate in the plasma. If η as a function of the current is known ($R_r = \alpha_r n$ would be most effective in dense glow mode), we could obtain V_c-J characteristics. However, minimum current density in the normal glow mode is not determined from equation (10). The thickness d_c is calculated from Child-Langmuir Law (equation (1)). Here, $F(n_{et})$ means the reciprocal of trapping factor of primary electrons and strongly depends on the pressure ($F(n_{et}) >> 1$ for a obstructed mode at very low pressure). Even if the pressure is very low, the low impedance discharge could be realized by sufficient trapping ($F(n_{et}) \sim 1$) of primary electrons by using a magnetic field (Magnetron discharge [11]) or an electrostatic field (Hollow cathode discharge [12]). This explanation can be applied to a dense glow mode or the abnormal glow mode.

4.2 *Characteristics of normal glow mode*

In the normal glow mode, V_c takes a minimum value V_{cn} and d_c takes a maximum value d_{cn}. If the ionization occurs most efficiently ($\eta \sim 1$ and $F(n_{et}) \sim 1$), V_{cn} is given by

$$V_{cn} \approx \frac{\phi_i}{\gamma}. \qquad (11)$$

This indicates that the energy gain of a primary electron is required to be $e\phi_i/\gamma$ for generating $1/\gamma$ particles of ions, which release one electron from the cathode. Aassuming that any ionization does not occur in the sheath ($d_c \lesssim 1/Q_i$), d_{cn} is determined as

$$d_{cn} . p \approx \frac{p}{Q_i(V_{cn})} = \frac{1}{Q_{i0}(V_{cn})} \qquad (12)$$

where $1/Q_i$ is the mean ionization length of primary electrons. Equations (11) and (12) seem to agree with the experimental results (see figures 2 and 3)
The minimum current density J_n is calculated from Child-Langmiur Law using V_{cn} and d_{cn}.

$$\frac{J_n}{p^2} \approx \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{M}}\frac{V_{cn}^{3/2}}{(d_{cn}p)^2}(1+\gamma), \qquad (13)$$

However, J_n/p^2 is about ten times as large as the experimental results and the linear variation of the field is not explained.

In minimum current density, d_c becomes largest value and then some ions collide with the molecules. The scattered ions drift very slowly and accumulate in the sheath. The slow ions make a uniform space-charge layer. On the other hand, the fast ions carry the ion current.

We observed the motion of ions accelerated by uniform field in the intermediate range between the mobility limit and free fall limit and the deformation of ion energy distribution function by ion-neutral collisions. The typical results are shown in figure 4. The intensity of beam component decreases exponentially with number of ion-neutral collisions and the scattered ions form the group of slow ions. The mean velocity v_{is} of slow ions would be given by the mobility concept, $v_{is}=\mu E$, because of their large cross section compared with the fast ions. In the case of weak ion-neutral collisions d_c/λ_ι in the cathode sheath, the space charge density $n_i(x)$ is would be determined by the slow ions as follows,

$$n_i(x) = \frac{J_B}{ev_{is}(x)}\left(1-\exp\left[-\frac{d_{cn}}{\lambda_i}\left(1-\frac{x}{d_{cn}}\right)\right]\right) + \frac{J_B}{ev_{if}(x)}\exp\left[-\frac{d_{cn}}{\lambda_i}\left(1-\frac{x}{d_{cn}}\right)\right]$$
$$\sim \frac{1}{2}n\frac{\sqrt{kT_e/M}}{v_{is}(x)}\left\{\frac{d_{cn}}{\lambda_i}\left(1-\frac{x}{d_{cn}}\right)\right\} \qquad (14)$$

Solving the Poisson's equation, the field $E(x)$ is written by .

$$E(x) \approx \left(\frac{n\sqrt{kT_e/M}}{2\varepsilon_0\mu}\frac{d_{cn}^2}{\lambda_i}\right)^{\frac{1}{2}}\left(1-\frac{x}{d_{cn}}\right) = E(0)\left(1-\frac{x}{d_{cn}}\right) \qquad (15)$$

Equation (15) shows the linearly decaying field with x/d_{cn} is established and the space charge density becomes uniform. The space charge density is up to $n/2$, when $v_{is}(0) = (d_{cn}/\lambda_i).(kT_e/M)^{1/2.}$ The potential structure in the cathode sheath is determined by the space charge effect due to the slow ions and the current is carried by the fast ions. Therefore, J_n is determined as follows,

$$\frac{J_n}{p^2} \approx 2\varepsilon_0\sqrt{\frac{kT_e}{M}}\frac{V_{cn}}{(d_{cn}p)^2}(1+\gamma) = \frac{9}{2}\sqrt{\frac{kT_e}{2eV_{cn}}} \times \text{equation (13)} \qquad (16)$$

