

## CHAPTER 1

### MATHEMATICAL FORM OF THE SIMPLE MODEL

#### 1.1 Introduction.

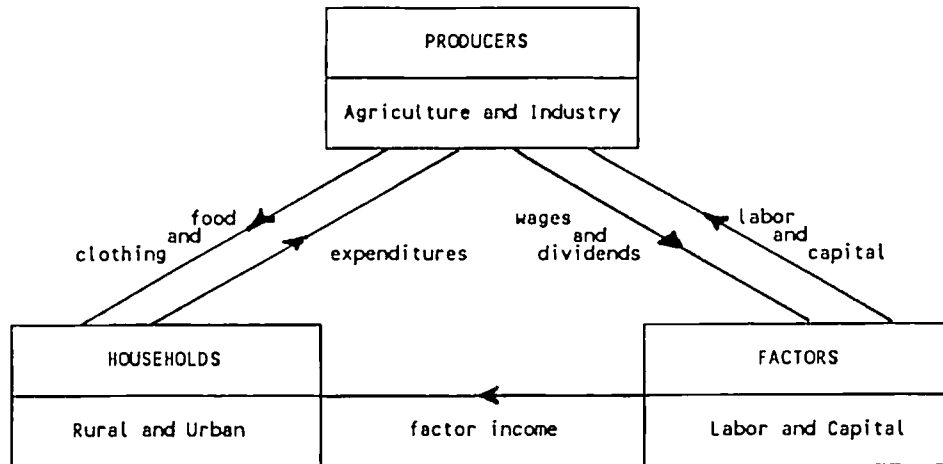
This book describes HERCULES. HERCULES is both an approach to organizing and building economy-wide models and a supporting software system, and the book describes both aspects. The HERCULES approach is quite different from the standard mathematically oriented approach to model building and the first three chapters describe the relationships between the two. We do this in stages. In chapter 1 we develop a small simple model with 2 production sectors, 2 factors, and 2 households. The model is based on standard economic assumptions and written using standard mathematical notation. In Chapter 2 we show how this model can be organized into a matrix form. We discuss Social Accounting Matrices (SAM) and show that all payment variables constitute a SAM. Furthermore, we show how the equations can be organized into a Specification Matrix, a matrix with the same structure as the SAM. This organization of the variables and equations into a SAM and a Specification Matrix forms the conceptual basis for the HERCULES approach. Chapter 3 covers the last step from the conceptual basis to a practical computer implementation of the model.

#### 1.2 The Economy.

Figure 1.1 illustrates the economy of the simple model used in this chapter. The economy consists of two producers, agriculture and industry, which produce food and clothing respectively, using labor and capital inputs. The labor and capital inputs are supplied by two sets of households, rural and urban. All output is consumed by the households, which use their wage and dividend income to pay their consumption expenditures.

An economy-wide model like this is normally described by a set of nonlinear equations that simultaneously determine prices, quantities, and money flows. The number of equations can be quite large so it is useful to have an organizing principle when developing the model. We will distinguish between two types of equations: (1) sets of equations or submodels that describe economic agents such as producers, factors, and households, and (2) linkage equations that describe the connections between the agents through, e.g., material balances or income-expenditure balances.

Figure 1.1  
The Flow of Money, Goods, and Factors



The following subsections will develop the submodels for the agents, and will then describe the linkage equations. First, however, we describe our conventions for variables.

### 1.3 Local and Global Variables.

It is useful to distinguish between variables that are used in several submodels (global variables) and variables that are used primarily in one submodel (local variables). Prices of goods and factors will in general appear in the submodels of all users of the good or factor and are therefore global variables. In the current model the following are global variables:

- $p_s$  = price of output in sector  $s$ ,
- $p_f$  = price of factor  $f$ , e.g., wage rate for labor and rental rate for capital, and
- $p_h$  = consumer price index for household  $h$ .

We have included the consumer price indices to be systematic: all prices are global variables.

The sector index  $s$  is used to denote discrete categories,  $a$  (agriculture) and  $i$  (industry). Similarly,  $f$  can take the values  $l$  (labor) and  $c$  (capital), and  $h$  can take the values  $r$  (rural) and  $u$  (urban). There is thus no overlap between the possible values of

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s, f, and h and therefore no overlap between the three price subscripts. Although we have assumed that there are two sectors, two factors and two household groups the model development will still be valid for S sectors, F factors, and H households, and we will use S, F, and H when we count variables and equations.

All physical and monetary flows will usually relate to two submodels, a supply submodel and a demand submodel. The supply submodel describes total supply without explicitly considering the sources of supply and how they behave: similarly, in the demand submodel one need not consider the sources explicitly. This restricted view in the submodels makes it possible to develop each submodel independently of the others, and physical flows and the corresponding monetary flows can be considered local variables. The linkage equations defined in section 1.7 will ensure that total supply is equal to total demand, and they will do so without matching individual suppliers with individual demanders.

#### 1.4 Producers.

The equations for a producer relate prices, quantities and total revenue of the goods produced to the prices, quantities and total costs of inputs, and therefore total cost. It is customary to break the description of the producer into two parts: a description of the technology, i.e., the technically feasible combinations of inputs and outputs, and a description of the rules used to select one of these feasible combinations.

To make the mathematical formulae easier it is assumed here that the feasible combinations of inputs and outputs for each producer are represented by Cobb-Douglas production functions of the form

$$(1q) \quad q_s = b_s \prod_f c_{fs}^{a_{fs}} \quad \text{all } s$$

[output] [factor inputs]

where

- $q_s$  = quantity of output in sectors,
- $c_{fs}$  = quantity of factor input f into sector s,
- $a_{fs}$  = share parameter for factor f in sector s, satisfying  $\sum_f a_{fs} = 1$ ,
- $b_s$  = scale parameter, and
- $\prod_s$  indicates the product over all sectors s.

A note on the numbering scheme for equations is appropriate here. All equations defining quantities have a letter q appended, equations defining payments

$$\begin{aligned} \text{Max, Prod. t.} & \quad p_s q_s - \sum_f p_f c_{fs} \\ \frac{\partial \text{Prod. t.}}{\partial c_{fs}} & \quad p_s b_s a_{fs} c_{fs}^{a_{fs}-1} \prod_{f \neq f} c_{fs}^{a_{fs}} - p_f = 0 \\ & \quad = p_s a_{fs} q_s / c_{fs} - p_f = 0 \quad (2.9) \end{aligned}$$

(or derived from ones defining payments) have a letter y-appended and definitional equations relating payments to quantities and prices have a letter d-appended. Equations that describe the same part of the economy have the same number. Eq. (1q) is, for example, the production quantity equation, Eq. (1y) is derived from the corresponding payments equations, and Eq. (1d) relates income for the producer to price and quantity sold. A similar trichotomy into prices (p), quantities (q and c), and payments (y and t) is also applied to the variables of the model.

It is assumed that the producer chooses the quantities of inputs and outputs in order to maximize profits under the assumption of perfect competition, i.e., under the assumption that the producer cannot influence prices. This means, that although prices are endogenous in the overall model, they are considered exogenous by the producer. Maximization of the profit function subject to the production function constraint yields a set of first order optimality conditions that define demands for factor inputs, viz. [See Dixon, Bowles, and Kendrick (1980), pp. 235-236]:

$$(2q) \quad c_{fs} = a_{fs} \frac{p_s q_s}{p_f} \quad \text{all } f,s$$

[factor demand]    [share parameter]    [output price]    [output quantity]    [factor price]

Thus the demand for each factor varies directly with the price and quantity of the commodity being produced and inversely with the price of the factor.

Equations (1q), and (2q), define quantities. The corresponding payments can be determined from the following identities:

$$(1d) \quad y_s = p_s q_s \quad \text{all } s$$

[revenue]    [price times quantity]

$$(2d) \quad t_{fs} = p_f c_{fs} \quad \text{all } f,s$$

[revenue]    [price times quantity]

where

$$y_s = \text{total revenue in sector } s,$$

$$t_{fs} = \text{factor payment from sector } s \text{ to factor } f.$$

Equations (1q), (2q) (1d) and (2d) define one version of the producer submodel that we will refer to as the quantity submodel because the key relationships, (1q) and (2q), define quantities. As we will see in the next section, it is often more convenient to define the key relationships in terms of payments, so we will derive a payment submodel as well. The factor demand equation (2q) combined with (2d) and (1d) gives the following factor payments equation

(2y)

$$t_{fs} = a_{fs} \frac{y_s}{y_s}$$

all  $f, s$

The payments version of the production function (1q) is

$$y_s = p_s \prod_f (t_{fs} / p_f)^{a_{fs}}$$

all  $s$

that by introduction of (2y) can be reduced to

(1y)

$$p_s = \beta_s \prod_f p_f^{a_{fs}}$$

all  $s$

where

$$\beta_s = \prod_f a_{fs}^{-a_{fs}} / b_s$$

all  $s$

So the "payment version" of the production function turns out to be equivalent with the so called "cost equation" that determines the price of the output from the prices of the inputs.

Table 1.1  
Equations in the producer submodels

| Name      | quantity <sup>q</sup>               | Payment <sup>y</sup>                 | Definition <sup>d</sup> |
|-----------|-------------------------------------|--------------------------------------|-------------------------|
| 1. Output | $q_s = b_s \prod_f c_{fs}^{a_{fs}}$ | $p_s = \beta_s \prod_f p_f^{a_{fs}}$ | $y_s = p_s q_s$         |
| 2. Input  | $c_{fs} = a_{fs} q_s p_s / p_f$     | $t_{fs} = a_{fs} y_s$                | $t_{fs} = p_f c_{fs}$   |

The equations for the producer submodel are summarized in Table 1.1, and the variables are shown in Table 1.2. There are  $3(S+FS)$  equations ( $S$  = number of sectors,  $F$  = number of factors) and  $2(S+FS)$  variables, but as already noted, the equations are not independent. Assuming the definitional equations are always included, then we can choose either a  $q$ - or a  $y$ -equation, but not both. We will usually choose all  $y$ -equations to get a payment submodel, but other submodels with a mixture of  $q$ - and  $y$ -equations are also valid. Note that although the submodels may look very different, they are mathematically equivalent. On the other hand, the ease with which the overall model can be solved may depend on which submodel formulation is chosen.

Table 1.2  
Variables in the producer submodel

| Name      | Quantity<br>q and c | Payment<br>y and t |
|-----------|---------------------|--------------------|
| 1. Output | $q_s$               | $y_s$              |
| 2. Inputs | $c_{fs}$            | $t_{fs}$           |

### 1.5 Factors of Production.

The submodel for a factor of production relate total factor income to the quantity of supply and the price of the factor, and explains the distribution of factor income to households.

Our model will assume that the total amount of each factor,  $q_f$ , is fixed and that the price of the factor,  $p_f$ , will adjust to reach full employment. Implicit in the assumption of a single price per factor is an assumption that each factor is homogenous and perfectly mobile between sectors.

We will also assume that each household group gets a share of each type of factor income proportional to the household's ownership of the factor.

The definitional equations that describe the relationships between factor variables are

$$(3d) \quad y_f = p_f q_f \quad \text{all } f$$

[factor income] [factor prices times total factor input]

where

$$y_f = \text{total income of factor } f.$$

The payment equations describing the distribution of income are

$$(4y) \quad t_{hf} = a_{hf} y_f \quad \text{all } f, h$$

[factor income paid to households] [share of total factor income]

where

$t_{hf}$  = payment from factor  $f$  to household  $h$ , and

$a_{hf}$  = share of factor  $f$  owned by household  $h$ .

The equations for the factor submodel are summarized in Table 1.3 and the variables are shown in Table 1.4.

Table 1.3  
Equations for the factor submodel

| Name        | Quantity<br>$q$ | Payment<br>$y$        | Definition<br>$d$ |
|-------------|-----------------|-----------------------|-------------------|
| 3. Income   |                 |                       | $y_f = p_f q_f$   |
| 4. Transfer |                 | $t_{hf} = a_{hf} y_f$ |                   |

Table 1.4  
Variable for the factor submodel

| Name        | Quantity<br>$q$ and $c$ | Payment<br>$y$ and $t$ |
|-------------|-------------------------|------------------------|
| 3. Income   | $q_f$                   | $y_f$                  |
| 4. Transfer |                         | $t_{hf}$               |

As Table 1.4 shows, there are  $2F + HF$  local variables associated with factors: quantities, incomes and payment to household. Also, as Table 1.3 shows, there are  $F + HF$  equations associated with factors: income and transfer. So there are  $F$  degrees of freedom in the factor submodel. We will see that these degrees of freedom are absorbed in the complete model by making  $q_f$  exogenous for all  $f$ .

## 1.6 Households.

The equations for households describe their consumption pattern and define consumer price indices and real consumption. It is customary to assume that households consume so as to maximize their utility, subject to their budget constraint, i.e.,

$$\begin{aligned} \max U_h(c_{sh}) \\ \text{s.t. } \sum_s P_s c_{sh} \leq y_h \end{aligned}$$

where

$$\begin{aligned} U_h(.) &= \text{utility function for household } h, \\ c_{sh} &= \text{quantity of good } s \text{ consumed by household } h \\ y_h &= \text{income for household } h. \end{aligned}$$

The first order optimality conditions for this optimization problem give the consumption system that defines consumption as a function of income and prices.

In our particular model we will assume that the utility function is

$$U_h(.) = \sum_s a_{sh} \log(c_{sh})$$

where

$$a_{sh} = \text{weight of good } s. \text{ We assume for convenience that } \sum_s a_{sh} = 1 \text{ for all } h$$

$$U = \prod_s c_{sh}^{a_{sh}}$$

or,  $\log U = \sum_s a_{sh} \log c_{sh}$

The consumption system derived from this utility function is

$$(5q) \quad c_{sh} = a_{sh} y_h / P_s \quad \text{all } s, h$$

The quantity consumed of each commodity varies directly with income and inversely with the price of the commodity. The definitional equation relating price and quantity to expenditure is

$$(5d) \quad \frac{t_{sh}}{\text{[expenditure]}} = \frac{P_s}{\text{[price times quantity consumed]}} \frac{c_{sh}}{\text{[quantity consumed]}} \quad \text{all } s, h$$

where

$$t_{sh} = \text{expenditure on good } s \text{ by household } h.$$

The consumption system can be derived in payments instead of quantities by combining (5q) and (5d), in which case we get

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(5y) 
$$t_{sh} = a_{sh} y_h \quad c_{sh} = \lambda \frac{a_{sh}}{p_s} \quad \text{all } s, h$$

The expenditure on each good is a fixed share of total income. Since the shares add to one, all income is used. A consumer price index and a real consumption equation are also associated with a utility function and the corresponding consumption system. For our utility function these equations are:

(6y) 
$$p_h = \sum_s p_s a_{sh} \quad \text{all } h$$

(6q) 
$$q_h = \alpha_h \sum_s c_{sh} a_{sh} \quad \text{all } h$$

where.

$q_h$  = real consumption for household h, and  
 $\alpha_h = \sum_s a_{sh}^{-a_{sh}}$

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$p_h$  and  $q_h$  satisfy the standard definitional equation

(6d) 
$$y_h = p_h q_h \quad \text{all } h$$

The equations of the household submodel are summarized in Table 1.5 and the variables are shown in Table 1.6.

Table 1.5  
Equations in the household submodel

| Name       | Quantity 'q                           | Payment y                 | Definition d          |
|------------|---------------------------------------|---------------------------|-----------------------|
| 5. Consume | $c_{sh} = a_{sh} y_h / p_s$           | $t_{sh} = a_{sh} y_h$     | $t_{sh} = p_s c_{sh}$ |
| 6. CPI     | $q_h = \alpha_h \sum_s c_{sh} a_{sh}$ | $p_h = \sum_s p_s a_{sh}$ | $y_h = p_h q_h$       |

There are 3(SH+H) equations. Given (5d), however, Eq (5q) and (5y) are not independent and only one of the two sets can be included in the model. The use of Eq. (5q) provides a quantity submodel, and the use of Eq. (6y) provides a payments submodel. Also, given (6d), Eqs (6q) and (6y) are not independent and only one of

**Table 1.6**  
**Variables in the household submodel**

| Name       | Quantity<br>q and c | Payment<br>y and t |
|------------|---------------------|--------------------|
| 5. Consume | $c_{sh}$            | $t_{sh}$           |
| 6. CPI     | $q_h$               | $y_h$              |

them should be included. There are therefore a total of  $2(SH+H)$  independent equations in this submodel. As Table 1.6 shows, there are  $2(SH+H)$  local variables, so the household submodel provides no degrees of freedom.

### 1.7 Linkages or Balance Equations.

All of the boxes in Fig. 1.1 have now been described independently. The index sets of quantity and income variables used in one box are different from the index sets used in other boxes, even though the variables have the same names,  $y$ ,  $q$ ,  $t$ , and  $c$ . With this notation the producer submodel is independent of the household submodel and vice versa. We will now relate the boxes by defining the equations that characterize the linkages. In this model there are three groups of linkage equations corresponding to each of the three groups of agents.

For the producers, total supply of each commodity must be equal to total demand:

$$(7q) \quad q_s = \sum_h c_{sh} \quad \text{all } s$$

[total production quantity] [sum over households: consumption]

This equation is relatively simple in this model since there is only one category of demander (households), but the same type of material balance equation exists in models with many demanders. Eq. (7q) is the quantity linkage equation corresponding to the arrow from producers to households shown in Figure 1.1. The payments linkage equation that corresponds to the arrow from households to producers in Figure 1.1 states that for each good the total receipts of the suppliers must be equal to the total expenditures of the demanders:

$$(7y) \quad y_s = \sum_h t_{sh} \quad \text{all } s$$

[total producer revenue] [sum over households: expenditure]

Given the definitions in Eqs. (1d) and (5d), i.e.

$$(1d) \quad y_s = P_s Q_s$$

$$(5d) \quad t_{sh} = P_s c_{sh}$$

Eqs. (7q) and (7y) are dependent, so only one of them can be included in the overall model. The producer-household linkages will therefore add one equation, a quantity or a payments equation, for each sector or commodity.

The quantity equations for factors are similar: total supply of each factor must be equal to total demand:

$$(8q) \quad q_f = \sum_s c_{fs} \quad \text{all } f$$

[total factor quantity] [sum over factors: inputs in each sector]

The total income of each factor must also be equal to the sum of the factor payments:

$$(8y) \quad y_f = \sum_s t_{fs} \quad \text{all } f$$

[factor income] [sum over factors: cost in each sector]

Again, Eqs.(8q) and (8y) are dependent, so only one can be included in the final model for each factor. The production-factor linkages will therefore add one equation for each factor of production.

The last linkage equation relates factor payments to household incomes. It corresponds to the arrow from factors to households in Fig. 1.1. There is no corresponding physical flow so there is only a monetary version of the equation

$$(9y) \quad y_h = \sum_f t_{hf} \quad \text{all } h$$

[household income] [sum over factors: factor transfers]

and there is therefore one linkage equation for each household.

There are only linkage equations for quantities and payments where the linkage equations correspond to material balance equations. There are no linkage equations for prices.

**Table 1.7**  
**Linkage equations**

| Name          | Quantity<br>q         | Payment<br>y          | Definition<br>d |
|---------------|-----------------------|-----------------------|-----------------|
| 7. Producers  | $q_s = \sum_h c_{sh}$ | $y_s = \sum_h t_{sh}$ |                 |
| 8. Factors    | $q_f = \sum_s c_{fs}$ | $y_f = \sum_s t_{fs}$ |                 |
| 9. Households |                       | $y_h = \sum_f t_{hf}$ |                 |

The linkage equations are summarized in Table 1.7. Because of dependence the model may include either the quantity or payments equation of (7), (8), and (9). Since there are S equations of type (7), F of type 8 and H of type 9 this provides S+F+H equations.

**Table 1.8**  
**Global variables**

| Name                   | Price<br>P |
|------------------------|------------|
| Prices of good         | $P_s$      |
| Factor prices          | $P_f$      |
| Consumer price indices | $P_h$      |

By way of summary the global variables are displayed in Table 1.8. There are also S+F+H global variables, so the number of linkage equations balances with the number of global variables and this part of the model provides no degrees of freedom.

## 1.8 The Overall Model

The equations of the complete model are given in Table 1.9 and Table 1.10 lists the variables in the complete model.

**Table 1.9**  
**Equations of the complete model**

| Name              | Quantity<br>$q$                         | Payment<br>$y$                    | Definition<br>$d$     |
|-------------------|---|-----------------------------------|-----------------------|
| <u>Production</u> |   |                                   |                       |
| 1. Output         | $q_s = b_s \sum_f c_{fs}^{a_{fs}}$      | $p_s = \beta_s \sum_f p_f a_{fs}$ | $y_s = p_s q_s$       |
| 2. Inputs         | $c_{fs} = a_{fs} q_s p_s / p_f$         | $t_{fs} = a_{fs} y_s$             | $t_{fs} = p_f c_{fs}$ |
| <u>Factors</u>    |   |                                   |                       |
| 3. Income         |   |                                   | $y_f = p_f q_f$       |
| 4. Transfer       |   | $t_{hf} = a_{hf} y_f$             |                       |
| <u>Households</u> |   |                                   |                       |
| 5. Consume        | $c_{sh} = a_{sh} y_h / p_s$             | $t_{sh} = a_{sh} y_h$             | $t_{sh} = p_s c_{sh}$ |
| 6. CPI            | $q_h = \alpha_h \sum_s c_{sh}^{a_{sh}}$ | $p_h = \sum_s p_s^{a_{sh}}$       | $y_h = p_h q_h$       |
| <u>Linkage</u>    |   |                                   |                       |
| 7. Producers      | $q_s = \sum_h c_{sh}$                   | $y_s = \sum_h t_{sh}$             |                       |
| 8. Factors        | $q_f = \sum_s c_{fs}$                   | $y_f = \sum_s t_{fs}$             |                       |
| 9. Households     |   | $y_h = \sum_f t_{hf}$             |                       |

To summarize, the production and household submodels are balanced and the factor submodels have one degree of freedom per factor. The number of global variables also balances with the number of linkage equations. Therefore the overall model has  $F$  degrees of freedom as shown below

|  | Variables                   | Equations                   | Degrees of Freedom |
|--|-----------------------------|-----------------------------|--------------------|
| Production                               | $2S+2FS$                    | $2S+2FS$                    | 0                  |
| Factors                                  | $2F+HF$                     | $F+HF$                      | F                  |
| Households                               | $2H+2SH$                    | $2H+2SH$                    | 0                  |
| Global Variable<br>and Linkage Equations | $S+F+H$                     | $S+F+H$                     | 0                  |
| Total                                    | $3S+3F+3H$<br>$+2FS+HF+2SH$ | $3S+2F+3H$<br>$+2FS+HF+2SH$ | F                  |

Table 1.10  
Variables of the complete model

| Name                    | Quantity<br>q and c | Payment<br>y and t | Price<br>p |
|-------------------------|---------------------|--------------------|------------|
| <u>Production</u>       |                     |                    |            |
| 1. Output               | $q_s$               | $y_s$              |            |
| 2. Inputs               | $c_{fs}$            | $t_{fs}$           |            |
| <u>Factors</u>          |                     |                    |            |
| 3. Income               | $q_f$               | $y_f$              |            |
| 4. Transfer             |                     | $t_{hf}$           |            |
| <u>Households</u>       |                     |                    |            |
| 5. Consume              | $c_{sh}$            | $t_{sh}$           |            |
| 6. CPI                  | $q_h$               | $y_h$              |            |
| <u>Global Variables</u> |                     |                    |            |
| Prices of goods         |                     |                    | $P_s$      |
| Factor prices           |                     |                    | $P_f$      |
| Consumer price indices  |                     |                    | $P_h$      |

The model can apparently be fully determined by making F of the variables,  $q_f$  exogenous. "Apparently", because even though the model has the same number of variables as equations, it is not yet fully determined. First of all, there is one linear dependency in the overall system of equations as follows:

$$\begin{aligned} \sum_h y_h &= \sum_h \sum_f t_{hf} \\ &= \sum_f \sum_h a_{hf} y_f & (4y) \\ &= \sum_f y_f & (5y) \\ &= \sum_s \sum_f t_{fs} & (6y) \\ &= \sum_s y_s & (7y) \\ &= \sum_s \sum_f a_{fs} y_s & (8y) \\ &= \sum_h \sum_s a_{sh} y_h & (9y) \end{aligned}$$

$$\begin{aligned} \sum_h a_{hf} &= 1 & (4y) \\ \sum_f a_{fs} &= 1 & (8y) \\ \sum_s a_{sh} &= 1 & (5y) \end{aligned}$$

This dependency is related to Walras' law which is discussed in Baumol (1977) pp. 481-3. Thus there is one equation less than the summary table above suggests. To compensate, it is necessary to add one equation.

Also, the model only explains relative prices. If all prices and values in a solution are multiplied by a constant then the new prices and values together with the old quantities will also form a solution. Therefore, the equation to add is an equation that fix a price variable. This price variable is called the Numerator. The value of the numerator will determine the overall price level. (A price index or a value can also be fixed to determine the overall price level). In the present model, the consumer price index for urban households will be fixed and will serve as the numerator. So finally the model consist of  $3S + 3F + 3H + 2FS + HF + 2SH$  variables and independent constraints.

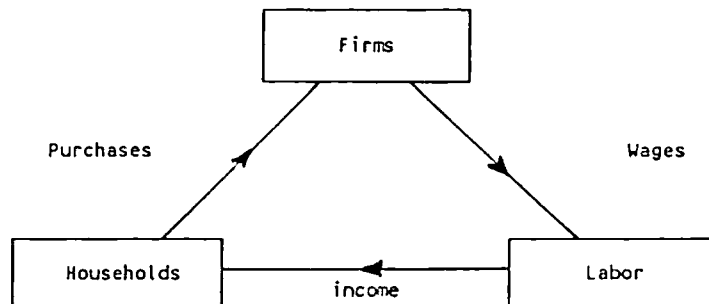
## CHAPTER 2

### THE CONCEPTUAL BASIS OF HERCULES

#### 2.1 Introduction

The previous chapter provided a mathematical statement of a simple two sector, two factor, and two household model. This chapter introduces the HERCULES approach to the formulation, estimation, and solution of this type of model. We begin by describing a Social Accounting Matrix (SAM). A SAM is a systematic and consistent way of representing all the payments in an economy, exemplified in this case by the payments from the small model described in Chapter 1. A demonstration is then given as to how the equations of the model fit into the SAM structure, and how most of them can be defined in a simple matrix form. The chapter ends with a few comments on how the SAM can be used to help with parameter estimation and model solution. The next chapter will then show how this information is input to the computer.

Figure 2.1  
The Circular Flow

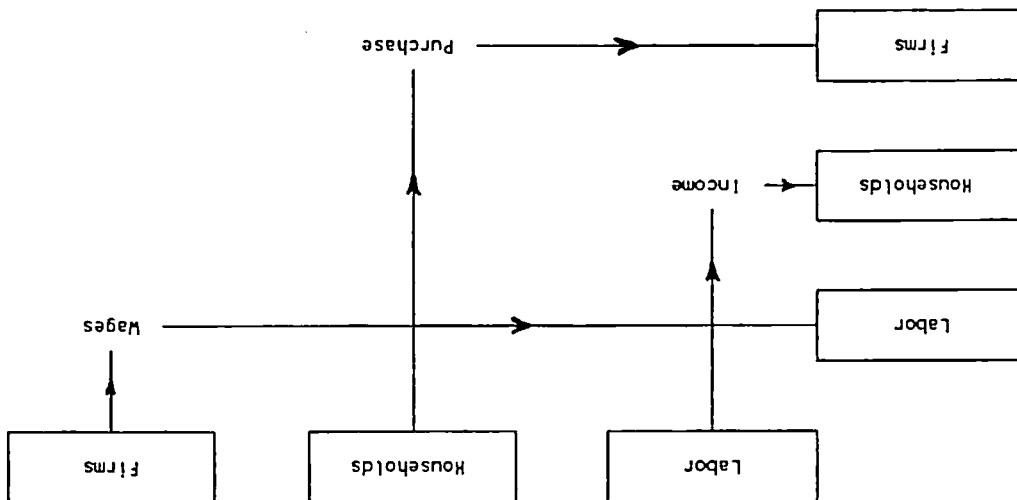


#### 2.2 The SAM as a Data Base

Social accounting matrices have been widely used by development economists, see for example Pyatt, Roe, et.al. (1977) or Pyatt and Round (1985). The SAM concept is less familiar to economists generally, however, and is therefore developed from first principles in this section. Consider first Figure 2.1, which is the familiar circular flow diagram of the kind used in the previous chapter. Firms pay wages to labor. Wage

earners then provide income to households. Finally, households close the circle by using the income to purchase goods from firms.

Figure 2.2  
Flows With Three Agents



This circular flow is fundamental to any economy-wide model and is the basic notion underlying the SAM. Next consider a rearrangement of the flow diagram to the table form shown in Figure 2.2. Here again, firms pay wages to laborers, who turn over their income to households, which make purchases from firms. The only change is that the figure is evolving toward the SAM tables, which will be used later in the chapter to present the data for the model. Figure 2.2 employs one of the basic conventions of SAMs - that columns make payments to rows.

In order to enrich the scheme further and get back to the framework of the previous chapter, we now supplement labor with a second factor, namely capital. Also, firms are now divided into farms (which produce food) and factories (which produce clothing), and households are divided into rural and urban households. Figure 2.3 is the resulting diagram. Here factors turn over labor and capital income to their owners, the two household groups. The households use the income to purchase food and clothing from the farms and factories, which in turn pay wages and profits to labor and capital.

These sets of transactions are shown numerically in Figure 2.4. Total labor income of 160 is divided, with 90 of it going to rural households and 70 to urban households; total capital income of 110 is also divided, with 30 and 80 going to rural and urban households, respectively. Total income of rural households is thus 120, and is used to

purchase 60 of food and 60 of clothing; total income of urban households is 150, and is used to purchase 65 of food and 85 of clothing. The food producers then take the 125 they have received from the households and pay 75 to labor and 50 to capital. Similarly the clothing producers use the 145 they receive from households to pay 85 to labor and 60 to capital. The SAM is completed in Table 2.1 by adding row and column totals. For example the row totals show the total income received by each set of agents. Thus in the first two rows labor receives 160 in wages and capital receives 110 in profits. Also, households receive 120 and 150 respectively in income from the labor and capital they provide. Finally, firms receive 125 in payments for food and 145 in payments for clothing. In contrast, the column totals indicate the total amounts paid out. Labor and capital pay 160 and 110 respectively to households and households make purchases of 120 and 150 respectively from firms. Finally, farms pay out 125 and factories pay out 145. This table embodies a second SAM property, namely that each row total must be equal to the corresponding column total. This is the basic conservation principle underlying the SAM concept.

Figure 2.3  
Flows With Six Accounts

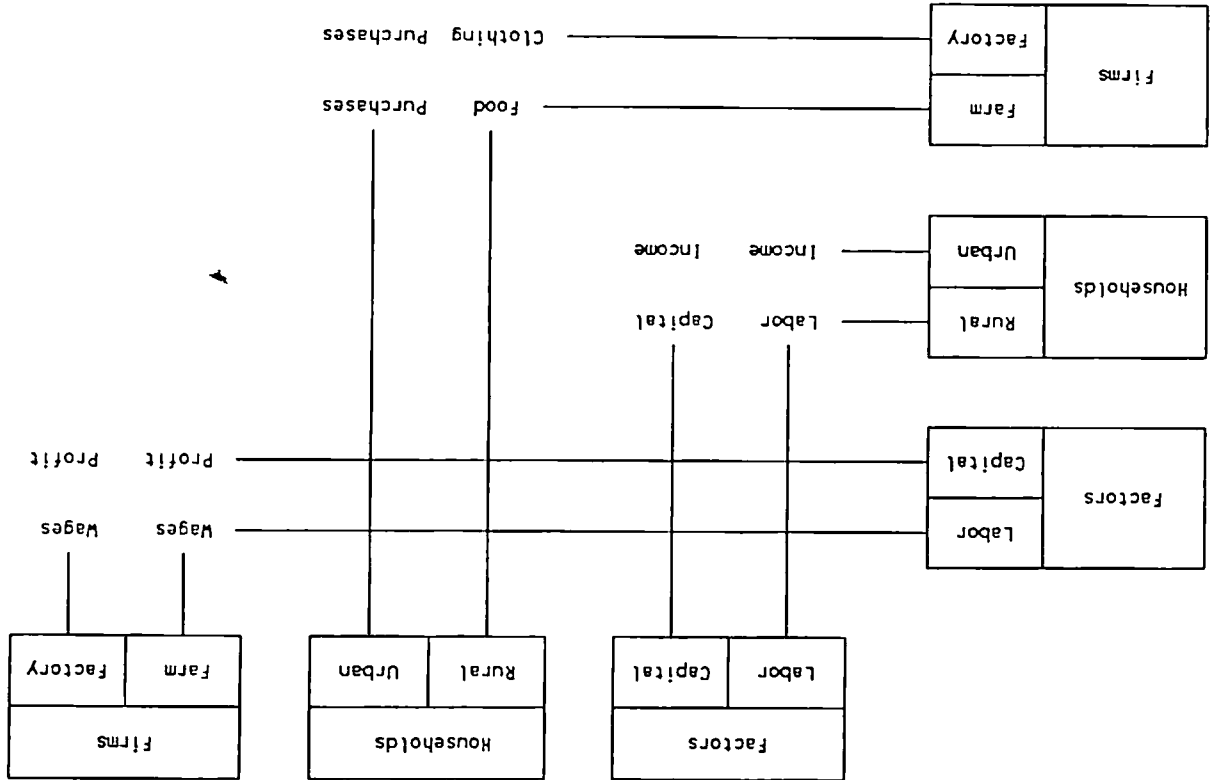


Figure 2.4  
Numerical Flows

|            |         | Factors |         | Households |       | Firms |         |
|------------|---------|---------|---------|------------|-------|-------|---------|
|            |         | Labor   | Capital | Rural      | Urban | Farm  | Factory |
| Factors    | Labor   |         |         |            |       | 75    | 85      |
|            | Capital |         |         |            |       | 50    | 60      |
| Households | Rural   | 90      | 30      |            |       |       |         |
|            | Urban   | 70      | 80      |            |       |       |         |
| Firms      | Farm    |         |         | 60         | 65    |       |         |
|            | Factory |         |         | 60         | 85    |       |         |

### 2.3 The Relationship between the SAM and the Model Variables and Equations

There is a strong relationship between the SAM and the variables and equations of the model constructed in Chapter 1. This relationship is important for understanding how HERCULES works. In this section, therefore, we explain how the variables and equations fit into the SAM structure.

From Table 1.10 the income and expenditure variables for the simple model are

$$y_s, t_{fs}, y_f, t_{hf}, y_h, \text{ and } t_{sh}$$

Table 2.2 shows the arrangement of these variables in the cells or as account totals of the SAM using the indices

Table 2.1  
A SAM

|            |         | Factors |         | Households |       | Firms |         | Totals |
|------------|---------|---------|---------|------------|-------|-------|---------|--------|
|            |         | Labor   | Capital | Rural      | Urban | Farm  | Factory |        |
| Factors    | Labor   |         |         |            |       | 75    | 85      | 160    |
|            | Capital |         |         |            |       | 50    | 60      | 110    |
| Households | Rural   | 90      | 30      |            |       |       |         | 120    |
|            | Urban   | 70      | 80      |            |       |       |         | 150    |
| Firms      | Farm    |         |         | 60         | 65    |       |         | 125    |
|            | Factory |         |         | 60         | 85    |       |         | 145    |
| Totals     |         | 160     | 110     | 120        | 150   | 125   | 145     |        |

l = labor  
 c = capital  
 r = rural  
 u = urban  
 a = agriculture  
 i = industry

Since columns pay rows, the variable  $t_{ja}$  in the upper right hand block of table 2.2 represents the payment from agriculture to labor.

Also from Table 1.10 the quantity variables of the model are

$q_s, c_{fs}, q_f, q_h,$  and  $c_{sh}$ .

These variables can be arranged in a SAM in a similar fashion to the income and expenditure variables as is shown in Table 2.3.

Note that some of the payments in Table 2.2 do not have a corresponding quantity in Table 2.3. As we will discuss shortly, all prices are price indices that we calibrate to be 1 in the base case. The measurement units for quantities are therefore such that the quantities are equal to their value when the price index is one, i.e. at base case prices. Table 2.3 is therefore also called the "Constant-Price SAM". It can be used to analyze changes in the economy in real terms.

**Table 2.2**  
**The Income and Expenditure Variables in the SAM**

|             | Households |          |          |          | Agriculture | Industry | Total |
|-------------|------------|----------|----------|----------|-------------|----------|-------|
|             | Labor      | Capital  | Rural    | Urban    |             |          |       |
| Labor       |            |          |          |          | $t_{la}$    | $t_{li}$ | $Y_l$ |
| Capital     |            |          |          |          | $t_{ca}$    | $t_{ci}$ | $Y_c$ |
| Households: |            |          |          |          |             |          |       |
| Rural       | $t_{rl}$   | $t_{rc}$ |          |          |             |          | $Y_r$ |
| Urban       | $t_{ul}$   | $t_{uc}$ |          |          |             |          | $Y_u$ |
| Agriculture |            |          | $t_{ar}$ | $t_{au}$ |             |          | $Y_a$ |
| Industry    |            |          | $t_{ir}$ | $t_{iu}$ |             |          | $Y_i$ |
| Total       | $Y_l$      | $Y_c$    | $Y_r$    | $Y_u$    | $Y_a$       | $Y_i$    |       |

**Table 2.3**  
**The Quantity Variables in the SAM**

|             | Households |         |          |          | Agriculture | Industry | Total |
|-------------|------------|---------|----------|----------|-------------|----------|-------|
|             | Labor      | Capital | Rural    | Urban    |             |          |       |
| Labor       |            |         |          |          | $c_{la}$    | $c_{li}$ | $q_l$ |
| Capital     |            |         |          |          | $c_{ca}$    | $c_{ci}$ | $q_c$ |
| Households: |            |         |          |          |             |          |       |
| Rural       |            |         |          |          |             |          | $q_r$ |
| Urban       |            |         |          |          |             |          | $q_u$ |
| Agriculture |            |         | $c_{ar}$ | $c_{au}$ |             |          | $q_a$ |
| Industry    |            |         | $c_{ir}$ | $c_{iu}$ |             |          | $q_i$ |
| Total       | $q_l$      | $q_c$   | $q_r$    | $q_u$    | $q_a$       | $q_i$    |       |

Finally, the prices can be associated with the accounts of the SAM as shown in Table 2.4.

Next consider the relations between the SAM and the equations constructed in Chapter 1. The simplest equations are those labeled d, in which a payment is defined as being equal to a physical flow times a price. There is exactly one physical flow and one of these equations for each account, and one for each of the non-empty cells in Table 2.3. The price is the price of the row, and the payment is the payment in the corresponding cell in Table 2.2. An example of a cell equation is Eq. (2d) for the

**Table 2.4**  
**Prices and Associated Accounts of the SAM**

| Accounts    | Prices |
|-------------|--------|
| Labor       | $P_l$  |
| Capital     | $P_c$  |
| Households: |        |
| Rural       | $P_r$  |
| Urban       | $P_u$  |
| Agriculture | $P_a$  |
| Industry    | $P_i$  |

capital row and agriculture column of both Table 2.2 and Table 2.3:  $t_{ca} = p_c c_{ca}$ . An example of an account equation is the labor version of Eq. (3d):  $y_l = p_l q_l$

The payments versions of the linkage equations, i.e. Eqs. (7y), (8y), and (9y) from Table 1.7 are

$$(7y) \quad y_s = \sum_h t_{sh}$$

$$(8y) \quad y_f = \sum_s t_{fs}$$

$$(9y) \quad y_h = \sum_f t_{hf}$$

These equations are accounting identities in the rows of the SAM. They simply say that the row total is equal to the sum of the elements in the row. There is one linkage equation for each row of the SAM, as shown under the row totals in Table 2.5.

The remaining equations are the behavioral equations. All these equations can be thought of as belonging either to cells or to columns of the SAM. As discussed in Chapter 1, one can often choose between a payments version or a quantity version of an equation, but the models are mathematically equivalent; in what follows, therefore, we shall work with the payments versions only. This choice gives us the behavioral equation shown in Table 2.5. Some of the column totals do not have an associated equation. This will be discussed later.

To summarize, Tables 2.2-2.5 above show the basic ideas of the HERCULES approach. All the variables and all the equations of our model can be associated with either the cells or the accounts of a SAM. Furthermore, all the definitional equations and the linkage equations can be derived from the structure of the SAM alone. The only information that is not implicit in the structure is the functional form of the equations contained in Table 2.5. This information can be defined through a 'Specifica-

**Table 2.5**  
**The Accounting Identities and the Behavioral Equations in the SAM**

|             |       |         | Households |       | Agriculture | Industry | Total |
|-------------|-------|---------|------------|-------|-------------|----------|-------|
|             | Labor | Capital | Rural      | Urban |             |          |       |
| Labor       |       |         |            |       | (2y)        | (2y)     | (7y)  |
| Capital     |       |         |            |       | (2y)        | (2y)     | (7y)  |
| Households  |       |         |            |       |             |          |       |
| Rural       | (4y)  | (4y)    |            |       |             |          | (8y)  |
| Urban       | (4y)  | (4y)    |            |       |             |          | (8y)  |
| Agriculture |       |         | (5y)       | (5y)  |             |          | (9y)  |
| Industry    |       |         | (5y)       | (5y)  |             |          | (9y)  |
| Total       |       |         | (6y)       | (6y)  | (1y)        | (1y)     |       |

tion Table'. The Specification Table for the simple model is shown in Table 2.6.

**Figure 2.6**  
**The Specification Table**

|             |       |         | Households |       | Agriculture | Industry |
|-------------|-------|---------|------------|-------|-------------|----------|
|             | Labor | Capital | Rural      | Urban |             |          |
| Labor       |       |         |            |       | CD          | CD       |
| Capital     |       |         |            |       | CD          | CD       |
| Households: |       |         |            |       |             |          |
| Rural       | IDIST | IDIST   |            |       |             |          |
| Urban       | IDIST | IDIST   |            |       |             |          |
| Agriculture |       |         | VSHR       | VSHR  |             |          |
| Industry    |       |         | VSHR       | VSHR  |             |          |

The cells contain acronyms that are abbreviations for the behavior represented by the cell: (1) CD stands for Cobb-Douglas and indicates that the functional income distribution will be determined by a Cobb-Douglas production function, (2) IDIST stands for income distribution and indicates that the overall income is allocated to income receivers in fixed proportions, and (3) VSHR stands for fixed value shares and indicates that funds are allocated for different consumption items in fixed shares.

The price equations in the production sectors, i.e. the (1p) equations, are defined implicitly from the specification table. The system knows that if the individual payments

come from a Cobb-Douglas specification, then the overall production function must be implied. It also knows that there are no column equations associated with columns with the IDIST specifications. ✓

#### 2.4 The Account Table

One part of the model definition is still missing: the total quantities of the factors,  $q_f$ , must be declared exogenously, and the consumer price index for urban households,  $p_{ur}$ , must be defined as a numeraire. The system also requires information about the economic classification of the accounts. This information is used to determine which accounts have prices and which do not and to detect inconsistencies in the model. This account specific information is defined in an 'Account Table'. Table 2.7 shows the Account Table for the simple model. The Account Table has one row for each account. It has one column to define the type of the account and one column to define the variables which are fixed. In later chapters some parameter values will also be defined through the Account Table.