Figure 5 shows the comparison between the observed values of J_n/p^2 and the calculated values from equation (16)

Considering with the two-component velocity distribution of ions moving across the cathode sheath,

the characteristics in the normal glow mode are theoretically explained. The calculated values of V_{cn}. $d_{cn}\, p$ and $J_n\, /p^2$ are in generally good agreement with the experimental results as shown figures 2, 3 and 5.
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4.3 *Cathode sheath in externally heated hot-cathode discharge*

The sheath structure of hot-cathode discharge is obtained by introducing $V_c \gtrsim \phi_I$, the condition of double sheath formation $J_e/J_i = \gamma = (M/m)^{1/2}$ ($E(0)$~0, $E(d_c)$~0, Langmuir mode) and the equation of bipolar space-charge conduction [2.16,17] From $V_c \gtrsim \phi_I$, $F(n_{et}) \sim (M/m)^{1/2}$ is derived. This means that a large amount of primary electrons make it possible to sustain the plasma in extremely low pressure and to carry the discharge current across the electrodes. The role of ions is only the charge neutralization of the primary electron beam.

5. Conclusion

Most of theories of cathode sheath have some defects. The defects of the collisionless sheath model are corrected by supplement with the plasma balance equations and two-component velocity distribution of ions. The new theory explains clearly the structure of cathode sheath in both cold- and hot-cathode discharges.. Considering with the trapping of primary electrons, it is possible to clear the enhanced ionization in magnetron discharges, hollow cathode discharges, RF discharges.
The stable cathode sheath could not exist if the self-sustaining condition $\gamma(M$-!)=1 is applied to this region. The cathode sheath seems to be the region where the mechanism of the electron acceleration is organized without ionization and the virtual anode (the front of plasma column) does not move toward the cathode after starting the breakdown.

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Figure Caption

Figure 1. Potential profile in cold- and hot- cathode discharge, particle fluxes and plasma parameters

Figure 2. Potential drop V_{cin} of cathode sheath in normal glow mode. (The experimental results in various gases and cathode materials [1]. The lines show the theoretical values for $\gamma = 0.1$ and 0.15.)

Figure 3. Cathode sheath thickness $d_{cn}.p$ in normal glow mode [1] and mean ionization length $1/Q_{i0}$ calculated from reference [13]. Theoretically $d_{cn}\, p$ is nearly equal to $1/Q_{i0}$. as shown equation (12)

Figure 4. Deformation of energy distribution function of ions by ion-neutral collisions. (The ions move under the uniform acceleration. The distribution functions are normalize to be $\int_0^\infty (dn/d\varepsilon)d\varepsilon = const.$)

Figure 5. Current density J_n/p^2 in normal glow mode.. (Calculated values from equation (16) using observed values of d_{cn} and V_{cn} , and kT_e = 1eV)

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Figure 1

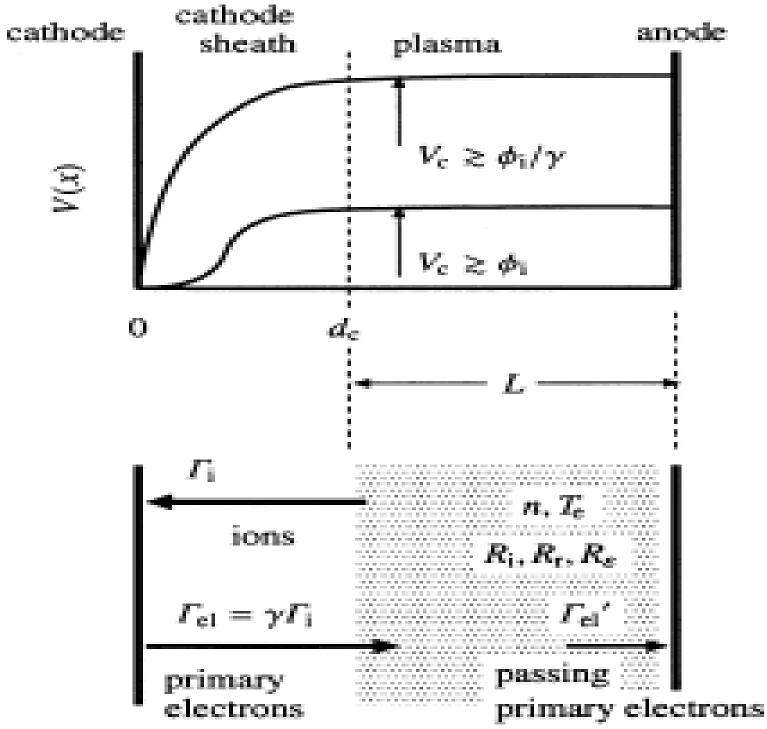


Fig.1 Potential profiles in cold- and hot-cathode discharges, particle fluxes and parameters of plasma