Table 2.7  
The Account Table

|             | Account Type | Fixed Variables |
|-------------|--------------|-----------------|
| Labor       | MF           | Q               |
| Capital     | MF           | Q               |
| Households: |              |                 |
| Rural       | INSTC        |                 |
| Urban       | INSTC        | NP              |
| Agriculture | AC           |                 |
| Industry    | AC           |                 |

The entries in the Account Table are again abbreviations or "acronyms. The acronyms for fixed variables are straightforward. P indicates a fixed price, Q a fixed quantity, Y a fixed payment, and PQ indicates that both price and quantity are fixed. The total for the account is referred to, so Q in the capital account means that the total quantity of capital available for use in both agriculture and in industry is fixed. An N in front, i.e., NP, NY or NPQ, indicates that the fixed P or Y is the numeraire of the model.

MF or 'Market Factor' accounts are accounts for factors of production with an associated price like labor and capital in the small model. INSTC or 'Institutions - Consumption' are accounts for consuming institutions like households and government;

households in the small model belong to this group. 'AC' or 'Activity or Commodity' denotes accounts for production or trading activities and their corresponding goods. Agriculture and industry belong to this group. Other account type acronyms will be explained as they are needed.

## 2.5 Parameter Values

The small model uses several sets of parameters: (1) the share parameter,  $a_{fs}$ , and the scale parameter,  $b_s$ , in the production function; (2) the share parameter,  $a_{hp}$  in the distribution of factor income; (3) the share parameter  $a_{sh}$  in the consumption system; (4) the quantities of factors available,  $q_f$ ; and (5) the numeraire,  $p_u$ . One must estimate these parameters before the model can be solved. Fortunately, if one makes the reasonable assumption that the base SAM is a solution to the model, then one can estimate all of these parameters and exogenous variables from the base SAM.

All  $a$ 's can be computed from values only. The parameters  $a_{ci}$  can, for example, be derived from Eq. (2y), given the base values for capital income,  $t_{ci}^0$ , and total revenue,  $y_i^0$  (a zero added as a superscript is used to indicate the value in the base SAM):

$$a_{ci} = t_{ci}^0 / y_i^0$$

The results using the data in the SAM in Table 2.1 are  $a_{ci} = 50/125$ .

The computation of the  $b_s$ 's requires quantities, but the SAM contains values only. One can get around this apparent problem by observing that prices and quantities depend on the measurement unit; they are really price and quantity indices. The measurement units are chosen such that

### ALL PRICES IN THE BASE CASE ARE ONE

The emphasis is added because this treatment of prices is very important. The quantities measured in this unit are therefore equal in the base case to the values, i.e. to the numbers in the SAM. With this choice of units one can compute  $b_s$  by solving Eq. (1q) for  $b_s$  in the base case, i.e.:

$$b_s = q_s^0 / \sum_s (c_{fs}^0)^{afs} = y_s^0 / \sum_f (t_{fs}^0)^{afs}$$

The units for measuring the quantities of the factors are also such that the quantities in the base case are equal to the values, so that one gets:

$$q_f = y_f$$

and the numeraire in the base case is

$$p_u = 1$$

HERCULES has the estimation equations shown above built in, and they are automatically applied. It is only necessary to define parameters explicitly when they take values different from those in the base case, e.g. as a result of a policy change, or if they cannot be estimated from the base SAM. Examples of both cases will be given in latter chapters.

## 2.6 Conclusions

The chapter has shown that there is a close relationship between the circular flow in the economy, the SAM as a database, and the model variables and equation of our small example model. It has even been possible to represent the model with a few keywords in some tables. However, before we continue we must ask an important question: Is this convenient representation due to the very simple nature of our model? This is fortunately not the case. The following chapters will prove that a large number of different aspects of the economy can be represented with a SAM, a Specification Table, and an Account Table.

The number of different keywords that are needed is not even very large. As pointed out in Thorbecke (1985), the number of different mathematical formulations that are used in practice is not very large. The variety comes from the many ways they can be combined. And this is one of the major advantages of the HERCULES approach: you can change combinations of mathematical formulations simply by changing keywords in a table, and the changes in calibration and solution approach is done automatically.

## CHAPTER 3

### HERCULES REPRESENTATION OF A SIMPLE MODEL

#### 3.1 Introduction

The previous two chapters provided the mathematical statement and the conceptual framework for the simple model. This chapter will describe the complete HERCULES representation of the model. The chapter also includes an experiment. The input for the simple model will first be shown in components presented in an order that is helpful for explanatory purposes. The overall input is shown in proper sequence in Appendix 3.1.

HERCULES runs as a subsystem of the General Algebraic Modeling System, GAMS. GAMS supports linear and nonlinear simulation and optimization models written in algebraic form, and has report writing and extensive data manipulation capabilities. HERCULES applications use only GAMS' data manipulation and report writing capabilities, and users only interested in this aspect of GAMS will find most of the necessary information in this book; users who want to know more about GAMS' other capabilities should consult "GAMS - A User's Guide" by Brooke, Kendrick, and Meeraus (1988), in the following referred to as the "GAMS Manual".

GAMS runs in "batch-mode" from a single input file containing the complete model specification in a format that is very easy to read and understand. The main components of the GAMS input file for HERCULES model are:

- Account List
- Cell Array
  - Social Accounting Matrix
  - Specification Table
- Accounts Table
- Acronyms
- Model Statement
- Solve Statement
- Experiment Information

In the rest of this chapter, these components will first be discussed one by one; then the complete model representation will be presented, followed by a discussion of some results.

### 3.2 The Accounts Set and Some Basic GAMS

The first component in a HERCULES implementation of a models is the set of accounts. For the model in Chapter 1 the set is

Labor  
Capital  
Household-rural  
Household-urban  
Food  
Clothing

The GAMS statement needed to input this set is

```
SET ACC ACCOUNTS /  
    LABOR  
    CAPITAL  
    HHLD-RURAL  
    HHLD-URBAN  
    FOOD  
    CLOTHING /;
```

This SET declaration and definition statement introduces several aspects of the GAMS language. We will give a short overview of its basic features; details can be found in the GAMS Manual.

Case: Input can be in a mixture of lower case and upper case, but GAMS translates all text into upper case before it is analysed. Therefore, "SET" is equivalent to "set". Throughout this text we will show GAMS input in upper case.

GAMS Keywords: The word SET is a GAMS keyword and must be entered exactly as shown. The SET keyword indicates that we are going to declare one or more sets. (Other GAMS keywords we will use frequently to declare quantities in GAMS are ACRONYM, PARAMETER, TABLE and MODEL).

Identifiers: The word ACC is an identifier. Identifiers are used to name quantities in GAMS such as SETS, ACRONYMS, PARAMETERS, VARIABLES, EQUATIONS, and MODELS. They must be distinct. For example, a SET cannot have the same name as a MODEL. In this case we declare a set with the name ACC. Identifiers are chosen freely by the modeler, subject to the following restrictions:

cannot be a GAMS keyword

|                               |                    |
|-------------------------------|--------------------|
| number of <u>characters</u> : | 1 to 10            |
| <u>first character</u> :      | letter             |
| <u>other characters</u> :     | letters or numbers |

Documentation Text: The text string ACCOUNTS is a documentation text that becomes associated with the set ACC. All identifiers can have an associated documentation text. The documentation text must be entered on the same line as the identifier. It starts at the first non-blank character after the identifier, and ends at the last non-blank before the next separator (see below). It can include blanks. The restrictions for general documentation text are:

|                       |  |
|-----------------------|--|
| number of characters: | 0-80                                       |
| legal characters:     | all except separators and quotes (" or '). |

You may surround the text by quotes (matching pairs of " or ') in which case you also can include the separators comma (,) and slash (/) in the documentation text.

Initial Values: Initial values may be supplied between a pair of slashes (/). Here it is used to define the elements of the set.

GAMS Labels: The next six names, LABOR, CAPITAL, ..., between the pair of slashes define the elements of the set ACC. Elements of sets and indices in general are called GAMS labels. Labels can be chosen freely by the modeler subject to the following restrictions:

|                         |                           |
|-------------------------|---------------------------|
| number of characters:   | 1 to 10                   |
| permissible characters: | letters, numbers, + and - |

You can include special characters such as \*, &, ! and / in a label if you surround the label by quotes (" or '). The collection of identifiers is distinct from the collection of labels and the same name can therefore be used both for an identifier and a label.

Separators: There are several levels of separators in the GAMS language. At the highest level is the semicolon (;), which is used to separate GAMS' statements. The semicolon at the end of the last line of the SET statement above separates this statement from the next one. (The semicolon is not necessary if the next statement starts on a new line and starts with a GAMS keyword, but we recommend that you include it). The second level of separators contains the slash (/), that demarcates a list of initial values, such as a list of elements in a

SET, or a list of components in a MODEL. The third level of separators comprises the comma (,) and 'end-of-line.' This type of separator is used between elements in a set, and between multiple identifiers in a SET, ACRONYM, PARAMETERS or MODEL declaration statement. In our example the end-of-line is used to separate the labels. We could have used commas instead and written the following equivalent version of the statement:

```
SET ACC ACCOUNTS/LABOR, CAPITAL, HHLD-RURAL, HHLD-URBAN, FOOD,CLOTHING/;
```

This concludes the description of the set ACC. In order to access both rows and columns of the SAM independently, one needs a copy of the set ACC. This copy is created with the next GAMS statement

```
ALIAS (ACC,ACCP);
```

ALIAS is a GAMS keyword. It must be followed by a list of identifiers in parenthesis. Exactly one of the identifiers must have been declared as a SET before and the others must not have been declared. Thus a set of "accounts prime", i.e., ACCP, is created which is exactly the same as the set of accounts, ACC.

### 3.3 The Cell Array

In the previous chapter the model was essentially defined through two tables, the SAM and the Specification Table, and we must therefore enter these tables. This is done with the GAMS TABLE statement.

The SAM is entered as shown below:

```
TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX
      LABOR  CAPITAL  HHLD-RURAL  HHLD-URBAN  FOOD  CLOTHING
LABOR                                75      85
CAPITAL                             50      60
HHLD-RURAL      90      30
HHLD-URBAN      70      80
FOOD                                60      65
CLOTHING                                60      85;
```

The word TABLE is a GAMS keyword. The TABLE statement has two functions. It declares a parameter, i.e., a multidimensional array, and it initializes it with numbers defined in a tabular form. The name SAM is an identifier, the name of the parameter.

It is not a keyword and any other identifier can be used. The characters in parentheses, (ACC,ACC), are the domain of the table. That is, labels defining the rows and the columns of the table must both belong to the set of accounts, ACC. The use of domains is not mandatory in GAMS; it is highly recommended, however. If the domain is defined, the GAMS systems will check the labels and flag the error if the user misspells one of the account labels across the top or along the side of the table. The labels are not checked if the domain is not defined, or if it is represented by a \*, i.e., if the table line is changed to

```
TABLE SAM SOCIAL ACCOUNTING MATRIX
```

or

```
TABLE SAM (*,*) SOCIAL ACCOUNTING MATRIX
```

The text string "SOCIAL ACCOUNTING MATRIX" is the documentation text associated with the identifier SAM.

The input is free format and you may add blank lines to improve the readability. The numbers do not have to be in specific columns as long as they are entered in a 'readable form'. This means that the numbers must be in the field under their column heading. The field is initially the columns occupied by the column heading. It is widened if some of the material extends beyond the right or left edge of the field. The fields of adjacent columns must not grow so much that they start to overlap; if they do, GAMS will flag the error.

The specification table is input in almost the same way as the social accounting matrix. The only difference is that the numbers in the SAM are replaced by specification acronyms.

```
TABLE SPEC(ACC,ACC) SPECIFICATIONS TABLE
```

|            | LABOR | CAPITAL | HHLR-RURAL | HHLR-URBAN | FOOD | CLOTHING |
|------------|-------|---------|------------|------------|------|----------|
| LABOR      |       |         |            |            | CD   | CD       |
| CAPITAL    |       |         |            |            | CD   | CD       |
| HHLR-RURAL | IDIST | IDIST   |            |            |      |          |
| HHLR-URBAN | IDIST | IDIST   |            |            |      |          |
| FOOD       |       |         | VSHR       | VSHR       |      |          |
| CLOTHING   |       |         | VSHR       | VSHR;      |      |          |

In this case the name of the parameter is SPEC. Any other identifier could be used. The specification acronyms CD, IDIST, and VSHR are HERCULES keywords and must be input exactly as they are. Acronyms are discussed later in this chapter, and a

full list of the specification acronyms and their underlying economics and mathematics is given in Appendix A.

The SAM and the specification table have been discussed separately but they are passed on to HERCULES as planes in a three dimensional array called the cell table. The cell table is first declared in GAMS with the following statements:

```
PARAMETER CT(ACC,ACC,*) CELL TABLE;
```

The word PARAMETER is a GAMS keyword. It declares a multi-dimensional array in the same way as TABLE, but it does not initialize the array. The name CT is an identifier, and any legal identifier could have been used instead. The cell table CT is three dimensional like a box of cards, as shown in Figure 3.1. Each card has rows and

Figure 3.1  
The Cell Table CT for the Example

|            |    | S P E C S |         |            |            |      |          |
|------------|----|-----------|---------|------------|------------|------|----------|
|            |    | LABOR     | CAPITAL | HHLD-RURAL | HHLD-URBAN | FOOD | CLOTHING |
| LABOR      |    |           |         |            |            | CD   | CD       |
|            |    |           |         |            |            |      | CD       |
|            |    | T B A S E |         |            |            |      |          |
|            |    | LABOR     | CAPITAL | HHLD-RURAL | HHLD-URBAN | FOOD | CLOTHING |
| LABOR      |    |           |         |            |            | 75   | 85       |
| CAPITAL    |    |           |         |            |            | 50   | 60       |
| HHLD-RURAL | 90 | 30        |         |            |            |      |          |
| HHLD-URBAN | 70 | 80        |         |            |            |      |          |
| FOOD       |    |           | 60      | 65         |            |      |          |
| CLOTHING   |    |           | 60      | 65         |            |      |          |

columns which are labeled with the account names. The labels for the third dimension define the base SAM, the specification table, and the parameters. Thus the domains of CT are (ACC,ACC,\*). The third dimension is listed with an asterisk to indicate that the domain of this dimension is not defined, and domain checking is therefore not done on the labels in this dimension.

The SAM is copied into the first plane in the cell array using the assignment statement:

```
CT(ACC,ACCP,"TBASE") = SAM(ACC,ACCP);
```

The use of sets as indices has a special meaning in GAMS. The sets ACC and ACCP on the left hand side means that the assignment statements is executed for all combinations of elements in ACC and in ACCP. The sets ACC and ACCP on the right hand side are 'driven' by the sets on the left, i.e. a cell in the SAM is copied to the corresponding cell in the CT table. The quote marks, " ", around TBASE means that the third dimension is the single label "TBASE" and not a set like the first two dimensions. TBASE is a HERCULES keyword. The letter T comes from the fact that we denoted the cell in the *i*th row and the *j*th column of the SAM  $t_{ij}$  and BASE indicates that the SAM contains the base period information.

The copy of the set of accounts, ACCP, is used in this statement to show that the second index is independent of the first index. The statement

```
CT(ACC, ACC, "TBASE") = SAM(ACC, ACC);
```

would only copy the diagonal.

The second plane in the cell array is filled with the statement

```
CT(ACC, ACCP, "SPECS") = SPEC(ACC, ACCP);
```

This statement is used to copy the specification table into the cell array 'box' as the second card in the box and to give the second card the name SPECS. SPECS is a HERCULES keyword.

The first two planes in the cell array have now been explained. The remaining planes are parameter values associated with the functional specifications. Examples of such parameters are the exponents in Cobb-Douglas production functions. It is not necessary to input these exponent parameters separately, however, since their values can be calibrated from the data in the SAM as discussed in Chapter 2. It is only necessary to include parameter tables in cases where function specifications include parameters that cannot be computed from the data in the SAM and in cases where you want to define parameter values that are different from the values that can be calibrated. Since the present example does not include any specifications that require parameter tables, we shall postpone more detailed discussion of these tables until later.

This completes the discussion of the cell table. So far two components of the input have been described, namely the list of accounts and the cell array. Furthermore, three parts of the cell table have been discussed, namely the social accounting matrix, the specification table, and the parameter tables. We will now consider the next component of the input, the account table.

### 3.4 Account Table

The Account Table was introduced in Chapter 2. It is input as shown below:

```
TABLE AT(ACC,*) ACCOUNT TABLE
                                TYPE    FIX
LABOR                           MF      Q
CAPITAL                          MF      Q
HHLD-RURAL                       INSTC
HHLD-URBAN                       INSTC  NP
FOOD                              AC
CLOTHING                         AC;
```

The name of the account table is here AT, but it can be chosen freely. The row domain for the table is the set of accounts ACC. The column domain includes the labels TYPE and FIX. TYPE and FIX are HERCULES keywords and must be entered as shown. Since many other labels are allowed as column labels, we do not want to force domain checking on this index; the asterisk is therefore used to disable domain checking. The contents of the table are the acronyms MF, INSTC, AC, Q, and NP, which were explained in chapter 2. Note that there is not a space between the N and the P in the HHLD-URBAN row and FIX column. Thus the acronym NP is used to indicate that the price P is fixed as a numeraire.

### 3.5 Acronyms

The next component of the input consists of the acronyms. They are the abbreviations that are used to specify the model. Although they are HERCULES keywords they are not reserved word to GAMS. Therefore they must be declared so GAMS can distinguish them from other user-defined identifiers. For example the acronyms used in the present model are declared to GAMS with the statement:

Table 3.1  
Display Before Solve

| ---- 68 ACCOUNT AND CELL TABLES BEFORE SOLVE: |             |               |       |
|---|-------------|---------------|-------|
| ---- 68 PARAMETER AT                          |             | ACCOUNT TABLE |       |
|   | TYPE        | FIX           |       |
| LABOR   | MF          | Q             |       |
| CAPITAL                                       | MF          | Q             |       |
| HHLD-RURAL                                    | INSTC       |               |       |
| HHLD-URBAN                                    | INSTC       | NP            |       |
| FOOD  | AC          |               |       |
| CLOTHING                                      | AC          |               |       |
| ---- 68 PARAMETER CT                          |             | CELL TABLE    |       |
|   |             | TBASE         | SPECS |
| LABOR   | .FOOD       | 75.000        | CD    |
| LABOR   | .CLOTHING   | 85.000        | CD    |
| CAPITAL                                       | .FOOD       | 50.000        | CD    |
| CAPITAL                                       | .CLOTHING   | 60.000        | CD    |
| HHLD-RURAL                                    | .LABOR      | 90.000        | IDIST |
| HHLD-RURAL                                    | .CAPITAL    | 30.000        | IDIST |
| HHLD-URBAN                                    | .LABOR      | 70.000        | IDIST |
| HHLD-URBAN                                    | .CAPITAL    | 80.000        | IDIST |
| FOOD  | .HHLD-RURAL | 60.000        | VSHR  |
| FOOD  | .HHLD-URBAN | 65.000        | VSHR  |
| CLOTHING                                      | .HHLD-RURAL | 60.000        | VSHR  |
| CLOTHING                                      | .HHLD-URBAN | 85.000        | VSHR  |

The cell table has also several new planes, namely TSOL, QCSOL, and A-USED. TSOL returns the solution SAM, i.e.,  $t_{ij}$ , and QCSOL returns the corresponding constant price SAM,  $c_{ij}$ . A-USED returns the  $a_{ij}$  parameters that have been calibrated from the base SAM.

Note that PSOL = 1, QSOL = YSOL = YBASE, and TSOL = QCSOL = TBASE, i.e., the solution to the model reproduces the base SAM.

Table 3.2  
Display After Solve

| ---- 73 ACCOUNT AND CELL TABLES AFTER FIRST SOLVE: |              |               |       |         |         |  |
|--|--------------|---------------|-------|---------|---------|--|
| ---- 73 PARAMETER AT                               |              | ACCOUNT TABLE |       |         |         |  |
|  | TYPE         | FIX           | PSOL  | QSOL    | YSOL    |  |
| LABOR  | MF           | Q             | 1.000 | 160.000 | 160.000 |  |
| CAPITAL  | MF           | Q             | 1.000 | 110.000 | 110.000 |  |
| HHL-D-RURAL  | INSTC        |               | 1.000 | 120.000 | 120.000 |  |
| HHL-D-URBAN  | INSTC        | NP            | 1.000 | 150.000 | 150.000 |  |
| FOOD   | AC           |               | 1.000 | 125.000 | 125.000 |  |
| CLOTHING   | AC           |               | 1.000 | 145.000 | 145.000 |  |
|  | + YBASE      |               |       |         |         |  |
| LABOR  |              |               |       | 160.000 |         |  |
| CAPITAL  |              |               |       | 110.000 |         |  |
| HHL-D-RURAL  |              |               |       | 120.000 |         |  |
| HHL-D-URBAN  |              |               |       | 150.000 |         |  |
| FOOD   |              |               |       | 125.000 |         |  |
| CLOTHING   |              |               |       | 145.000 |         |  |
| ---- 73 PARAMETER CT                               |              | CELL TABLE    |       |         |         |  |
|  |              | TBASE         | SPECS | TSOL    | QCSOL   |  |
| LABOR  | .FOOD        | 75.000        | CD    | 75.000  | 75.000  |  |
| LABOR  | .CLOTHING    | 85.000        | CD    | 85.000  | 85.000  |  |
| CAPITAL  | .FOOD        | 50.000        | CD    | 50.000  | 50.000  |  |
| CAPITAL  | .CLOTHING    | 60.000        | CD    | 60.000  | 60.000  |  |
| HHL-D-RURAL  | .LABOR       | 90.000        | IDIST | 90.000  |         |  |
| HHL-D-RURAL  | .CAPITAL     | 30.000        | IDIST | 30.000  |         |  |
| HHL-D-URBAN  | .LABOR       | 70.000        | IDIST | 70.000  |         |  |
| HHL-D-URBAN  | .CAPITAL     | 80.000        | IDIST | 80.000  |         |  |
| FOOD   | .HHL-D-RURAL | 60.000        | VSHR  | 60.000  | 60.000  |  |
| FOOD   | .HHL-D-URBAN | 65.000        | VSHR  | 65.000  | 65.000  |  |
| CLOTHING   | .HHL-D-RURAL | 60.000        | VSHR  | 60.000  | 60.000  |  |
| CLOTHING   | .HHL-D-URBAN | 85.000        | VSHR  | 85.000  | 85.000  |  |
|  | + A-USED     |               |       |         |         |  |
| LABOR  | .FOOD        | 0.600         |       |         |         |  |
| LABOR  | .CLOTHING    | 0.586         |       |         |         |  |
| CAPITAL  | .FOOD        | 0.400         |       |         |         |  |
| CAPITAL  | .CLOTHING    | 0.414         |       |         |         |  |
| HHL-D-RURAL  | .LABOR       | 0.562         |       |         |         |  |
| HHL-D-RURAL  | .CAPITAL     | 0.273         |       |         |         |  |
| HHL-D-URBAN  | .LABOR       | 0.437         |       |         |         |  |
| HHL-D-URBAN  | .CAPITAL     | 0.727         |       |         |         |  |
| FOOD   | .HHL-D-RURAL | 0.500         |       |         |         |  |
| FOOD   | .HHL-D-URBAN | 0.433         |       |         |         |  |
| CLOTHING   | .HHL-D-RURAL | 0.500         |       |         |         |  |
| CLOTHING   | .HHL-D-URBAN | 0.567         |       |         |         |  |

There are many possible parameters to the GAMS call. See the GAMS help file on your computer or contact your GAMS administrator for further information on your particular system.

**Table 3.3**  
**Solution Summary from Model Experiment**

| SOLUTION SUMMARY |       |         |         |         |            |
|------------------|-------|---------|---------|---------|------------|
|                  | PSOL  | QSOL    | YSOL    | YBASE   | RESIDUAL   |
| LABOR            | 1.040 | 160.000 | 166.341 | 160.000 |            |
| CAPITAL          | 0.945 | 121.000 | 114.360 | 110.000 |            |
| HHL-D-RURAL      | 1.000 | 124.745 | 124.756 | 120.000 | -2.6949E-5 |
| HHL-D-URBAN      | 1.000 | 155.945 | 155.945 | 150.000 | -3.3182E-5 |
| FOOD             | 1.001 | 129.858 | 129.954 | 125.000 | -0.142     |
| CLOTHING         | 0.999 | 150.833 | 150.747 | 145.000 | -0.167     |

### 3.10 Results of the Model Experiment

The detailed results of the model experiment defined in section 3.8 can be seen by displaying AT and CT after the last SOLVE. To start with, we will look at the solution summary that is always printed by HERCULES. It is shown in Table 3.3. This is not the first solution summary but rather the second, i.e. the summary which is after the first experiment. (Other parts of the standard output will be discussed later.)

Table 3.3 shows the solution values that are printed as the solution summary and are also returned in the account table. First note, that the price in the HHL-D-URBAN account is 1. This was the numeraire. The numeraire remains at one in all solutions. The increased availability of capital results in a relative fall in its price to 0.945. At the same time, labor becomes relatively more scarce and its price increases to 1.040. The commodity prices are functions of the prices of the inputs used to produce them, and since food has a high labor content its price increases slightly, while the price of clothing decreases slightly due to its lower labor content.

The column QSOL shows the now quantities in all accounts. The quantity of labor exogenous and fixed to its base value, and a comparison of QSOL and YBASE confirms this. And the exogenous quantity of capital has prescribed grown by 10% from 110 to

**Table 3.4**  
**The Complete Cell Table After the Experiment**

| ----       |             | 83 PARAMETER CT | CELL TABLE |        |        |
|------------|-------------|-----------------|------------|--------|--------|
|            |             | TBASE           | SPECS      | TSOL   | QCSOL  |
| LABOR      | .FOOD       | 75.000          | CD         | 77.973 | 75.000 |
| LABOR      | .CLOTHING   | 85.000          | CD         | 88.369 | 85.000 |
| CAPITAL    | .FOOD       | 50.000          | CD         | 51.982 | 55.000 |
| CAPITAL    | .CLOTHING   | 60.000          | CD         | 62.378 | 66.000 |
| HHLD-RURAL | .LABOR      | 90.000          | IDIST      | 93.567 |        |
| HHLD-RURAL | .CAPITAL    | 30.000          | IDIST      | 31.189 |        |
| HHLD-URBAN | .LABOR      | 70.000          | IDIST      | 72.774 |        |
| HHLD-URBAN | .CAPITAL    | 80.000          | IDIST      | 83.171 |        |
| FOOD       | .HHLD-RURAL | 60.000          | VSHR       | 62.378 | 62.332 |
| FOOD       | .HHLD-URBAN | 65.000          | VSHR       | 67.576 | 67.526 |
| CLOTHING   | .HHLD-RURAL | 60.000          | VSHR       | 62.378 | 62.414 |
| CLOTHING   | .HHLD-URBAN | 85.000          | VSHR       | 88.369 | 88.419 |
|            |             | +               | A-USED     |        |        |
| LABOR      | .FOOD       | 0.600           |            |        |        |
| LABOR      | .CLOTHING   | 0.586           |            |        |        |
| CAPITAL    | .FOOD       | 0.400           |            |        |        |
| CAPITAL    | .CLOTHING   | 0.414           |            |        |        |
| HHLD-RURAL | .LABOR      | 0.562           |            |        |        |
| HHLD-RURAL | .CAPITAL    | 0.273           |            |        |        |
| HHLD-URBAN | .LABOR      | 0.437           |            |        |        |
| HHLD-URBAN | .CAPITAL    | 0.727           |            |        |        |
| FOOD       | .HHLD-RURAL | 0.500           |            |        |        |
| FOOD       | .HHLD-URBAN | 0.433           |            |        |        |
| CLOTHING   | .HHLD-RURAL | 0.500           |            |        |        |
| CLOTHING   | .HHLD-URBAN | 0.567           |            |        |        |

121. The column YSOL shows the new incomes in all accounts. As the prices, their overall level is determined by the numeraire, and only their relative size is affected by the experiment.

The last column, labeled RESIDUAL, will be explained shortly.

The detailed cell-specific results are returned in the cell table, CT. Table 3.4 shows the DISPLAY of CT after the experiment has been solved. The planes with results are labeled TSOL, QCSOL and A-USED.

The TSOL plane is displayed in a different and easier to read format in Table 3.5. (The way to make GAMS create this table will be explained in chapter 4.) The TSOL matrix is the solution SAM; row and column sum are the same and equal to the YSOL column in the account table.

**Table 3.5**  
**The TSOL Plane from the Cell Table**

| ----       | 90 PARAMETER TSOL |         | CURRENT PRICE SOLUTION SAM |            |        |
|------------|-------------------|---------|----------------------------|------------|--------|
|            | LABOR             | CAPITAL | HHLD-RURAL                 | HHLD-URBAN | FOOD   |
| LABOR      |                   |         |                            |            | 77.973 |
| CAPITAL    |                   |         |                            |            | 51.982 |
| HHLD-RURAL | 93.567            | 31.189  |                            |            |        |
| HHLD-URBAN | 72.774            | 83.171  |                            |            |        |
| FOOD       |                   |         | 62.378                     | 67.576     |        |
| CLOTHING   |                   |         | 62.378                     | 88.369     |        |
|            | + CLOTHING        |         |                            |            |        |
| LABOR      | 88.369            |         |                            |            |        |
| CAPITAL    | 62.378            |         |                            |            |        |

Table 3.6 shows QCSOL combined with the QSOL and RESIDUAL column of the account table. The QCSOL plane contains the 'constant prices SAM' corresponding to the transactions measured at base case prices. First note that some cells with SAM values do not have a QCSOL value, namely the income transfer cells with specification IDIST. The row and column totals of QCSOL are not the same, not even in accounts that have QCSOL such as FOOD and CLOTHING, and QCSOL is therefore not a SAM. Next, note that the sums of the QCSOLs in each row are equal to QSOL. This is in general the case in rows with QCSOL values, except in rows that contain the CET transformation specification discussed in chapter 12. The RESIDUAL column holds the differences between row and column totals in the Constant Price SAM, or between account totals and QSOL in case QCSOL has empty rows or columns. It measures the effect of the adjustment to changes in prices, in real terms. For example, the input into food production measured at base case prices (or the input quantities) is  $75 + 55 = 130$ , and the output is 129.858. The gain or the income effect of the adjustment in the economy is therefore  $129.858 - 130 = -0.142$  in real terms, which is exactly the number shown in RESIDUAL.

It is instructive to compare the nominal and real payments in the model by comparing the TSOL and QCSOL results. Table 3.7 provides this comparison for the

**Table 3.6**  
**The Constant Price SAM (QCSOL)**  
**and Its Relationship to QSOL and RESIDUAL**

|             | LABOR   | CAPITAL | HHL-D-RURAL | HHL-D-URBAN | FOOD    | CLOTHING | QSOL    |
|-------------|---------|---------|-------------|-------------|---------|----------|---------|
| LABOR       |         |         |             |             | 75.000  | 85.000   | 160.000 |
| CAPITAL     |         |         |             |             | 55.000  | 66.000   | 121.000 |
| HHL-D-RURAL |         |         |             |             |         |          | 124.000 |
| HHL-D-URBAN |         |         |             |             |         |          | 155.945 |
| FOOD        |         |         | 63.332      | 67.526      |         |          | 129.858 |
| CLOTHING    |         |         | 62.414      | 88.419      |         |          | 150.833 |
| RESIDUAL    |         |         | 0.000       | 0.000       | -0.142  | -0.167   |         |
| TOTAL       |         |         | 124.746     | 155.945     | 129.858 | 150.833  |         |
| QSOL        | 160.000 | 121.000 | 124.745     | 155.945     | 129.858 | 150.833  |         |

factor payments. Since payments in a SAM are always from a column to a row the labor-food pair in the first line of the table represents payments from the food industry to labor.

The first two lines of Table 3.7 show that although the nominal value of labor in each industry increases to 78.0 and 88.4 respectively that the real value does not change because the total amount of labor available has not changed in this experiment, and there has been no re-allocation of labor between sectors. In contrast, the real capital increased by ten percent in each sector but the nominal value of the capital increased by only 3.96 percent since the price of the capital fell when there was a capital increase in a comparative statics economy.

Table 3.8 provides a similar comparison of the payments from factors to households. There is no real payment in this case because there is no physical flow which corresponds to the transfer from factors to households.

Table 3.9 shows that both nominal and real consumption of food and clothing grows by about 4 percent. However, there is a slightly higher growth rate of real consumption of clothing (4.02) than for food (3.89) due to the fact that clothing prices decline relative to food prices.

The uniform growth in all nominal payments in the model economy results from the assumptions (1) that production functions are Cobb-Douglas so that nominal factor shares remain constant, (2) that the percentage distribution of capital and labor

**Table 3.7**  
**Comparisons of TSOL and QCSOL for Factor Payments**

|         |          | TBASE | TSOL | QCSOL | TSOL<br>% | QCSOL<br>% |
|---------|----------|-------|------|-------|-----------|------------|
| labor   | food     | 75.0  | 78.0 | 75.0  | 3.96      | 0.00       |
| labor   | clothing | 85.0  | 88.4 | 85.0  | 3.96      | 0.00       |
| capital | food     | 50.0  | 52.0 | 55.0  | 3.96      | 10.00      |
| capital | clothing | 60.0  | 62.4 | 66.0  | 3.96      | 10.00      |

payments to households are unaffected by increases in income, and (3) that nominal consumption percentages across commodities are unaffected by income changes. Thus one can see that the results are caused by the selection of CD, IDIST, and VSHR in the specification matrix that makes all nominal shares constant. Other specifications that we will see later will give different results.

**Table 3.8**  
**Comparisons of TSOL and QCSOL for Factor to Household Transfers**

|                    | TBASE | TSOL | QCSOL | TSOL<br>% | QCSOL<br>% |
|--------------------|-------|------|-------|-----------|------------|
| hhld-rural labor   | 90.0  | 93.6 | -     | 3.96      | -          |
| hhld-rural capital | 30.0  | 31.2 | -     | 3.96      | -          |
| hhld-urban labor   | 70.0  | 72.8 | -     | 3.96      | -          |
| hhld-urban capital | 80.0  | 83.2 | -     | 3.96      | -          |

Table 3.9  
Comparisons of TSOL and QCSOL for Consumption

|          |            | TBASE | TSOL | QCSOL | TSOL<br>% | QCSOL<br>% |
|----------|------------|-------|------|-------|-----------|------------|
| food     | hhld-rural | 60.0  | 62.4 | 62.3  | 3.96      | 3.89       |
| food     | hhld-rural | 65.0  | 67.6 | 67.5  | 3.96      | 3.89       |
| clothing | hhld-urban | 60.0  | 62.4 | 62.4  | 3.96      | 4.02       |
| clothing | hhld-urban | 85.0  | 88.4 | 88.4  | 3.96      | 4.02       |

MORE COMPLEX MODELS

PART II

## CHAPTER 4

### GOVERNMENT AND TAXES

#### 4.1 Introduction.

In this and the following chapters we will gradually add new components to our small two sector economy, and we will use the same procedure with each addition. We will first show a small SAM, which includes the new types of payments, a Data SAM. We will then make some assumptions about the behaviour of these new payments and about that of the old payments affected by them, i.e., we will make some modeling assumptions, and we will show how a larger SAM, a Model SAM, can capture these assumptions. Next we will show the new specification matrix and discuss the new types of specifications. Finally we will discuss the overall implementation of the model, new aspects of GAMS used by the model, and the model results.

Governments play an important role in all economies, so the first expansion of our simple model is the addition of a government sector. This sector has a direct influence on disposable household income (through direct taxes), on prices of goods (through indirect taxes), and on overall demand (through its consumption of goods). In this chapter, we will look at the government as a collector of direct and indirect taxes, and as a consumer of goods; its role in generating savings is discussed in Chapter 5, and its role as regulator of prices and quantities will be left to Chapter 10.

Table 4.1  
The Data SAM for a Small Economy With a Government

|            | Labor | Capital | Household | Government | Food | Clothing | Total |
|------------|-------|---------|-----------|------------|------|----------|-------|
| Labor      |       |         |           |            | 95   | 65       | 160   |
| Capital    |       |         |           |            | 30   | 80       | 110   |
| Household  | 160   | 110     |           |            |      |          | 270   |
| Government |       |         | 30        |            | 30   | 10       | 70    |
| Food       |       |         | 140       | 15         |      |          | 155   |
| Clothing   |       |         | 100       | 55         |      |          | 155   |
| Total      | 160   | 110     | 270       | 70         | 155  | 155      |       |

## 4.2 The Data SAM

The SAM describing our two sector, two factor economy with a government sector added is shown in Table 4.1. To avoid unmanageable SAMs and models we have aggregated rural and urban households into one household group, so we still have a six account SAM. Eight of the payments in the SAM are similar to the ones in Chapters 2 and 3, but the numerical values have been changed slightly so the overall SAM is balanced. The economy still has the same structure, however, with a labor intensive agricultural sector and a capital intensive industrial sector. Only the five numbers in the row and column of the Government account are new. These new payments are:

**Households to Government:** This payment of 30 shows direct taxes (or income taxes) paid by households, and it appears as one of the items on which the household spends its income.

**Food and Clothing to Government:** These two payments, of 30 and 10 respectively, represent indirect taxes (or sales taxes) on food and clothing, and they are shown as part of the expenditures incurred in bringing the goods to market.

**Government to Food and Clothing:** These two payments, of 15 and 55 respectively, represent the governments consumption of goods.

Note that the composition of demand differs for the two consumer sectors. Government consumption has a high percentage of industrial goods, while household consumption has a high percentage of agricultural goods.

## 4.3 Modeling Assumptions and the Model SAM

In order to build a model that reproduces this economy, we must make some assumptions about how these government-related payments are determined, and how they affect other payments. We will make the following assumptions:

**Direct taxes:** The household sector pays a fixed percentage of its income in direct taxes, and the remaining disposable income is used for consumption in the same way as in the previous model, i.e., the consumption choice depends only on disposable income and on market prices of goods.

**Indirect taxes:** Producers must pay a fixed, sector-specific percentage of their sales revenues as indirect taxes. When determining the use of factors in the production process, the producer acts as described in our previous model, except that he uses

the price excluding indirect tax, i.e., the price that he actually gets, as the price of the output. Consumers, on the other hand, react to the price including indirect tax.

**Government consumption:** In this initial model that omits savings and investment we will assume that the government (like the household sector) spends all its income on goods in fixed value shares.

In order to highlight these assumptions it is useful to disaggregate the data SAM as shown in Table 4.2. We have split the household account into two, a household income (hhld-incm) account that shows total income and how it is split into direct taxes and disposable income, and a disposable income (hhld-cons) account that shows how disposable income is spent on goods. The payment from account hhld-incm to account hhld-cons is simply derived from accounting identities as either the sum of consumption (140+100) or as total income minus direct taxes (270-30). This disaggregation of the household account makes the model assumptions explicit: total income is first divided between direct taxes and disposable income, and the disposable income is afterwards spend on the two consumption items.

Table 4.2  
The Expanded Model SAM

|            | LABOR | CAPITAL | HHLD-<br>INCOM | HHLD-<br>CONS | GOVERN-<br>MENT | INDR-<br>TAX | ACT-<br>FOOD | ACT-<br>CLOTH | COM-<br>FOOD | COM-<br>CLOTH |
|------------|-------|---------|----------------|---------------|-----------------|--------------|--------------|---------------|--------------|---------------|
| LABOR      |       |         |                |               |                 |              | 95           | 65            |              |               |
| CAPITAL    |       |         |                |               |                 |              | 30           | 80            |              |               |
| HHLD-INCOM | 160   | 110     |                |               |                 |              |              |               |              |               |
| HHLD-CONS  |       |         | 240            |               |                 |              |              |               |              |               |
| GOVERNMENT |       |         | 30             |               |                 | 40           |              |               |              |               |
| INDR-TAX   |       |         |                |               |                 |              |              |               | 30           | 10            |
| ACT-FOOD   |       |         |                |               |                 |              |              |               | 125          |               |
| ACT-CLOTH  |       |         |                |               |                 |              |              |               |              | 145           |
| COM-FOOD   |       |         |                | 170           | 15              |              |              |               |              |               |
| COM-CLOTH  |       |         |                | 100           | 55              |              |              |               |              |               |

We have made similar disaggregations on the production side to make the modeling assumptions explicit. For each sector we have an activity account (Act-food and Act-cloth) and a commodity account (Com-food and Com-cloth). The activity accounts show production expenditures per se and the prices associated with these accounts will be producer prices, i.e. prices at the factory gate. The commodity accounts buys the output of the production activities, pays the indirect tax, and sells to the consumers. The prices associated with the commodity accounts will therefore be market prices, i.e. the prices the consumers have to pay. Note that the households and the government buys from the

commodity accounts. One can think of the activity accounts as representing the factories, while the commodity accounts represent the marketing branches of the sectors. The numerical values of the payments from commodities to activities are derived from accounting identities in the same way as was done with the household account, i.e. the 125 paid from COM-FOOD to ACT-FOOD is computed as the sum of the payments in column ACT-FOOD.

The last change is the disaggregation of the government sector into a government account and an account for indirect taxes (INDR-TAX) where the indirect tax account pays all its income to the government. The numerical value of 40 has been computed as the sum of the two indirect tax payments of 30 and 10. This change has been included to reflect the special role of indirect taxes. Unlike direct taxes, indirect taxes influence prices, and net indirect taxes play a special role among the national accounting aggregates. We will get back to this point in chapter 8.

Table 4.3

The Account Table for the Two Sector Model with Government

|            |       |    |
|------------|-------|----|
| TYPE       | FIX   |    |
| Labor      | MF    | 0  |
| Capital    | MF    | 0  |
| Hhld-incm  | INST  |    |
| Hhld-cons  | INSTC | NP |
| Government | INSTC |    |
| Indr-tax   | TAX   |    |
| Act-food   | AC    |    |
| Act-cloth  | AC    |    |
| Com-food   | AC    |    |
| Com-cloth  | AC    |    |

4.4 The Account Table and Specification Table

HERCULES has two types of institutional accounts, INSTC and INST. Accounts of type INSTC are Institutional Consumption accounts. They can only spend money on goods and services, i.e., on accounts of type AC, and there is a price index, the consumer price index, associated with the account. Accounts of type INST are Institutional income transfer accounts. They can only pay to other institutions, i.e., accounts of type INST and INSTC and, as we shall see later, to accounts of type ROW, rest of the world accounts. There is no price associated with INST accounts.

The account table for the new model is shown in Table 4.3. We saw the INSTC account type in the Household accounts of the previous model. The government account is of the same type because the government spends all its income on consumption. In the new model we also need the INST account type to describe the Hhld-incm account. It tells us that the account is an institutional income transfer account without consumption, and without a price index.

The other new account type is TAX found in the indirect tax account. It defines the account as a tax account, which means that it can only receive income in the form of taxes; it must pay its income to an institutions account, in this case to the Government account.

**Table 4.4**  
**The Account Specific Variables Defined for Each Account**

|            | Defined variables |   |   |
|------------|-------------------|---|---|
| Labor      | P                 | Q | Y |
| Capital    | P                 | Q | Y |
| Hhld-incm  |                   |   | Y |
| Hhld-cons  | P                 | Q | Y |
| Government | P                 | Q | Y |
| Indr-tax   |                   |   | Y |
| Act-food   | P                 | Q | Y |
| Act-cloth  | P                 | Q | Y |
| Com-food   | P                 | Q | Y |
| Com-cloth  | P                 | Q | Y |

Both the activity and the commodity accounts are of type AC, activity or commodity. They all represents goods in some processed form, there is a price and a quantity associated with each of them, and they are therefore all treated in the same way. Activities represent goods at the factory gate, and commodities represent goods at the final goods market.

The account specific variables implied by the account table are shown in Table 4.4. Out of the 30 possible variables only 26 are actually defined.