Figure 2

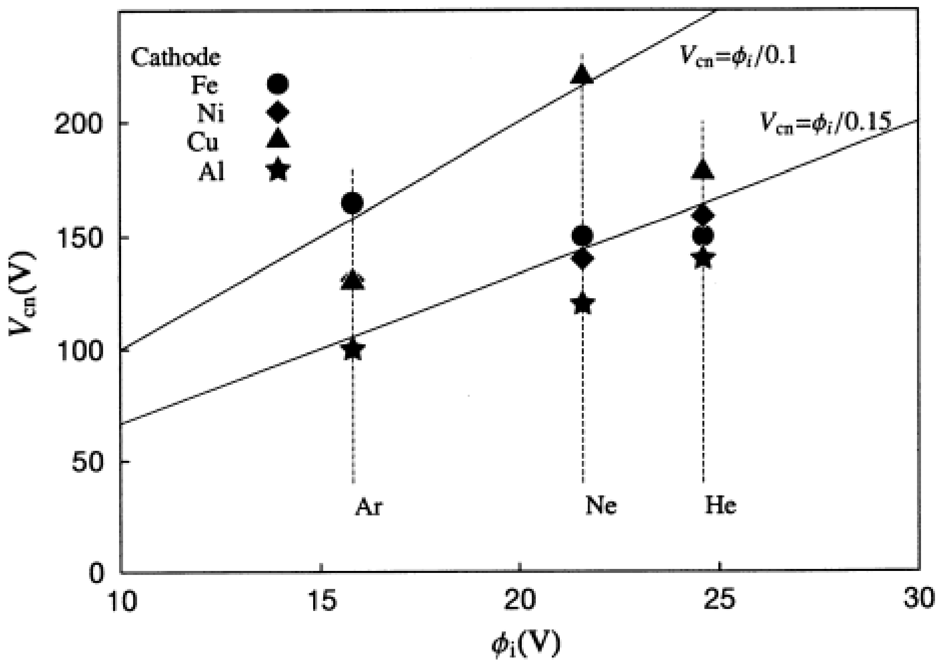

Cathode
Fe
Ni
Cu
Al
$V_{cn}=\phi_i/0.1$
$V_{cn}=\phi_i/0.15$
Ar
Ne
He
0
50
100
150
200
10
15
20
25
30
V_{cn}(V)
ϕ_i(V)