The specification table for our new model is shown in Table 4.5. The modeling of expenditure for the factor accounts, the household consumption account, and the activity accounts is exactly as in our previous model, and government consumption is modeled in the same way as household consumption. The only new features are in the household

income account, the indirect tax account, and the commodity accounts. The IDIST specifications now used in the household income account and in the indirect tax account were also used in the previous model. They indicate that overall household income is split in exogenous proportions between direct taxes and consumption, and that indirect tax proceeds are paid to the government account (the fixed proportion in IDIST is 100%). The only new specifications are ITAX and IO, in the commodity columns. ITAX defines that a fixed percentage of the account total excluding tax is paid as indirect tax. IO indicates that the quantities of input(s) are proportional to the quantity of output which correspond to a Leontief production function. In this case, it simply means that to sell a unit of a good including tax we must buy one unit from the activity account.

Table 4.5  
The Specification Matrix

|                |       |         |                |               |                          |              |              |               |              |               |
|----------------|-------|---------|----------------|---------------|--------------------------|--------------|--------------|---------------|--------------|---------------|
|                | LABOR | CAPITAL | HHLD-<br>INCOM | HHLD-<br>CONS | HHLD-<br>GOVERN-<br>MENT | INDR-<br>TAX | ACT-<br>FOOD | ACT-<br>CLOTH | COM-<br>FOOD | COM-<br>CLOTH |
| LABOR          |       |         |                |               |                          |              | CD           | CD            |              |               |
| CAPITAL        |       |         |                |               |                          |              |              |               |              |               |
| HHLD-<br>INCOM | IDIST | IDIST   |                |               |                          |              |              |               |              |               |
| HHLD-<br>CONS  |       |         | IDIST          |               |                          |              |              |               |              |               |
| GOVERNMENT     |       |         |                | IDIST         |                          |              |              |               |              |               |
| INDR-<br>TAX   |       |         |                |               |                          | IDIST        |              |               |              |               |
| ACT-<br>FOOD   |       |         |                |               |                          |              |              | IO            | ITAX         |               |
| ACT-<br>CLOTH  |       |         |                |               |                          |              |              |               |              | IO            |

#### 4.5 The GAMS Implementation of the Model

In this and the remaining chapters we will refer to the complete GAMS listing in the appendices in the separate volume, in this case in Appendix 4.1. We will refer to parts of it during the discussion by mentioning the line numbers printed at the beginning of each line of the full listing as produced by GAMS. The model is also available with HERCULES as MODELB.GMS.

Most of the input file is similar to the input for the previous model. The only changes in the account set is the addition and substitution of new account names. The new acronyms are of course included in the list of acronyms from line 20 to 33, but the format is the same. The first new feature is the table continuation '+' found in line 49. Whenever a table has too many columns to fit across a page it is 'sliced' into subtables

that are entered after each other with a '+' on the column header line of the second and later subtables to indicate the continuation. As before, rows and columns can come in any order, with the only restriction that columns can only be defined once, i.e., a column can only appear in one subtable.

The definition and implementation of the experiments will be explained in the next subsection. The last new feature in this input file is the rudimentary report defined in lines 136 to 145. In order to get a two dimensional printout of the solution SAM and the corresponding constant price SAM, like the ones shown in Chapter 3, we define the two dimensional parameters, CURSAM and CONSAM, in line 139 and 140, assign the proper solution values to them in line 142 and 143, and display them in line 145. The assignment statements are similar to the statements that move the SAM and SPEC tables into the cell table: we are now simply moving information in the opposite direction. The printout produced by these statements is discussed in the next subsection.

#### 4.6 Experiments and Results

The base SAM in Table 4.1 shows an economy in which the agricultural tax rate is about three times as high as the tax rate on industrial goods. We would like to see what happens if we reverse this situation by reducing the agricultural tax rate by a factor of three and increasing the industrial tax rate by a factor of three.

We proceed as in chapter 3 by solving the model in the base case (line 117). The cell table produced by the DISPLAY after the SOLVE is shown in Table 4.6. It contains one new plane labeled "THETA-USED". The numbers represent the tax rates used in the solution, i.e., the tax rates calibrated from the base SAM. For example, the agricultural tax is shown as 24%, which is consistent with the SAM figures 30/125. Note again that the tax rate is computed by dividing with on the value excluding tax. The tax rates for the experiment are computed from THETA-USED and saved in a plane of the cell table labeled "THETA". This is done with the assignment statements in line 126 to 129. Note that assignment statements can span more than one line but must be separated by semicolons (;).

The summary solution to the experiment is shown in Table 4.7. Many results can be read from this figure. Agricultural commodities have, as expected, become cheaper (PSOL = 0.908) and industrial commodities have become more expensive (PSOL = 1.145). Before we interpret other prices, we should remember that the model mainly describes relative prices. The overall price level is determined by the numeraire, in this case the consumer price index for households. The consumer price index for government grows because the government consumes a higher percentage of the more expensive industrial goods than households do. The change in relative prices shifts

Table 4.6  
The Cell Table After Solving the Model in the Base Case

|            |             | TBASE   | SPECS      | TSOL    | QCSOL   |
|------------|-------------|---------|------------|---------|---------|
| LABOR      | .ACT-AGRCLT | 95.000  | CD         | 95.000  | 95.000  |
| LABOR      | .ACT-INDSTR | 65.000  | CD         | 65.000  | 65.000  |
| CAPITAL    | .ACT-AGRCLT | 30.000  | CD         | 30.000  | 30.000  |
| CAPITAL    | .ACT-INDSTR | 80.000  | CD         | 80.000  | 80.000  |
| HHL-D-INCM | .LABOR      | 160.000 | IDIST      | 160.000 |         |
| HHL-D-INCM | .CAPITAL    | 110.000 | IDIST      | 110.000 |         |
| HHL-D-CONS | .HHL-D-INCM | 240.000 | IDIST      | 240.000 |         |
| GOVERNMENT | .HHL-D-INCM | 30.000  | IDIST      | 30.000  |         |
| GOVERNMENT | .INDR-TAX   | 40.000  | IDIST      | 40.000  |         |
| INDR-TAX   | .COM-AGRCLT | 30.000  | ITAX       | 30.000  | 30.000  |
| INDR-TAX   | .COM-INDSTR | 10.000  | ITAX       | 10.000  | 10.000  |
| ACT-AGRCLT | .COM-AGRCLT | 125.000 | IO         | 125.000 | 125.000 |
| ACT-INDSTR | .COM-INDSTR | 145.000 | IO         | 145.000 | 145.000 |
| COM-AGRCLT | .HHL-D-CONS | 140.000 | VSHR       | 140.000 | 140.000 |
| COM-AGRCLT | .GOVERNMENT | 15.000  | VSHR       | 15.000  | 15.000  |
| COM-INDSTR | .HHL-D-CONS | 100.000 | VSHR       | 100.000 | 100.000 |
| COM-INDSTR | .GOVERNMENT | 55.000  | VSHR       | 55.000  | 55.000  |
|            | +           | A-USED  | THETA-USED |         |         |
| LABOR      | .ACT-AGRCLT | 0.760   |            |         |         |
| LABOR      | .ACT-INDSTR | 0.448   |            |         |         |
| CAPITAL    | .ACT-AGRCLT | 0.240   |            |         |         |
| CAPITAL    | .ACT-INDSTR | 0.552   |            |         |         |
| HHL-D-INCM | .LABOR      | 1.000   |            |         |         |
| HHL-D-INCM | .CAPITAL    | 1.000   |            |         |         |
| HHL-D-CONS | .HHL-D-INCM | 0.889   |            |         |         |
| GOVERNMENT | .HHL-D-INCM | 0.111   |            |         |         |
| GOVERNMENT | .INDR-TAX   | 1.000   |            |         |         |
| INDR-TAX   | .COM-AGRCLT |         | 0.240      |         |         |
| INDR-TAX   | .COM-INDSTR |         | 0.069      |         |         |
| ACT-AGRCLT | .COM-AGRCLT | 0.806   |            |         |         |
| ACT-INDSTR | .COM-INDSTR | 0.935   |            |         |         |
| COM-AGRCLT | .HHL-D-CONS | 0.583   |            |         |         |
| COM-AGRCLT | .GOVERNMENT | 0.214   |            |         |         |
| COM-INDSTR | .HHL-D-CONS | 0.417   |            |         |         |
| COM-INDSTR | .GOVERNMENT | 0.786   |            |         |         |

demand toward agriculture, as can be seen from the increased quantity in the commodity account (QSOL = 174.990 and QBASE = YBASE = 155) and away from industry (QSOL = 137.533 and QBASE = YBASE = 155). Since agriculture is more labor intensive, demand for labor goes up relative to demand for capital and the labor price grows (from 1 to 1.064) while the rental rate for capital decreases (from 1 to 0.975), even though the factor supplies are constant. The relatively more expensive labor makes the producer prices in the labor intensive agricultural sector slightly higher than the producer prices in the capital intensive industrial sector (PSOL(ACT-AGRCLT) = 1.042 and PSOL(ACT-INDSTR) = 1.014). Finally, the overall consumption of households grows (from 240 to 246.732), and the consumption of government falls (from 70 to 63.887).

To understand the relationship between the activity and commodity accounts, we need to return to the price and quantity indices. The physical flow (e.g. in tons) through the ACT-AGRCLT account is the same as the physical flow through the COM-AGRCLT account, and the absolute price of the COM-AGRCLT is derived from the absolute price of ACT-AGRCLT by adding the indirect tax. But our price indices are calibrated to be one in the base case, and our quantity indices are derived from these price indices. Therefore, QSOL in account ACT-AGRCLT cannot be compared directly with QSOL in account COM-AGRCLT. However, growth rates of quantities (QSOL/YBASE) and price changes (PSOL) can be compared directly. The growth of ACT-AGRCLT ( $141.121/125 = 1.129$ ) is the same as the growth of COM-AGRCLT ( $174.990/155 = 1.129$ ). Also, the change in the relative prices of marketed and produced agricultural goods:  $PSOL(COM-AGRCLT)/PSOL(ACT-AGRCLT) = 0.908/1.042 = 0.871$ , reflects the changed tax:  $(1+THETA)/(1+THETA0) = (1+0.080)/(1+0.240) = 0.871$ , where THETA and THETA0 are the tax rates in the solution and bases cases, respectively.

The more detailed changes in the economy can be read from the current and constant price solution SAMs (CURSAM and CONSAM). As already noted, they are produced by line 136 to 145 of the GAMS program; the results are shown in Tables 4.8 and 4.9. Before commenting on the numbers, let us look at the format. The displayed tables are written out in the same format used to enter tables into GAMS, and tables that have too many columns to fit across a page are split into subtables (with the '+' sign used to indicate continuation). Within each subtable, empty rows are suppressed; thus, even though CURSAM has ten rows, there are only five non-empty rows in the first subtable and six non-empty rows in the second subtable. Empty columns are also suppressed. In constant price SAMs in general, and in CONSAM in particular, the columns for factors (account type MF), institutions without consumption (account type INST), and indirect taxes (account type TAX) are empty; these columns are therefore not included in Table 4.9.

Table 4.7  
Solution Summary for Model Experiment

|            | PSOL  | QSOL    | YSOL    | YBASE   | RESIDUAL |
|------------|-------|---------|---------|---------|----------|
| LABOR      | 1.064 | 160.000 | 170.277 | 160.000 |          |
| CAPITAL    | 0.975 | 110.000 | 107.296 | 110.000 |          |
| HHLD-INCM  |       |         | 277.574 | 270.000 |          |
| HHLD-CONS  | 1.000 | 246.732 | 246.732 | 240.000 | -1.600   |
| GOVERNMENT | 1.090 | 63.887  | 69.607  | 70.000  | -0.304   |
| INDR-TAX   |       |         | 38.765  | 40.000  |          |
| ACT-AGRCLT | 1.042 | 141.121 | 147.077 | 125.000 | -0.099   |
| ACT-INDSTR | 1.014 | 128.660 | 130.496 | 145.000 | -0.120   |
| COM-AGRCLT | 0.908 | 174.990 | 158.843 | 155.000 |          |
| COM-INDSTR | 1.145 | 137.533 | 157.496 | 155.000 |          |

The constant price SAM shows, among other things, that overall growth in household consumption is composed of a large increase in consumption of agricultural goods, of 18.558 (from 140 to 158.558) and a smaller reduction in consumption of industrial goods, of 10.226 (from 100 to 89.774). Although the overall increase in consumption seems to be  $8.332 = 18.558 - 10.226$ , it was only shown as 6.732 in the solution summary. The overall consumption quantity in QSOL is computed from equation (6q) (see chapter 1) that was derived from the utility function. The difference between the sum of the individual consumption quantities and the overall consumption quantity is -1.600. This number is equal to the RESIDUAL in the solution summary and it reflects the loss of utility due to changes in the relative shares of agriculture and industry in the consumption basket.

There is one new type of number in the constant price SAM, the constant price tax payments in row INDR-TAX and columns COM-AGRCLT and COM-INDSTR. These numbers are in HERCULES defined as the tax payments that would materialize with base period tax rates and base period prices but with solution quantities, i.e.,  $QCSOL(i,j) = THETA0(i,j) * QSOL(j)$ .



## CHAPTER 5

### INTERMEDIATE INPUTS

#### 5.1 Introduction

One common way of describing SAMs is to characterize them as input-output matrices, see Leontief (1953), expanded to include institutions such as consumers and government as intermediaries between the supply of factors and the different components of final demand. In terms of that view, this chapter is the heart of the matter, since it deals with the input-output system. As in the last chapter, we will start by showing a Data SAM that includes the new flows, i.e., flows of goods into the production sectors. Also, we will make assumptions about the mechanisms determining the new flows and their relationships to some of the old flows. This will give rise to a more detailed discussion of how to model substitution possibilities in the production system. We implement a model and run some experiments, and we show some new examples of how to create summary report tables. Finally, we look at how the same Data SAM with a new set of assumptions can give rise to a new model, and we show some details from the implementation of this model.

#### 5.2 The Data SAM

The basis for this chapter is the SAM in Table 5.1. The new payments are those in the submatrix where the rows and columns of the food and clothing accounts intersect; some of the other payments have been adjusted so that the overall SAM remains balanced, but the matrix has the same accounts and the same basic features as the SAM in Table 4.1.

The food sector purchases 40 units from itself to be used as inputs and it purchases 35 units of inputs from the clothing sector. This use of food inputs in the production of food requires some comment. So far we have simply assumed that each sector produces one good. In practice, however, we always work with bundles of similar goods. For example, the food sector produces both bread and meat for final consumption as well as animal feed to be used as inputs in meat production. The fact that some of the SAM accounts cover more than one good does not invalidate the SAM as a database: the receipts of multi-good accounts are just the sum of the receipts of the individual goods. However, when building models we must assume that each account covers one good only or that the goods within each account are homogenous for the purposes of the model. We will return to this homogeneity principle in Chapter 6.

Table 5.1  
The Data SAM with Intermediate Inputs

|            | Labor | Capital | Household | Government | Food | Clothing | Total |
|------------|-------|---------|-----------|------------|------|----------|-------|
| Labor      | 160   |         |           |            | 65   |          | 160   |
| Capital    |       | 110     |           |            | 30   |          | 110   |
| Household  |       |         | 160       |            |      |          | 160   |
| Government |       |         |           | 30         |      |          | 30    |
| Food       |       |         |           | 145        | 15   |          | 160   |
| Clothing   |       |         |           |            | 55   | 35       | 90    |
| Total      | 160   | 110     | 160       | 30         | 160  | 90       | 610   |

### 5.3 Modeling Assumptions and the Model SAM

Before we can build a model that incorporates the new flows we must make some assumptions about the underlying economic mechanisms. These are summarized below:

**Intermediate Inputs:** We need to assume something about prices and quantities of intermediate inputs. On the price side, we will assume that these inputs are bought in the market for final goods and therefore that their price includes taxes. In the terminology of the last chapter, intermediate inputs are bought from the commodity accounts. On the quantity side, we will assume that the quantity of intermediate inputs is proportional to the quantity of output.

**Indirect Taxes:** These are assumed to be computed as a percentage of the factory gate value of output, which in turn will reflect the value of all inputs, i.e., intermediate as well as labor and capital.

**Factor Inputs:** In the production system used in Chapters 1 to 4, all inputs were factor inputs; we modeled total output as a Cobb-Douglas production function of factor inputs, which implied a possibility for substituting labor and capital with a substitution elasticity of 1. We will maintain the assumption that capital and labor can be substituted for each other, and augment it with an assumption that the aggregate factor input must be combined with intermediate inputs in fixed proportions, measured in quantity units. This also means that the quantity of the aggregate factor input is proportional to the quantity of the output.

Table 5.2  
The Model SAM

|                  | Labor | Capital | H-hold<br>Income | H-hold<br>Cons | Govern-<br>ment | Indr-<br>tax | VA-<br>Food | VA-<br>Clothing | ACT<br>Food | ACT<br>Clothing | COM<br>Food | COM<br>Clothing |
|------------------|-------|---------|------------------|----------------|-----------------|--------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|
| Labor            |       |         |                  |                |                 |              | 95          | 65              |             |                 |             |                 |
| Capital          |       |         |                  |                |                 |              | 30          | 80              |             |                 |             |                 |
| H-hold<br>Income | 160   | 110     |                  |                |                 |              |             |                 |             |                 |             |                 |
| H-hold<br>Cons   |       |         | 240              |                |                 |              |             |                 |             |                 |             |                 |
| Government       |       |         | 30               |                |                 | 40           |             |                 |             |                 |             |                 |
| Indr-Tax         |       |         |                  |                |                 |              |             |                 |             |                 | 30          | 10              |
| VA-Food          |       |         |                  |                |                 |              |             |                 | 125         |                 |             |                 |
| VA-Clothing      |       |         |                  |                |                 |              |             |                 |             | 145             |             |                 |
| ACT-Food         |       |         |                  |                |                 |              |             |                 |             |                 | 200         |                 |
| ACT-Clothing     |       |         |                  |                |                 |              |             |                 |             |                 |             | 235             |
| COM-Food         |       |         |                  | 145            | 15              |              |             |                 | 40          | 30              |             |                 |
| COM-Clothing     |       |         |                  | 95             | 55              |              |             |                 | 35          | 60              |             |                 |

These assumptions have been used to disaggregate the data SAM into the model SAM shown in Table 5.2. Most of the table is similar to Table 4.2. The household and government accounts have been disaggregated exactly as before. The difference is the two new accounts in the production part labeled VA-food and VA-clothing, where VA indicates Value Added. The production accounts illustrate the story told above: labor and capital are combined into an aggregate called value added; this value added is then in the activity accounts combined with intermediate inputs to create a set of goods at the factory gate; and these are finally in the commodity accounts transformed into goods valued at market prices by paying the indirect tax.

#### 5.4 The Account Table and Specification Table

In this and the following chapters we will only show selected parts of the GAMS input file. The overall file is shown in one of the appendices; for this chapter in Appendix 5.1. The models can also be found among the files distributed with HERCULES. The model for this chapter can for the MS-DOS version be found in a file named MODEL.C.GMS, and the name is similar on other machines.

There are no new features in the account table, shown in lines 83 to 97 in the GAMS listing. The two new value added accounts represents semi processed goods and they are therefore of type AC, i.e., activity or commodity.

Table 5.3  
The Production Columns of the  
Specification Matrix

|              | VA-<br>Food | VA-<br>Clothing | ACT-<br>Food | ACT-<br>Clothing | COM-<br>Food | COM-<br>Clothing |
|--------------|-------------|-----------------|--------------|------------------|--------------|------------------|
| Labor        | CD          | CD              |              |                  |              |                  |
| Capital      | CD          | CD              |              |                  |              |                  |
| Indr-Tax     |             |                 |              |                  | ITAX         | ITAX             |
| VA-Food      |             |                 | IO           |                  |              |                  |
| VA-Clothing  |             |                 |              | IO               |              |                  |
| ACT-Food     |             |                 |              |                  | IO           |                  |
| ACT-Clothing |             |                 |              |                  |              | IO               |
| COM-Food     |             |                 | IO           | IO               |              |                  |
| COM-Clothing |             |                 | IO           | IO               |              |                  |

Table 5.3 summarizes the production columns of the specification matrix; since the factor and institutions columns of the specification matrix are exactly as in Chapter 4, they are not shown. The table confirms the pattern described: labor and capital are combined in the VA-food and VA-clothing columns using a Cobb-Douglas production function (specification CD), so the relative amounts of labor and capital react to changes in relative prices. The combined factor input is then combined with intermediate inputs in the ACT-food and ACT-clothing columns using Input/Output or Leontief production technology (specification IO), so no substitution is possible at this second stage. The output of the production process is finally made ready for market by adding the indirect tax (specification ITAX) in the COM-food and COM-clothing columns. The IO specification in the ACT- rows and COM- columns indicate that one marketable unit requires one production unit.

## 5.5 The GAMS Implementation of the Model

The GAMS implementation of the model is shown in Appendix 5.1. The definition of the account set and the acronyms is exactly as before, and the first new feature appears in the SAM in lines 37 to 55: we have left out the empty rows in the subtables. It is not necessary to mention empty rows; GAMS automatically assumes that all cells in a table that are not mentioned explicitly are zero or empty. Internally, GAMS only retains non-zero numbers, so a zero is treated in the same way as an empty cell and it is not stored. This has important implications, as we shall see in a moment.

We have also eliminated the empty rows in the specification table in lines 57 to 75 and saved 10 input lines. The saving in these small examples is not dramatic, but it grows with the size of the SAM and the specification tables; for 100-account models, it can be considerable.

The last part of the GAMS file shows an example of how to create a report that pulls results from several experiments together into two summary tables. We are interested in comparing the effect on output by sector of changing the indirect tax rate by 10 percent in agriculture with the effect of changing it by 10 percent in industry. We therefore define a set with the accounts we are interested in with the statement

```
SET CA(ACC) COMMODITY ACCOUNTS / COM-AGRCLT, COM-INDSTR /
```

in line 108. The name of the set is CA. (ACC) defines the elements of CA to be elements of the set of accounts ACC, i.e., CA is a subset of ACC. We say that ACC is the domain of CA. "COMMODITY ACCOUNTS" is an associated documentation text, and the two labels between the slashes are the two elements in the set CA. Note that we have both labels on one line, and thcom between the two labels is therefore important; if the comma was omitted, the COM-INDSTR would be treated as a documentation text associated with the label COM-AGRCLT. It is important to define the domain of CA to be ACC, because we declared the domain of the first index in AT to be ACC in the heading of the account table (in line 83) and we are going to extract information from the account table AT using CA in the first index position (in line 113). If CA was not declared as a subset of ACC GAMS would issue a 'Domain Violation Error'.

We are interested in collecting solution values from several experiments and making computations on the results. We therefore define a set of cases with the statement

```
CASES EXPERIMENT CASES / BASE-CASE, "+10%AG-TAX", "+10%IN-TAX"/;
```

in line 109. The statement in line 108 was not terminated with a semicolon so line 109 is a continued SET statement that defines another set named CASES. The two lines could have been written as

```
SET CA(ACC) COMMODITY ACCOUNTS / COM-AGRCLT, COM-INDSTR /
SET CASES EXPERIMENT CASES / BASE-CASE, "+10%AG-TAX", "+10%IN-TAX" /;
```

with or without a semicolon at the end of the first line.

The label "+10%AG-TAX" contains the special character % and the label has therefore been enclosed in quotes. The quotes are not part of the label, so its length stays within the limit of 10 characters. You can use single or double quotes as long as you use the same opening and closing quote for each label.

The next two statements in line 110 and 111 defines to two parameters, COMM and ELAST, each with one row per commodity and one column per experiment case. The production quantities from the base case solution are stored in COMM in line 113. Lines 115 to 123 define the first experiment, solve it and store the production quantities in a new column of COMM. Lines 125 to 134 are similar: they define the second experiment, solve it and store the results. Only line 130 is new:

```
CT("INDR-TAX", "COM-AGRCLT", "THETA") - 0.;
```

If this line were omitted, then the second experiment would contain both the change in tax rate for industry defined in lines 128 to 129 for the second experiment, and the old change in tax rate for agriculture inherited from the first experiment. Line 130 gets rid of the effect of the first experiment by setting the agricultural tax rate to zero. This mechanism for returning parameters to their base values is based on the following two principles:

**GAMS Default Principle:** The default value of a position in a parameter is zero. Zero values are not stored, and zero cannot be distinguished from 'not defined'.

**HERCULES Default Principle:** A parameter that can be calibrated from the base SAM is calibrated if and only if it is 'not defined', i.e., if its value in GAMS is zero. Otherwise, the parameter is assigned the non-zero value in the account or cell table.

If you really want to define a HERCULES parameter to be zero then you must define it with the special GAMS value EPS (epsilon), that for most purposes is equal to zero.

The last lines in the GAMS file compute and display the elasticities of commodity supply with respect to changes in tax rates as the relative changes in supply ( $COMM(CA,CASES) / COMM(CA,"BASE-CASE") - 1$ ) over the relative change in tax rate (0.1).

Table 5.4  
The DISPLAY of the Summary Table of Commodity Balance

| -----      | 139 PARAMETER COMM |            | SUMMARY TABLE OF COMMODITY BALANCE |  |
|------------|--------------------|------------|------------------------------------|--|
|            | BASE-CASE          | +10%AG-TAX | +10%IN-TAX                         |  |
| COM-AGRCLT | 230.000            | 227.916    | 230.027                            |  |
| COM-INDSTR | 245.000            | 246.912    | 244.975                            |  |

Table 5.5  
The DISPLAY of the Elasticity of Commodity  
Supply with Respect to Changes in Tax Rates

| ----       | 139 PARAMETER ELAST |            | ELASTICITY OF COMMODITY SUPPLY W.R.T.<br>CHANGES IN TAX RATES |  |
|------------|---------------------|------------|---|--|
|            | +10%AG-TAX          | +10%IN-TAX |   |  |
| COM-AGRCLT | -0.091              | 0.001      |   |  |
| COM-INDSTR | -0.078              | 0.001      |   |  |

## 5.6 Results

The display of COMM and ELAST are shown in Tables 5.4 and 5.5. Notice that the change in the industrial tax rate has a much smaller impact on the composition of supply than the change in the agricultural rate, even allowing for the fact that the base for the tax on agriculture is three times as large as the base for the tax on industry. The reasons can be found by examining the details of the solutions. (Exercise: change the GAMS input file to include DISPLAYs of the account and cell tables, run the model, and verify the following explanation by examining the output.) The added agricultural tax reduces the share of agricultural goods in both household and government consump-

tion. Simultaneously, the income distribution shifts in favor of the government, which has a lower base share of agricultural demand. The combined effect is that overall agricultural demand and therefore also supply falls sharply. The added tax on industry also reduces the share of industrial goods in both household and government consumption, and it increases the government's share of overall income; but the reallocated income is mainly used on industrial goods, so the two effects work in opposite directions and the overall effect is marginal.

### 5.7 An Alternative Model

A SAM-based model can be considered an interpretation of observed data, and will be based on both the numerical data and assumptions about the underlying behavior. In this section we will make some alternative behavioral assumptions and show how the same Data SAM can support an alternative model. The changed assumption relates to indirect taxes and is now as follows:

**Indirect Taxes:** These are assumed to be value added taxes and they are computed as a percentage of value added, i.e., the value of the output at the factory gate output minus the value of intermediate inputs (or the value of factor payments)

The production columns of the Model SAM that supports this assumption is shown in Table 5.6. The value added accounts (VA-) are as before, but taxes are added in the value added tax accounts (VAT-), before intermediate inputs are added in the commodity accounts (COM-). The corresponding columns of the Specification Matrix are shown in Table 5.7. The overall GAMS implementation of the model is distributed with the HERCULES system as MODEL C1; try to run it and compare the results with those of MODEL C above.

**Table 5.6**  
**The Production Columns of the Model SAM**  
**for an Alternative Model.**

|              | VA-<br>Food | VA-<br>Clothing | VAT-<br>Food | VAT-<br>Clothing | COM-<br>Food | COM-<br>Clothing |
|--------------|-------------|-----------------|--------------|------------------|--------------|------------------|
| Labor        | 95          | 65              |              |                  |              |                  |
| Capital      | 30          | 80              |              |                  |              |                  |
| Indr-Tax     |             |                 | 30           | 10               |              |                  |
| VA-Food      |             |                 | 125          |                  |              |                  |
| VA-Clothing  |             |                 |              | 145              |              |                  |
| VAT-Food     |             |                 |              |                  | 155          |                  |
| VAT-Clothing |             |                 |              |                  |              | 155              |
| COM-Food     |             |                 |              |                  | 40           | 30               |
| COM-Clothing |             |                 |              |                  | 35           | 60               |

**Table 5.7**  
**The Production Columns of the Specification Matrix**  
**for an Alternative Model**

|              | VA-<br>Food | VA-<br>Clothing | VAT-<br>Food | VAT-<br>Clothing | COM-<br>Food | COM-<br>Clothing |
|--------------|-------------|-----------------|--------------|------------------|--------------|------------------|
| Labor        | CD          | CD              |              |                  |              |                  |
| Capital      | CD          | CD              |              |                  |              |                  |
| Indr-Tax     |             |                 | ITAX         | ITAX             |              |                  |
| VA-Food      |             |                 | 10           |                  |              |                  |
| VA-Clothing  |             |                 |              | 10               |              |                  |
| VAT-Food     |             |                 |              |                  | 10           |                  |
| VAT-Clothing |             |                 |              |                  |              | 10               |
| COM-Food     |             |                 |              |                  | 10           | 10               |
| COM-Clothing |             |                 |              |                  | 10           | 10               |

## CHAPTER 6

### FOREIGN TRADE, SAVINGS, AND INVESTMENTS

#### 6.1 Introduction

This chapter will add two more components to our model, namely (1) foreign trade, and (2) savings and investment. The reason both components are added at the same time is that one of the important sources of savings in many countries is foreign savings, which in turn is closely related to the foreign trade account. The chapter is organized in the same way as its two predecessors, with a description of a small Data SAM, followed by assumptions for a model, a GAMS/HERCULES implementation of the model, and results. Because both the economic content of this chapter and the number of new HERCULES features are extensive, we will introduce relatively few new GAMS features.

#### 6.2 The Data SAM

Table 6.1 shows a SAM that includes foreign trade and savings and investment. Foreign trade is represented by the Rest of the World account (labeled R-World), and savings and investment are represented by one account labeled Savings.

The foreign trade cells in the SAM are:

**Food and Clothing to R-World:** These two entries, 50 and 100 respectively, represent payments to the rest of the world for goods received, i.e. imports. Imports enlarge the pool of goods available in the domestic economy.

**R-World to Food and Clothing:** These two entries, 75 and 55 respectively, represent payments from the rest of the world for goods sold, i.e. exports. Exports diminish the pool of goods available in the domestic economy.

The savings and investment cells in the SAM are:

**Household and Government to Savings:** The numbers, both 40, represent savings by households and government, respectively.

**Savings to Food and Clothing:** These entries, 15 and 85 respectively, show the uses of overall savings. They represent investment, and since the numbers show pur-

chases from the food and clothing sectors, they represent investment by sector of origin. There is nothing in these numbers to tell us how the increase in capital stock is distributed between the two sectors, i.e. investment by sector of destination. Investment is only shown as a user of goods. The long term capital creation effect is not shown.

The last new cell is related to both the rest of the world account and the savings account:

**R-World to Savings:** The 20 in this cell represents foreign savings, i.e. savings by foreign institutions and individuals in our economy. It is equal to total nominal investment minus domestic savings ( $100 - (40 + 40) = 20$ ). Foreign savings are in this case also equal to payments for imports ( $50 + 100 = 150$ ) minus payments for exports ( $75 + 55 = 130$ ) or the trade deficit. (Different types of transfers may mean that trade deficit and foreign savings are different. We will see an example of this in chapter 8).

Table 6.1  
The Initial SAM with Foreign Trade, Savings, and Investment

|            | Labor | Capital | Household | Government | Savings | Food | Clothing | R-World | Total |
|------------|-------|---------|-----------|------------|---------|------|----------|---------|-------|
| Labor      |       |         |           |            |         | 95   | 65       |         | 160   |
| Capital    |       |         |           |            |         | 30   | 80       |         | 110   |
| Household  | 160   | 110     |           |            |         |      |          |         | 270   |
| Government |       |         | 20        |            |         | 55   | 35       |         | 110   |
| Savings    |       |         | 40        | 40         |         |      |          | 20      | 100   |
| Food       |       |         | 130       | 15         | 15      | 40   | 30       | 75      | 305   |
| Clothing   |       |         | 80        | 55         | 85      | 35   | 60       | 55      | 370   |
| R-World    |       |         |           |            |         | 50   | 100      |         | 150   |
| Total      | 160   | 110     | 270       | 110        | 100     | 305  | 370      | 150     |       |

Investment can be measured on a gross or net basis, the difference being the value of depreciation of existing capital stock. Input-Output tables and SAMs normally measure gross investment. This indirectly implies that intermediate inputs only cover direct inputs into production, while replacement of capital stocks is included as part of investment. We will therefore assume that the investment figures in the SAM represent gross investment. Depreciation is not included in the SAM, and the payments to the capital factor are gross income before depreciation.

The numbers representing indirect taxes in earlier SAMs must now be reinterpreted. In the previous SAMs, we only had one source of supply for each commodity and two homogenous sources of demand, and indirect taxes were therefore relatively easy to relate to supply. We could only interpret them as sales taxes or value added taxes. In this SAM, we have two sources of supply, domestic production and imports, and two different types of demand, domestic demand and export demand. Therefore, we cannot relate the overall tax figures to these components of supply and demand. The SAM in Table 6.1 is therefore inadequate for model building; we need extra figures that split the tax payments into their individual components. This is done in the Data SAM in Table 6.2, where indirect taxes are distributed between three accounts, Domestic-tax for taxes on domestic production used domestically, Import-tax for import duties, and Export-tax for taxes on domestic production used for exports.

Table 6.2  
The Data SAM

|              | Labor | Capital | House-<br>hold | Govern-<br>ment | Savings | Domestic<br>tax | Import<br>tax | Export<br>tax | Food | Clothing | R-World |
|--------------|-------|---------|----------------|-----------------|---------|-----------------|---------------|---------------|------|----------|---------|
| Labor        |       |         |                |                 |         |                 |               |               | 95   | 65       |         |
| Capital      |       |         |                |                 |         |                 |               |               | 30   | 80       |         |
| Household    | 160   | 110     |                |                 |         |                 |               |               |      |          |         |
| Government   |       |         | 20             |                 |         | 30              | 40            | 20            |      |          |         |
| Savings      |       |         | 40             | 40              |         |                 |               |               |      |          | 20      |
| Domestic tax |       |         |                |                 |         |                 |               |               | 20   | 10       |         |
| Import tax   |       |         |                |                 |         |                 |               |               | 20   | 20       |         |
| Export tax   |       |         |                |                 |         |                 |               |               | 15   | 5        |         |
| Food         |       |         | 130            | 15              | 15      |                 |               |               | 40   | 30       | 75      |
| Clothing     |       |         | 80             | 55              | 85      |                 |               |               | 35   | 60       | 55      |
| R-World      |       |         |                |                 |         |                 |               |               | 50   | 100      |         |

### 6.3 Model Assumptions and the Model SAM

The new payments in the SAM require a large number of modeling decisions, e.g., how imports and exports are determined, how total income is split between savings and consumption, and how overall investment levels are determined. At this stage, we will make the following assumptions about foreign trade:

**Import supply:** Import supply is assumed to be perfectly elastic, i.e., any amount can be bought at a fixed world price, measured in foreign exchange. The landed

price reflects the world price converted at the current exchange rate, plus import duties.

Import demand: The SAM has no data that describe the use of import in detail (which is often the case in practice). We know for example that total food imports are 50 and import duties are 20, so the landed value of food imports is 70. But we do not know how these 70 are distributed among household consumption (total 130), government consumption (total 15), investment (total 15), and intermediate use (40 and 30, respectively). We will therefore assume that the import share is the same in all components of demand (except in exports, where it is zero), and we will model demand for imports in two stages: In stage one we model total demand for goods from a particular sector, and in stage two we model the share of total demand that is covered by imports. Stage one is simple: total demand is derived implicitly from the balancing equations of the SAM combined with consumers' consumption patterns and producers' production functions. For stage two we make the assumption that the import share depends on the relative prices of landed imports compared to the (taxed) prices of domestic products on the basis of the formula:

$$\text{import-share} = \text{base-import-share} * ( P(\text{import}) / P(\text{domestic}) )^{-\text{sigma}}$$

where import-share is the share of imports measured in some quantity unit, base-import-share is the same share in a base case,  $P(\text{import})$  and  $P(\text{domestic})$  are the market prices of imported and domestic goods, respectively, and sigma is an elasticity of substitution. This formulation was first suggested by Armington, (1969). It builds on the assumption that imports and domestic goods are imperfect substitutes whose share of total consumption can be determined on the basis of a CES function with elasticity of substitution sigma. A high value of sigma indicates that the imported good is very similar to the domestic good and the import share is therefore very price responsive; the imported good and the domestic good are competitive. A low value of sigma, on the other hand, indicates that the share of imported goods is relatively stable independent of prices; the imported good and the domestic good are complementary. For a discussion of CES functions and the corresponding optimality conditions, see Brown (1966).

Export demand: We will assume that demand from the rest of the world for exports from a particular sector depends on the price of exported goods measured in foreign exchange units, relative to some world price level:

$$\text{export-demand} = \text{base-export-demand} * ( WP / P(\text{export}) )^{\text{eta}}$$

where export-demand is a quantity index for export demand, base-export-demand is the base value of this index, WP is the world price level,  $P(\text{export})$  is the export

price, measured in foreign exchange, and  $\eta$  is a demand elasticity. A high value of  $\eta$  indicates that the demand curve is almost horizontal, and a low value indicates that the demand curve is steep.

**Export supply:** In this initial model we will assume that producers are indifferent between sales on domestic and export markets as long as they receive the same price. There is therefore no supply function for exports as such, but rather a supply function for domestic and export output as a whole, derived from producers' production function. We have also implicitly assumed that the producer receives the same price, i.e. that domestic and export market prices are identical before tax.

The above summary outlines our assumptions about foreign trade at this stage. The assumptions made here are consistent with those made in many trade models, see for example Whalley (1985). Our assumptions for savings and investments are:

**Household consumption and savings:** We will assume that total income is split between consumption, taxes, and savings in fixed proportions. Total consumption is then split between products as before. This simple assumption is useful if detailed data and a more sophisticated savings model are not available.

**Government consumption and savings:** The government is assumed to have an activity plan for the period, and the quantity of consumption is therefore fixed, independent of prices. Savings is a residual, namely total income from direct and indirect taxes minus consumption expenditures.

**Investment:** The total quantity of investment is assumed to be exogenous, and investment by sector of origin is derived from total investment using a fixed coefficient model, measured in quantity units.

Finally we make the following assumptions about foreign savings and the exchange rate:

**Foreign savings:** The foreign savings variable is a residual, derived both as the difference between fixed investment and domestic savings, and as the difference between imports and exports. It may seem as if we are asking foreign savings to do more than its share in balancing the economy by expecting it to bring both the foreign trade account and the domestic savings-investment balance into equilibrium. However, if all but one of the accounts of a SAM are balanced (e.g., all but the rest of the world account), then this last item will be balanced automatically; thus, if foreign savings adjusts to the savings-investment balance, it will automatically also balance the trade account.

**Exchange rate:** The exchange rate is assumed to be fixed, and will act as the numeraire of the model, i.e. all prices will be measured relative to a world price level. The consumer price index for households is no longer fixed.

#### 6.4 The GAMS Implementation

All the assumptions from the previous section have been used to set up the Model SAM. Due to the size of the SAM, we will not present a separate table here; instead, we refer the reader to the GAMS implementation shown in Appendix 6.1 and in file MODEL.D.GMS. By now we have seen several GAMS implementations of SAMs, and the reader should have no difficulty reading the computer listing or the GAMS file.

Before looking at the SAM itself, it is useful to look at the accounts. Many of them are similar to the ones we had in previous models. Savings and investment are represented with one new institutional account, and rest of the world is also represented with one account. The main difference is the disaggregation of the commodities accounts, where we have domestic commodities (domestic products including domestic taxes), imported commodities (goods bought from the rest of the world including import duties), exported commodities (domestic products including export taxes), and composite commodities (a combination of domestic and imported commodities used to represent total domestic demand as explained above).

Four accounts for each sector may be considered excessive so we will consider that for a moment. HERCULES is based on the following important

**HERCULES Homogeneity Principle:** Goods represented by one account are implicitly assumed to be homogeneous.

This principle implies that goods in one account have the same price, face the same demand, and are traded on the same market. The same physical good sold with two different tax rates must therefore be represented by two accounts.

The specification table for the model is shown in lines 98-139 of the computer listing. It shows many new specifications, all of which are declared as ACRONYMS prior to their use in lines 32-53. In this and the following chapters we will not go in detail with the many new specifications, so the reader should consult Appendix A where all specifications are described, with details about the underlying economic assumptions, the mathematical equations, and the necessary parameters.

The first new specification, QEXO, is in the government consumption column. It models the consumption of a fixed quantity, regardless of price. The second new

specification, QSHR, is in the investment column. It states that investment buys agricultural and industrial goods in fixed quantity ratios. The new foreign trade specifications, IMPORT and EXPORT, model payments to the rest of the world for imports and from the rest of the world for exports. The CES specifications in the composite commodity columns describes the substitution between domestic and imported commodities according to constant elasticity of substitution (CES) production functions. One can think of the account as representing a 'producer' that produces the composite commodity from domestic and imported commodities. The producer minimizes the cost of the composite good subject to a CES production function, given the prices of domestic and imported goods. The last new specification is UNSPEC in the savings-investment row and the rest of the world column. It defines a residual payment that adjusts to balance the accounts as explained above.

The cell table is defined in line 148, and the SAM and the specification matrix are moved into the cell table in lines 150 and 151, respectively. Line 152 defines a new plane in the cell table, labeled ETA. This is an example of a cell specific parameter value that is defined through the cell table. All parameters are described in Appendix C, sorted into those related to accounts and those related to cells. Please review the entry for ETA in the section on Cell Specific Parameters in Appendix C and note the following: The specifications that use the parameter are listed under 'USE'. The legal values are listed under 'RESTRICTIONS', and they indicate that ETA must lie between zero and infinity, both inclusive. The value that is used if the parameter has the GAMS value 'not defined' (which is zero) is listed under 'DEFAULT' and we see that ETA does not have a default value. The item under 'ATCUAL VALUE' will be reviewed shortly.

ETA is the price elasticity of export demand used in the export equations above. So as to be able to manipulate the export accounts easily, we define in line 141 the set of exported commodities, ACCEX, with the elements COM-EXP-AG and COM-EXP-IN. The ACC in parenthesis after ACCEX tells us that the elements of ACCEX must be in ACC, i.e., that ACCEX must be a subset of ACC, and the labels we enter are checked. The numerical values of ETA are entered into a one dimensional parameter named ETAS as shown in lines 143-144, and the assignment statement in line 152 moves ETAS into the exported commodity rows, the REST-WORLD column, and the ETA plane of the cell table.

The account table for the model is shown in lines 154-177. All the new commodity accounts are of type AC (activity or commodity). So as not to produce too many types of accounts, we use the term 'consumption' to denote all forms of final use and 'consumption systems' to denote all allocation rules for final use, so the savings-investment account is classified INSTC: institutions consumption account. The rest of the world account has a new account type: ROW. ROW accounts have two associated variables:

the total transaction volume measured in domestic currency, YSOL, and the exchange rate, PSOL. The exchange rate is treated as a price, the price of foreign exchange measured in domestic currency. An increase in PSOL is therefore equivalent to a devaluation. Since we always use price indices instead of actual prices, PSOL is an exchange rate index that is one in the base case. In this model, the exchange rate is fixed; in fact, it is the numeraire (FIX is NP) and all domestic prices are therefore measured relative to a world price level.