Figure 3

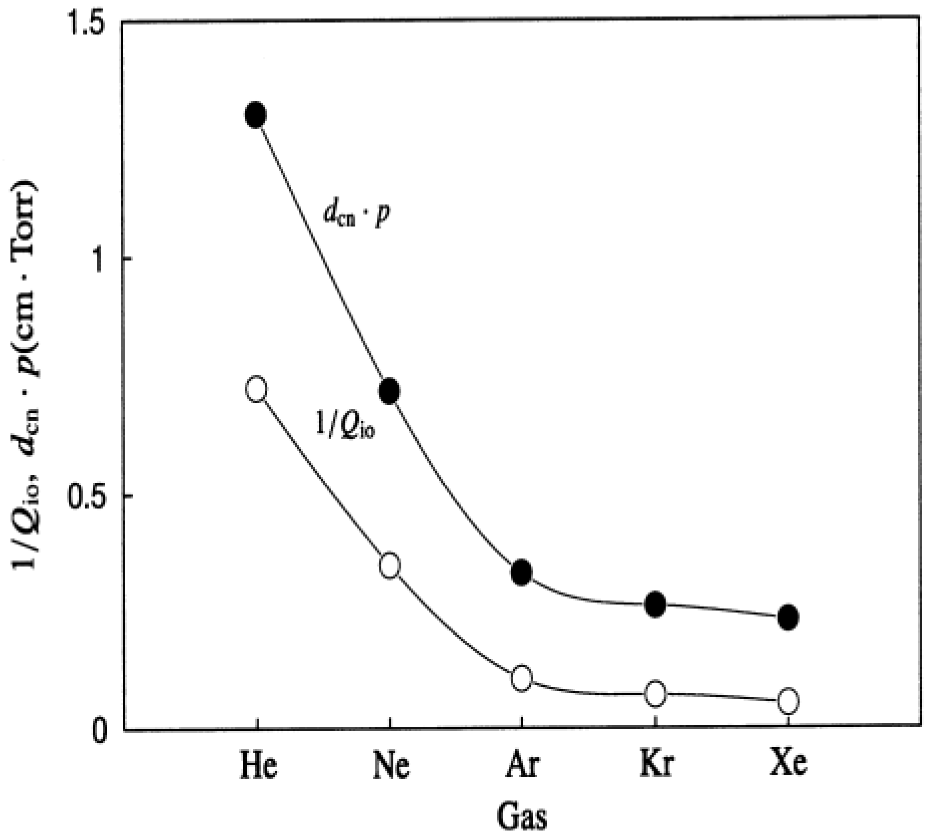


Figure 4

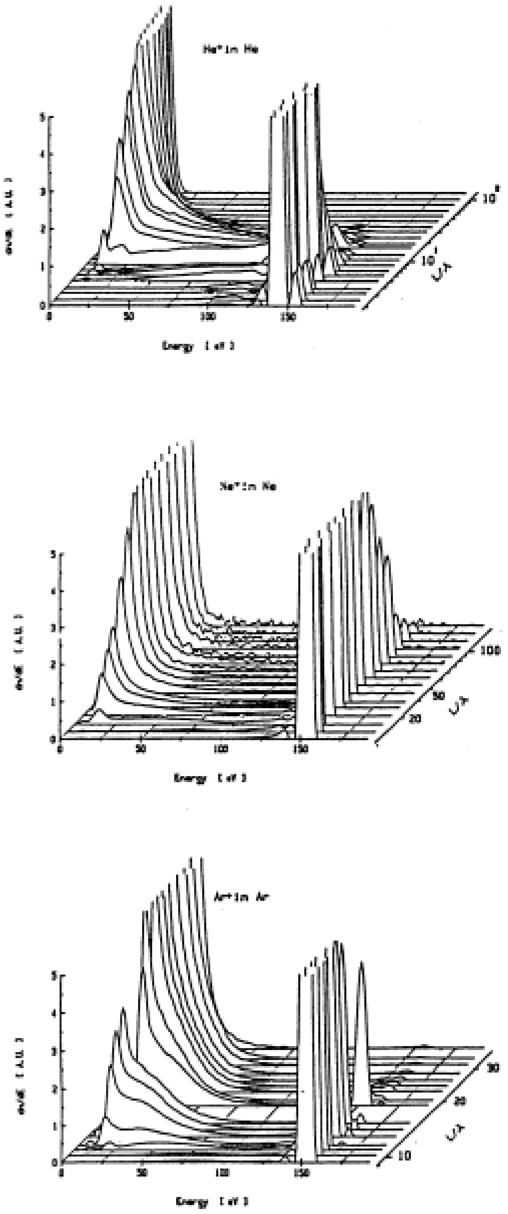
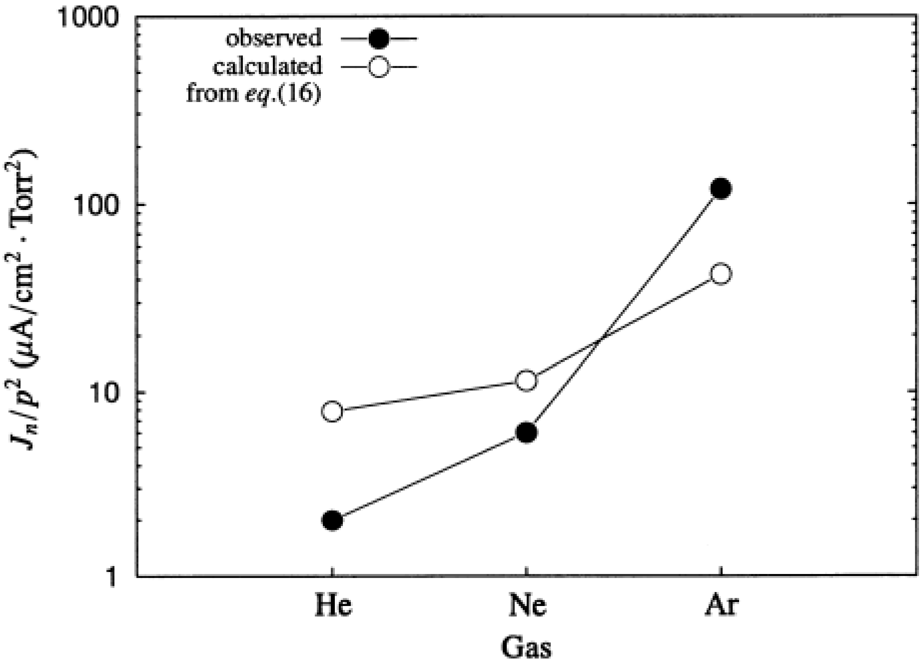


Figure 5



1000
100
10
1
observed
calculated
from *eq*.(16)
J_n/p^2 (μA/cm$^2\cdot$Torr^2)
He
Ne
Ar
Gas