In the current model, we have assumed that the total quantity of investment is exogenous, which is shown with a Q in the FIX column. The fixed quantity of investment in the account table, combined with the fixed quantity shares, QSHR, in the specification table, effectively determines the quantities of each commodity used for investment. An alternative and equivalent formulation would therefore be to change the two QSHR specifications onto QEXO which sets the quantities exogenously, and to remove the Q in the account table. Or we could also have modeled government consumption with a fixed Q and QSHR instead of QEXO.

The last new component of the account table is the column labeled SIGMA. Like ETA above, SIGMA is a model parameter, this time one that depend on the account. It is described in the section on Account Specific Parameters in Appendix C. SIGMA contains the elasticities of substitution used by the CES specifications in the corresponding columns of the specification matrix. The elasticity of substitution between domestically produced and imported agricultural goods is shown as 3.0 in the composite commodity account. It indicates that the goods are very close substitutes. The import share will therefore be quite price responsive. The elasticity of substitution between the two types of industrial goods is only 0.5, indicating that the goods are quite different and that little substitution will take place.

Like ETA above, SIGMA is one of the parameters that cannot be calibrated from the base SAM: the modeler must therefore provide it. The legal values are from zero to INF (plus infinity). A CES specification with SIGMA = 0 is equivalent to an IO specification, CES with SIGMA = 1 is equivalent to a CD specification, and CES with SIGMA = INF indicates that the input goods are perfect substitutes. This can also be represented by specification SUBST. In these special cases, you may choose either a CES specification combined with the definition of SIGMA, or one of the specialized specifications, but within each column of the specification matrix you must use the same specification.

HERCULES will complain that SIGMA is needed but has not been defined if you define SIGMA as zero (0.) in the account table. This is related to the GAMS and HERCULES principles for defining default values mentioned in chapter 3. GAMS does not store zeroes and it cannot distinguish between zero and 'not defined'. And HER-

CULES complains about 'not defined' for parameters that it cannot calibrate. To overcome this problem you should always use the special GAMS value EPS instead of zero when a parameter value of zero must be provided by the modeler.

Lines 179-186 show how to compute the row and column totals of the SAM in order to check if it is balanced. The first assignment statement

```
TOTALS(ACC, "ROW-TOTAL") = SUM(ACCP, SAM(ACC, ACCP));
```

computes for each row account (the set ACC on the left hand side) the sum over all column accounts (the set ACCP) of the SAM values, SAM(ACC,ACCP), and saves the value in column ROW-TOTAL of the parameter TOTALS. Since the rows and columns are traversed independently, we need two copies of the same set, ACC and ACCP. The next statement is similar, but with the role of rows and columns reversed, and the last statement

```
TOTALS(ACC, "DIFFERENCE") = TOTALS(ACC, "ROW-TOTAL") -  
TOTALS(ACC, "COL-TOTAL");
```

simply computes the difference for all accounts, ACC. A check of this kind can be very useful during the initial development of the SAM before the specification matrix has even been entered. Once the SAM is balanced, the check can be removed and the specification matrix can be created with exactly the same pattern as the SAM. ✓

The MODEL statement in lines 188-189 finalizes the definition of the model.

## 6.5 Experiments and Results

We are interested in the effects on the economy of changes in world prices, specifically an increase in agricultural prices and a decrease in industrial prices, both by 10 percent. We assume that both import and export prices are changed by the same amount.

The world price indices are defined in a plane labeled WP in the cell table. To find the name of the relevant parameter, first review the entries for EXPORT and IMPORT in Appendix A. EXPORT depends on the three parameters FQ, WP and ETA, and IMPORT depends on parameter WP. Then review the entries for the parameters in Appendix C. You will see that FQ is the base-export-demand mentioned in the export demand function in section 6.3 above, and that WP represents the world price. In the 'DEFAULT' section for parameter WP you will note that the default value, which is used if the modeler has not defined a value, is the same as the default

for all other price indexed, namely one. The item 'ACTUAL VALUE' tells us that the world price index that was used for the model solution is returned in the cell table in a plane named 'WP-USED'. This means that after a SOLVE the plane named WP-USED will contain one if WP is 'not defined' and the value of WP otherwise.

The experiment is defined in lines 210-213, where WP is assigned non-default values in the cells with specification IMPORT or EXPORT, and the basic experiment is solved in line 215.

Some of the elasticities are uncertain, and we are interested in the sensitivity of the results to changes in the elasticities. We will therefore perform two other experiments, with the same changes in world prices but with other numerical values of the elasticities. In line 230 we change the elasticity of export demand for agricultural goods, ETA, to infinity (INF), which means that the demand from the rest of the world is perfectly elastic or that the rest of the world will buy any amount of agricultural exports at a fixed world price. The behavior of the model is now quite different. Since the producer demands to get the same price from sales to exports as to the domestic market (excluding tax), the domestic price will be determined by the export market. Production, domestic demand and import share will all be determined by this price, and the exported quantity becomes a residual, namely production minus the domestic component of domestic demand.

The other sensitivity experiment is defined in lines 243-248. But before we define the new experiment, we must 'undo' the previous one by resetting ETA to its original value. This is done in line 243. If we do not 'undo' experiments they will 'accumulate' so later experiments contain all changes made so far. Actually, the changes in world prices are not undone so they are still part of the next experiment. Try to run MODEL D with statements to DISPLAY AT and CT after each SOLVE and observe the values of SIGMA in AT and ETA, WP, FQ-USED and WP-USED in CT.

The actual experiment is defined in line 248. This time, we change the elasticity of substitution between domestic and imported agricultural goods, SIGMA, to infinity (INF). This implies that domestic and imported goods essentially becomes perfect substitutes and the price of the domestic good must be the same as the price of the imported good, both inclusive of taxes. Production, domestic demand, and exports are determined from the import price, and this time the imported quantity becomes a residual, namely equal to domestic demand plus export demand minus domestic production.

The remaining GAMS statements define two small report tables, REPORTQ and REPORTP, where the quantities and prices of all commodities are summarized. For that purpose we define the set of commodities, COM, as a subset of the set of accounts

in lines 194-196. The parameters REPORTQ and REPORTP are declared in lines 198-199. Both report tables are two dimensional; the first dimension is the set of commodities and the second dimension defines the different experiments. The report collection statements are shown in lines 222-223, 236-237 and 254-255 for the base experiment and the two sensitivity experiments, respectively. The report numbers for the base SAM are defined in lines 203-204; we could have solved the model and extracted the solution without defining an experiment, but instead we use the account totals computed previously, and the knowledge that prices in the base are one. The reports are shown in Tables 6.3 and 6.4, and are displayed with the statement in line 259.

**Table 6.3**  
**Report over quantities in the alternative models**

| ----       | 259 PARAMETER REPORTQ | QUANTITIES OF ALL COMMODITIES |            |            |
|------------|-----------------------|-------------------------------|------------|------------|
|            | BASE-CASE             | BASE-SOLUT                    | ETA-AG=INF | SGM-AG=INF |
| COM-DM-AG  | 160.000               | 165.652                       | 162.028    | 175.977    |
| COM-DM-IN  | 195.000               | 192.973                       | 192.719    | 192.719    |
| COM-IMP-AG | 70.000                | 65.041                        | 70.887     | 56.938     |
| COM-IMP-IN | 120.000               | 128.034                       | 130.071    | 130.071    |
| COM-CMP-AG | 230.000               | 230.600                       | 232.915    | 232.915    |
| COM-CMP-IN | 315.000               | 320.578                       | 322.142    | 322.142    |
| COM-EXP-AG | 75.000                | 83.569                        | 90.258     | 75.000     |
| COM-EXP-IN | 55.000                | 43.885                        | 41.690     | 41.690     |

**Table 6.4**  
**Report over prices in the alternative models**

| ----       | 259 PARAMETER REPORTP | PRICES OF ALL COMMODITIES |            |            |
|------------|-----------------------|---------------------------|------------|------------|
|            | BASE-CASE             | BASE-SOLUT                | ETA-AG=INF | SGM-AG=INF |
| COM-DM-AG  | 1.000                 | 1.061                     | 1.100      | 1.100      |
| COM-DM-IN  | 1.000                 | 1.046                     | 1.083      | 1.083      |
| COM-IMP-AG | 1.000                 | 1.100                     | 1.100      | 1.100      |
| COM-IMP-IN | 1.000                 | 0.900                     | 0.900      | 0.900      |
| COM-CMP-AG | 1.000                 | 1.072                     | 1.100      | 1.100      |
| COM-CMP-IN | 1.000                 | 0.989                     | 1.011      | 1.011      |
| COM-EXP-AG | 1.000                 | 1.061                     | 1.100      | 1.100      |
| COM-EXP-IN | 1.000                 | 1.046                     | 1.083      | 1.083      |

## CHAPTER 7

# INSTITUTIONAL DISAGGREGATION AND INCOME DISTRIBUTION

### 7.1 Introduction

So far we have been dealing with the production system, with the role of government, with foreign trade, and with savings and investment. This chapter will look into the issues of income distribution and derived questions of institutional disaggregation. The SAMs and models we have seen so far have already contained distributional features, namely the distribution of income between factors of production (the factorial income distribution), and between households and government (the first sign of institutional income distribution). In this chapter we will make factorial income distribution a little more detailed by disaggregating capital, but our main objective is to look more closely at institutional income distribution. For this purpose we will introduce more institutional detail and will discuss the interactions between different institutional groups.

As usual, we will introduce a SAM that highlights the essential features we want to capture, make some assumptions about underlying economic behavior, and build and solve a small model based on these assumptions. The economics of the model are quite simple and need little explanation. Instead we will take the opportunity to discuss more GAMS report features.

Income distribution is an area where data often are verarse, and certain shortcuts are sometimes necessary. In the last section of the chapter we therefore repeat our model with simplified data for income distribution, and we show how certain distributional effects disappear.

### 7.2 The Data SAM

The Data SAM for this chapter is shown in Table 7.1. We have removed the foreign trade component introduced in Chapter 6 to keep the SAM manageable in size, but we have retained the savings and investment account.

The SAM has three accounts for factors of production, labor, agricultural capital, and industrial capital, compared to two in the previous SAMs. There is little new on the production side from the point of view of data. The value added earned by capital is written along a diagonal instead of in a row in the Goods-Ag and Goods-Ind columns

with the 30 and 80 entries, but the meaning of the individual numbers is the same. The difference shows up in the distribution of the income from capital. Agricultural capital income is allocated exclusively to households with two-thirds to rural households and one-third to urban households. Industrial capital income is allocated to urban households, government, and a institution called 'Companies'. The data thus show that the two types of capital are not homogenous from an ownership point of view. Later in the model we will also assume that they are difrent from a production point of view, but this assumption will not be directly attributed to the data SAM.

Table 7.1  
Data SAM with Institutional Disaggregation

|             | Labor | Capital-<br>Ag | Capital-<br>Ind | HHld-<br>Ru | HHld-<br>Ur | Compa-<br>nies | Govern-<br>ment | Saving/<br>Inv | Goods-<br>Ag | Goods-<br>Ind |
|-------------|-------|----------------|-----------------|-------------|-------------|----------------|-----------------|----------------|--------------|---------------|
| Labor       |       |                |                 |             |             |                |                 |                | 95           | 65            |
| Capital-Ag  |       |                |                 |             |             |                |                 |                | 30           |               |
| Capital-Ind |       |                |                 |             |             |                |                 |                |              | 80            |
| HHld-Ru     | 80    | 20             |                 |             | 5           |                | 10              |                |              |               |
| HHld-Ur     | 80    | 10             | 25              |             |             | 15             | 5               |                |              |               |
| Companies   |       |                | 45              |             |             |                |                 |                |              |               |
| Government  |       |                | 10              | 5           | 15          | 10             |                 |                | 35           | 15            |
| Saving/Inv  |       |                |                 | 10          | 15          | 20             | 15              |                |              |               |
| Goods-Ag    |       |                |                 | 80          | 60          |                | 15              | 10             | 40           | 30            |
| Goods-Ind   |       |                |                 | 20          | 40          |                | 45              | 50             | 35           | 60            |

The most important new account in the SAM is the account for 'Companies'. Companies are legal entities like corporations that own a share of the factor called capital for which they receive a rental, which in turn is used for savings, taxes, and dividends. Companies should not be confused with activities or producers. Activities represent the physical process of creating goods from capital and labor factors and from intermediate inputs. Industrial activity represents all the many physical plants that produce industrial goods. The industrial capital account represents the combined capital of all these plants, and the production data show the profit received by these plants in the industrial capital account row. The industrial capital account column shows that there are three types of industrial plants: some directly owned by urban households, some owned by the government, and some owned by corporations or 'Companies'. Although companies are ultimately owned by households (or by the government), a plant is only considered to be owned by a household if the profits from the plant are an integral part of the household's income, and the household decides on how to allocate total income, not just profits as such. Companies, on the other hand, allocate profits to

savings, taxes and dividends independently of the economy of the households that own them. It is this behavioral difference that makes the introduction of companies important.

Companies do not buy goods in the Data SAM. Since the activity accounts purchase intermediate inputs for the production process, the only types of goods companies can generally buy are investment goods. In this SAM companies do not buy investment goods directly; they pay their savings to a combined savings-investment account that takes care of investments.

The next important aspect of the Data SAM is that of payments between institutions. In earlier chapters we have seen payments of direct taxes from households to the government, and payments of savings from households and the government to a combined savings-investment account. This Data SAM shows four new types of payments between institutions:

**Urban Households to Rural Households:** This payment of 5 is a transfer of income, e.g., from workers in the city to family members in the country.

**Companies to Urban Households:** This payment of 15 represents profits paid to the owners of the companies.

**Companies to Government:** This payment of 10 represent direct taxes. In certain cases, it will also include profits paid on company shares owned by the government. Here, however, we will assume that the government's share of profit was accounted for in the industrial capital account column, so the payment is only for taxes.

**Government to Rural and Urban Households:** These payments of 10 and 5 respectively represent transfers from the government, e.g., in the form of social security payments or pensions.

Note that none of the inter-institutional transfers are related to goods or service transactions. In particular, subsidies on goods are not included as transfers; if present, they should be treated as negative taxes in the activity-commodity part of the SAM to make their price implications explicit.

### 7.3 Modeling Assumptions

In the previous chapters labor and capital have had one account each. Because of the HERCULES Homogeneity Principle the implicit assumption has been that both labor and capital are perfectly mobile across sectors. For long run models this may be a reasonable assumption, but most capital will be sector specific in the medium run. In the model in this chapter we will therefore assume that there are two capital stocks, one in agriculture and one in industry, and that they are immobil. On the other hand, labor is homogeneous and perfectly mobile between sectors.

Other aspects of the production system, e.g., the modeling of intermediate inputs, indirect taxes, and substitution between labor and capital, are as in Chapter 5.

The primary distribution of factor income to institutions is assumed to follow fixed shares, i.e., each institution is assumed to own a share of each factor, and income is split according to these ownership shares.

The redistribution of income through inter-institutional transfers is assumed to have the following features:

**Direct Taxes:** Direct taxes paid by households and companies are a fixed percentage of total disposable income. (Note below the definition of total disposable income used here.)

**Savings:** Savings by households and companies are a fixed percentage of total disposable income.

**Transfers:** Transfers from urban to rural households are a fixed percentage of total disposable income for urban households. Transfers from the government to rural and urban households are fixed in nominal terms in the government's budget.

**Profits:** Profits paid out by companies are a fixed percentage of income.

The term total disposable income is used above to denote the gross income from factors of production and from transfers, i.e., the account total in the SAM.

Final use is assumed to be determined as follows:

**Households:** Households spend their income after taxes, savings, and net transfers on goods in fixed value shares. This is the same assumption used in earlier chapters.

**Government:** The government has a fixed activity budget, i.e., it buys a fixed quantity of goods, independent of prices. This assumption is similar to the assumption in Chapter 6.

**Savings:** Total savings in the economy by households, companies, and the government are spent on investment goods in fixed ratios, measured in quantity terms. This assumption is different from the assumption in Chapter 6, where total investment was fixed. However, in this model we do not have the foreign savings component to bridge the gap between domestic savings and investment, so we assume that investment adjusts to available savings.

As in Chapter 6, the assumptions about the behavior of the government implicitly defines government savings as a residual determined as the difference between income from taxes and industrial capital, and expenditures on transfers and consumption.

Finally, the consumer price index for urban households is chosen as the numeraire.

#### 7.4 GAMS Implementation and Experiments

The first step in creating a HERCULES model from the Data SAM in Table 7.1 and the modeling assumptions above is to create a Model SAM. The disaggregation of the two goods accounts into activity-, value-added-, and commodity-accounts, and the disaggregation of the institutional accounts for households and government into consumption, income and transfer accounts, follows the principles from previous chapters. The GAMS implementation of the final Model SAM is shown in lines 48-89 in the computer listing in Appendix 7.1 and it can be found in file MODELE.GMS.

The specification table follows exactly the same format as the SAM and is shown in lines 93-134. The only new specification is TEXO, meaning Tsol EXOgenous. It is used in the cells where government makes transfers to households to represent a payment that is exogenous in value terms. The account table in lines 143-163 has no new features except that now there three factors, each with a fixed quantity.

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The experiments focus on the effects of various policy experiments on final use, e.g., household consumption, government consumption, and investment. The policy experiments are an increase in exogenous government consumption, an increase in agricultural taxes, and an increase in industrial taxes. The final summary tables should show the prices, quantities, and values of final use for each of the final users for each policy experiment, and the corresponding elasticities and they will therefore be three dimensional. Before the report tables are declared it is necessary to define the sets FINUSE,

CASES, and PQY, that describe the dimensions of the tables (in line 179-184). The two report tables REPORT and ELAS are then declared in lines 187-188. (35-186)

A special set, T, is defined as a subset of the set CASES to facilitate the development of the reports in the following way. In general we would write the base case report statements like

```
REPORT(.....,"BASE-CASE") - ...
```

where ... represents labels or expressions. The report statements for the government consumption experiment would be

```
REPORT(.....,"GOVCON+10%") - ...
```

where the ... labels and expressions would be the same, and the only change would be in the case label. To save typing we create the set T with only one element, the case label, and all report statements will become the same

```
REPORT(.....,T) - ...
```

By redefining T before each report one can then use the same report statements repeatedly. The report statements used here are shown in lines 208-210 and exactly the same lines are shown again in lines 228-230 and 248-250. The only remaining task is to define T. Sets can in GAMS be created or changed dynamically by assignments of the form

```
S(S1) - YES;
```

and

```
S(S2) - NO;
```

where S is a set and S1 and S2 represent sets or individual labels. The first statement makes all elements in S1 elements in S as well, and the second statement removes all elements in S2 from S (if they were in S before). We can therefore first define T with the statement

```
T("BASE-CASE") - YES;
```

as shown in line (34) 193. Before we use T for the government consumption experiment we must remove the label "BASE-CASE" from T and enter "GOVCON + 10%" instead. This is more easily done with the two statements

```
T(T) - NO; T("GOVCON+10%") - YES;
```

shown in line <sup>(205)</sup>207. The two statements are for convenience entered on one line and separated by a semicolon (;). The first statement removes any existing elements from T, i.e., it makes T the empty set, and the second statement makes GOVCON+10% an element of T. We could have used the statements

```
T("BASE-CASE") - NO; T("GOVCON+10%") - YES;
```

but the first is shorter, and it is independent of the order and number of the experiments.

The model is this time not solved in the base case. Instead we rely on the fact that in the base case all prices are one and quantities are equal to values that are equal to the SAM account totals. This is all shown in the special report statements in lines 194-196. This approach saves a little computer time by avoiding a model solution, but instead we need the statements that compute the account totals (here shown in lines 165-172) and the special report statements. It can only be recommended for cases where the only solution values needed are cell values or account totals. If calibrated parameter values are needed for the solution report, then it is easier to solve the model in the base case as well. ✓

In the first experiment, we increase government consumption by a factor (1+DELTA) over its value in the base case, as shown in line 210. DELTA is defined as a SCALAR in line 189. SCALARs are real numbers. They are similar to PARAMETERS with the only exception that they do not have any dimensions or indices. DELTA is initialized to 0.10 with the usual "/ value /" initialization format. The input line is longer than the output line and the printout in the appendix shows the last part of the input line, "EXPERIMENT /0.10/;", on a separate line. The line can be recognized as a continuation line because it does not have a line number.

The rest of the experiment definitions and the solution statements contain no new features, but they reemphasize two important rules:

Parameter values keep their value after an experiment, and the experiment specific values must therefore be removed explicitly when they are no longer wanted.

If a parameter is zero and can be calibrated by HERCULES, then it will be calibrated.

Combining these two rules, we see that the easiest way to remove an experiment is usually to set the parameter value that defined the previous experiment to zero. This is done at the beginning of the input for both experiment 2 (in line 219) and experiment 3 (in line 239).

The summary statistics are collected from the Account and Cell tables after each experiment. This is necessary because these tables are reused by the following SOLVE statements so AT and CT will at any time only contain the results of the last SOLVE. However, the elasticities are computed from the REPORT parameter and this can be done once for all experiments with the statement shown in lines 254-255. For all combinations of price, quantity, and value (PQY), final uses (FINUSE), and experiments (CASES), the elasticity is computed as the growth over the base case, divided by DELTA, the size of the policy change.

**Table 7.2**  
**The Price, Quantity, and Value of Final Use**  
**for the Policy Experiments**

|          |             | BASE-CASE | GOVCON+10% | AG-TAX+10% | IN-TAX+10% |
|----------|-------------|-----------|------------|------------|------------|
| PRICE    | .HOU-RU-CO  | 1.000     | 1.000      | 1.009      | 0.979      |
| PRICE    | .HOU-UR-CO  | 1.000     | 1.000      | 1.000      | 1.000      |
| PRICE    | .GOVERN-CO  | 1.000     | 0.999      | 0.984      | 1.038      |
| PRICE    | .SAVING-INV | 1.000     | 0.999      | 0.980      | 1.047      |
| QUANTITY | .HOU-RU-CO  | 100.000   | 100.064    | 90.781     | 95.597     |
| QUANTITY | .HOU-UR-CO  | 100.000   | 99.965     | 94.126     | 93.710     |
| QUANTITY | .GOVERN-CO  | 60.000    | 66.000     | 60.000     | 60.000     |
| QUANTITY | .SAVING-INV | 60.000    | 54.016     | 73.816     | 70.328     |
| VALUE    | .HOU-RU-CO  | 100.000   | 100.102    | 91.638     | 93.617     |
| VALUE    | .HOU-UR-CO  | 100.000   | 99.965     | 94.126     | 93.710     |
| VALUE    | .GOVERN-CO  | 60.000    | 65.956     | 59.034     | 62.301     |
| VALUE    | .SAVING-INV | 60.000    | 53.971     | 72.339     | 73.645     |

## 7.5 Results

The two result tables produced by GAMS/HERCULES are shown in Table 7.2 and 7.3. Three dimensional parameters are by default displayed with the first two dimensions on the row and the last dimension on the column. Note that Table 7.3 has one row less than Table 7.2. Because the consumer price index for urban households has been chosen as the numeraire all elasticities in the row PRICE.HOU-UR-CO are zero and the row is not displayed.

The change in government consumption reduces total investment (from 60 to 53.9) by approximately the amount of the extra government consumption (from 60 to 66), and

the influence on household consumption and prices is marginal. The key factors in this result are the specification of government savings as a residual, and the assumption that investment adjusts to available savings.

Table 7.3  
The Elasticity of Price, Quantity, and Value of  
Final Use with respect to the Policy Experiments

|                     | GOVCON+10% | AG-TAX+10% | IN-TAX+10% |
|---------------------|------------|------------|------------|
| PRICE .HOU-RU-CO    | 0.004      | 0.094      | -0.207     |
| PRICE .GOVERN-CO    | -0.007     | -0.161     | 0.383      |
| PRICE .SAVING-INV   | -0.008     | -0.200     | 0.472      |
| QUANTITY.HOU-RU-CO  | 0.006      | -0.922     | -0.440     |
| QUANTITY.HOU-UR-CO  | -0.004     | -0.587     | -0.629     |
| QUANTITY.GOVERN-CO  | 1.000      |            |            |
| QUANTITY.SAVING-INV | -0.997     | 2.303      | 1.721      |
| VALUE .HOU-RU-CO    | 0.010      | -0.836     | -0.638     |
| VALUE .HOU-UR-CO    | -0.004     | -0.587     | -0.629     |
| VALUE .GOVERN-CO    | 0.993      | -0.161     | 0.383      |
| VALUE .SAVING-INV   | -1.005     | 2.057      | 2.274      |

When agricultural taxes are increased both rural and urban household consumption decreases. Government income increases, and since government consumption is constant, savings and therefore investment also increase. The increase in agricultural taxes causes a shift in household consumption from agriculture to industry. In addition, the final use that is reallocated from consumption to investment is also more industry intensive, so there are two forces lowering overall demand for agricultural goods. This reduces the rental rate for agricultural capital compared to the rental rate for industrial capital, and the wage rate is reduced compared to the average capital rental rate because agriculture is more labor intensive. The total effect is a substantial decrease in the sources of income for rural households compared to urban households.

When industrial taxes are increased we see the same shift in overall consumption from rural and urban households towards investment. Also, the increase in industrial taxes shifts consumption from industry to agriculture, but the reallocation of demand from consumption to investment has the opposite effect. The demand composition and therefore the demand for factors does not change as much as in the previous case, and the total effect is that urban households only lose a little more than rural households.

## 7.6 An Alternative Model

A model with income distribution can often be difficult to build because the necessary data are not readily available. The crucial data are those that describe the details of the mapping from factor income to institutional income, i.e., how much of each factor's income is paid to each institution. For our initial model, this information has been extracted in Table 7.4. As we will show in this section, it is still possible to build a SAM and a model if this detail is not available, but the quality of the results will suffer.

Table 7.4  
The Income Distribution from Factors to Institutions

|                 | Labor | Capital-Ag | Capital-Ind |
|-----------------|-------|------------|-------------|
| Household-rural | 80    | 20         |             |
| Household-urban | 80    | 10         | 25          |
| Companies       |       |            | 45          |
| Government      |       |            | 10          |

The data that are often available are total factor income by factor and total income by institution. These items can be organized as shown in Table 7.5, where a new account called Factor-total has been added. The Factor-total account receives all factor income and redistributes it to institutions. This small change in data still gives us a balanced SAM. The natural way to build a model based on this new SAM is to make the new account an institutional account (TYPE = INST) and to use specification IDIST in all the non-zero cells in Table 7.5.

A model with these changes is distributed with HERCULES on a file named MODELE1.GMS. The summary report table for the new model is shown in Table 7.6. It can be directly compared with Table 7.2. The first experiment (increased government consumption) gives almost the same result, but the second (increased agricultural taxes) gives a very different outcome. The new model gives results for factor income that are similar to the results for the original model, but now rural households get a fixed share of the total factor income. The result is that rural households have a slightly higher nominal income than urban ones; only the fact that they consume a higher share of expensive agricultural output makes their real consumption slightly smaller than that of urban households. The lack of a mapping from factorial income distribution to institutional income distribution prevents changes in the composition of demand, and the resulting change in factorial income distribution, from showing up in the institutional side of the economy.

Table 7.5  
Simplified Income Distribution from Factors to Institutions

|                 | Labor | Capital-Ag | Capital-Ind | Factor-total |
|-----------------|-------|------------|-------------|--------------|
| Household-rural |       |            |             | 100          |
| Household-urban |       |            |             | 115          |
| Companies       |       |            |             | 45           |
| Government      |       |            |             | 10           |
| Factor-total    | 160   | 30         | 80          |              |

Table 7.6  
The Price, Quantity, and Value of Final Use for the  
Policy Experiments for the Model without Detailed  
Mapping of the Income Distribution from Factors to  
Institutions

|                     | BASE-CASE | GOVCON+10% | AG-TAX+10% | IN-TAX+10% |
|---------------------|-----------|------------|------------|------------|
| PRICE .HOU-RU-CO    | 1.000     | 1.000      | 1.010      | 0.979      |
| PRICE .HOU-UR-CO    | 1.000     | 1.000      | 1.000      | 1.000      |
| PRICE .GOVERN-CO    | 1.000     | 0.999      | 0.982      | 1.038      |
| PRICE .SAVING-INV   | 1.000     | 0.999      | 0.978      | 1.047      |
| QUANTITY.HOU-RU-CO  | 100.000   | 99.949     | 93.147     | 95.972     |
| QUANTITY.HOU-UR-CO  | 100.000   | 99.981     | 93.780     | 93.660     |
| QUANTITY.GOVERN-CO  | 60.000    | 66.000     | 60.000     | 60.000     |
| QUANTITY.SAVING-INV | 60.000    | 54.110     | 71.949     | 70.026     |
| VALUE .HOU-RU-CO    | 100.000   | 99.982     | 94.113     | 93.999     |
| VALUE .HOU-UR-CO    | 100.000   | 99.981     | 93.780     | 93.660     |
| VALUE .GOVERN-CO    | 60.000    | 65.961     | 58.941     | 62.283     |
| VALUE .SAVING-INV   | 60.000    | 54.071     | 70.371     | 73.303     |

## CHAPTER 8

### SAMS AND NATIONAL ACCOUNTS AGGREGATES

#### 8.1 Introduction

This chapter serves two purposes. It presents a SAM and a model with most of the SAM features that are interesting from a national accounting point of view. The SAM contains all the types of payments shown so far plus a few extra ones needed in practical modeling work, such as workers' remittances, unrequited transfers from and to the rest of the world, factor payments to the rest of the world, and interest payments. The chapter also explains the relationships between the SAM, the model specifications, and standard national accounts aggregates such as GDP. Some of these statistics are automatically printed out in a summary table by HERCULES, and the table is defined in this chapter. Further details of national accounting in this setting can be found in Taylor (1979).

The chapter also presents some more advanced features of GAMS. So far, all summary tables have extracted information from the solution with little further processing. This chapter shows how to create summary tables with aggregates from the SAM using mappings. We use the national accounts summary table produced by HERCULES as an example, thereby both showing the report generation features and defining the summary table.

#### 8.2 The Data SAM and the Model Implementation

The Data SAM for this chapter is shown in Table 8.1. There is one new account named FAC-Abr which is an abbreviation for Factor Income from Abroad. Most of the payments in the Data SAM have been explained already, but there are a few new ones:

**Rest of the World to Factor Income from Abroad:** This payment of 5 represents income from labor working abroad and capital owned by our country and invested abroad. The Data SAM shows that this foreign factor income all accrues to urban households.

**Rest of the World to Institutions:** The rest of the world pays 2, 6, and 5 to rural households, urban households, and the government, respectively. These payments represent unrequited transfers to households and the government; from a national

Table 8.1  
Data SAM with National Accounts Details

|             | Labor | Capital-<br>Ag | Capital-<br>Ind | Fac-<br>Abr | HHld-<br>Ru | HHld-<br>Ur | Compa-<br>nies | Govern-<br>ment | Saving | Agri-<br>cult. | Indus-<br>try | R-World |
|-------------|-------|----------------|-----------------|-------------|-------------|-------------|----------------|-----------------|--------|----------------|---------------|---------|
| Labor       |       |                |                 |             |             |             |                |                 |        | 95             | 65            |         |
| Capital-Ag  |       |                |                 |             |             |             |                |                 |        | 30             |               |         |
| Capital-Ind |       |                |                 |             |             |             |                |                 |        |                | 80            |         |
| Fac-Abr     |       |                |                 |             |             |             |                |                 |        |                |               | 5       |
| HHld-Ru     | 80    | 20             |                 |             |             | 5           |                | 10              |        |                |               | 2       |
| HHld-Ur     | 80    | 10             | 25              | 5           |             |             | 20             | 5               |        |                |               | 6       |
| Companies   |       |                | 45              |             | 5           |             |                | 7               |        |                |               |         |
| Government  |       |                | 5               |             | 5           | 15          | 17             |                 |        | 55             | 35            | 5       |
| Saving      |       |                |                 |             | 10          | 15          | 20             | 35              |        |                |               | 20      |
| Agricult.   |       |                |                 |             | 76          | 54          |                | 20              | 15     | 40             | 30            | 70      |
| Industry    |       |                |                 |             | 26          | 54          |                | 60              | 85     | 35             | 60            | 50      |
| R-World     |       |                | 5               |             |             |             | 3              |                 |        | 50             | 100           |         |

accounting point of view, they differ from the factor payments above. In practice, it may be difficult to distinguish the two types of payments when building a SAM because the necessary information is not available.

**Industrial Capital to Rest of the World:** This payment of 5 represent a share of profit that foreigners receive and repatriate because they own a part of the industrial capital.

**Institutions to Companies:** Urban households pays 5 and government pays 7 to companies. These payments do not represent payments for goods or services, since these transactions are recorded in the commodity rows; rather, they are transfers, i.e., interest payments on debt owed to the companies.

**Urban households to Rest of the World:** This payment of 3 is a transfer payment, e.g. to family members abroad. It is not a payment for consumer imports.

The Data SAM does not contain enough information in this case to build the model SAM. As in Chapter 6 we need additional information on the breakdown of indirect taxes into taxes on domestic products, import duties, and export taxes. This is done in Table 8.2.

Table 8.2  
Breakdown of Indirect Taxes

|                              | Agricult. | Industry  |
|------------------------------|-----------|-----------|
| Taxes on Domestic Production | 20        | 10        |
| Import Duties                | 20        | 20        |
| Export Taxes                 | 15        | 5         |
| <b>Total</b>                 | <b>55</b> | <b>35</b> |

We will make some very simple assumptions about the behavior of the new payments discussed above:

**Factor Payments and Remittances from Abroad:** These payments are assumed to be exogenous, measured in foreign exchange. This can be defined with specification FEXO. The equation is  $TSOL(i,j) = FV(i,j) * PSOL(j)$  where FV is exogenous and PSOL(j) is the exchange rate index.

**Industrial Capital to Rest of World:** This payment is modeled as a fixed share of the total income in the industrial capital account, where the share represents the ownership share of the rest of the world.

**Urban Households' Interest Payments and Transfers Abroad:** These payments are both assumed to be fixed percentages of overall income.

**Government's Interest Payment:** This payment is assumed to be exogenous.

The differences in assumptions about interest payments are made to make the model easy to implement, but the exact assumption is probably not important as long as the payments are small.

The model SAM can now be derived from the Data SAM and from these assumptions. It is shown in its GAMS implementation in Appendix 8.1 and it can also be found in file MODEL.F.GMS. There should be no surprises in the SAM and in the specification table. The Account Table contains one new account type, NMF. NMF stands for Non-Market Factor, and it represents a value added account where the underlying 'factor' only has an income; there is no price or quantity associated with it. In this case, the factor is labor and/or capital employed abroad; since we do not want to model the functioning of a foreign factor market, the income is made exogenous (in foreign ex-

change) and there is no associated price or quantity. NMF is also used in connection with specification RENT as we shall see in chapter 10.

The experiment is the same as the one in Chapter 5: world prices for agricultural goods rise by 10% and world prices for industrial goods fall by 10%. We have chosen this experiment to get a solution in which relative prices (and especially import and export prices) change a fair amount, so that we can talk about current and constant price aggregates.

### 8.3 Some National Accounts Aggregates

Before we start aggregating numbers from the SAM, it is useful to summarize the which types of payment can exist. This information is given in Table 8.3, that shows the possible payments from one type of account to another. There are eight different account types in HERCULES. We have already seen seven of the eight. The last, TNMF or Taxed Non-Market Factor, will be explained in Chapter 9. For now, we can think of it as equivalent to AC. Table 8.3 shows that out of the 64 combinations of accounts, only 27 can have payments in practice.

Table 8.3  
Possible Payments among Account Types

|       | MF | NMF | INST | INSTC | AC | TNMF | TAX | ROW |
|-------|----|-----|------|-------|----|------|-----|-----|
| MF    |    |     |      |       | X  |      |     |     |
| NMF   |    |     |      |       | X  | X    |     | X   |
| INST  | X  | X   | X    |       |    |      | X   | X   |
| INSTC | X  | X   | X    |       |    |      | X   | X   |
| AC    |    |     |      | X     | X  |      |     | X   |
| TNMF  |    |     |      |       | X  |      |     |     |
| TAX   | X  | X   | X    |       | X  | X    |     |     |
| ROW   | X  | X   | X    |       | X  |      |     |     |

One of the most frequently used aggregates is Gross Domestic Product (GDP). It comes in two versions, GDP at factor cost and GDP at market prices. GDP at factor cost is defined as total value added in the economy, or as total domestic factor payments. In terms of the possible payments, GDP at factor cost is the sum of the payments in the cells in the MF and NMF rows, and in the AC and TNMF columns as shown in Table 8.4.

Table 8.4  
Cells Defining GDP at Factor Cost

|       | MF | NMF | INST | INSTC | AC | TNMF | TAX | ROW |
|-------|----|-----|------|-------|----|------|-----|-----|
| MF    |    |     |      |       | +  |      |     |     |
| NMF   |    |     |      |       | +  | +    |     |     |
| INST  |    |     |      |       |    |      |     |     |
| INSTC |    |     |      |       |    |      |     |     |
| AC    |    |     |      |       |    |      |     |     |
| TNMF  |    |     |      |       |    |      |     |     |
| TAX   |    |     |      |       |    |      |     |     |
| ROW   |    |     |      |       |    |      |     |     |

Table 8.5  
Cells Defining GDP at Market Prices

|       | MF | NMF | INST | INSTC | AC | TNMF | TAX | ROW |
|-------|----|-----|------|-------|----|------|-----|-----|
| MF    |    |     |      |       |    |      |     |     |
| NMF   |    |     |      |       |    |      |     |     |
| INST  |    |     |      |       |    |      |     |     |
| INSTC |    |     |      |       |    |      |     |     |
| AC    |    |     |      |       | +  |      |     | +   |
| TNMF  |    |     |      |       |    |      |     |     |
| TAX   |    |     |      |       |    |      |     |     |
| ROW   |    |     |      |       |    |      |     |     |

One of the definitions of GDP at market prices is total domestic final use plus exports (f.o.b.) minus imports (c.i.f.). Total domestic final use is the sum of the payments in AC rows and INSTC columns, exports (f.o.b.) is the sum of the payments in AC rows and ROW columns, and imports (c.i.f.) is the sum of the payments in ROW rows and AC columns. The position of the relevant cells and their sign is shown in Table 8.5.

Net indirect taxes are defined as the sum of the payments in TAX rows and AC and TNMF columns. This definition means that the balancing equation (column sum

= row sum) for the combined AC and TNMF accounts becomes one of the well known national accounting identities:

$$\begin{aligned} & \text{Total value added} + \text{Net indirect Taxes} + \text{Imports (c.i.f.)} + \text{Intermediates} \\ & = \text{Total final use} + \text{Exports (f.o.b.)} + \text{Intermediates} \end{aligned}$$

or after cancelling intermediates and using the GDP definitions above:

$$\text{GDP at factor costs} + \text{Net indirect taxes} = \text{GDP at market prices}$$

Table 8.4  
The GDP Summary Table generated by HERCULES

|                       | GDP SUMMARY |                |                 |             |
|-----------------------|-------------|----------------|-----------------|-------------|
|                       | BASE        | SOLUTION       |                 |             |
|                       |             | CURRENT PRICES | CONSTANT PRICES | PRICE INDEX |
| GDP AT FACTOR COST    | 270.000     | 285.818        | 270.000         | 1.059       |
| NET INDIRECT TAXES    | 90.000      | 93.421         | 90.852          |             |
| PRICE EFFECT          |             |                | -.521           |             |
| FINAL USE             | 390.000     | 402.317        | 395.438         | 1.017       |
| EXPORTS               | 120.000     | 123.878        | 117.772         | 1.052       |
| IMPORTS               | -150.000    | -146.955       | -152.879        | .961        |
| GDP AT MARKET PRICES  | 360.000     | 379.239        | 360.331         | 1.052       |
| TERMS OF TRADE        |             |                | 11.100          |             |
| GROSS DOMESTIC INCOME | 360.000     | 379.239        | 371.431         |             |
| RESOURCE GAP          | 30.000      | 23.077         | 24.008          |             |

HERCULES can classify all payments using the account TYPES and it automatically computes and displays the aggregates mentioned above in the format shown in Table 8.6. The first column of the table, labeled 'BASE', shows the aggregates computed from the base SAM and the second column, labeled 'CURRENT PRICES', shows the same numbers computed from the solution SAM. The third column, labeled 'CONSTANT PRICES', shows the results of aggregating the constant price payments of the solution, i.e., the cell values in the QCSOL plane of the cell table. Remember that we defined payments of indirect taxes at constant prices as the tax payments that would take place on the basis of current quantities and base period prices, and therefore also at base period tax rates. We therefore also have a figure for net indirect taxes at constant prices.

The constant price SAM does not necessarily balance if relative prices have changed; as mentioned in Chapter 3, the differences between row and column totals are reported as 'RESIDUAL' in the account table. The national accounting identity

$$\text{GDP at factor costs} + \text{Net indirect taxes} = \text{GDP at market prices}$$

was derived from the accounting identities of the current price SAM, and is therefore not necessarily satisfied at constant prices. The difference is the sum of the RESIDUALS in the AC and TNMF accounts, and is here called the price effect; it is shown in an extra row labeled 'PRICE EFFECT' in the constant price column of the GDP summary table. So in constant prices we have the identity

$$\text{GDP at factor costs} + \text{Net indirect taxes} + \text{Price effect} = \text{GDP at market prices}$$

A few comments about these constant price aggregates are needed. First, they do not depend on an underlying model as long as all factors have prices. Given any new SAM,  $TNEW(i,j)$ , and a set of price indices,  $P(i)$ , we can compute  $QCSOL(i,j)$  as  $TNEW(i,j)/P(i)$ , or with a similar equation based on base period tax rates for constant price tax payments. From  $QCSOL$  we can easily derive the constant price aggregates.  $TNEW(i,j)$  and  $P(i)$  need not be derived from a model; they could equally well be observed values from a real economy.

Second, in the type of model used here, with fixed quantities of all factors, GDP at constant factor prices will always be the same, independently of policy changes. GDP at constant factor prices is simply the sum of the quantities of the factors, with the quantity index based on unit prices in the base case.

Third, GDP at constant factor cost is often computed using the so-called double deflation method. In this method GDP at constant factor cost is computed as gross output of production deflated by their individual output prices minus the sum of intermediate inputs deflated by their respective prices. This double deflated GDP at constant factor cost is equal to GDP at constant factor prices as defined here, plus the price effect. The double deflation method avoids measurement problems with factors, especially rent on capital, and GDP at constant market prices becomes by definition equal to GDP at constant factor cost plus net indirect taxes without an adjustment term; on the other hand, the double deflation method cannot distribute GDP at constant factor cost across factors while this is easy with the method used here since explicitly recognizes changes in factor use.

Fourth, note that there can be RESIDUALs in accounts other than the AC and TNMF accounts, namely in INSTC accounts, but they are not included in the price effect as defined here.

Finally, we could have defined total final use in constant prices as the sum of QSOL over INSTC accounts rather than using the definition employed here, the sum of QCSOL in the individual cells. The definition based on QSOL has the advantage of being more in line with consumers' utility; it does, however, depend on an underlying model and we have therefore chosen not to use it. The difference between these two definitions of final use is a second type of price effect, equal to the sum of the RESIDUALs in the INSTC accounts.

The GDP summary table contains some price indices or GDP deflators in a fourth column. The price indices are computed as the ratio of the current price aggregates over the corresponding constant price aggregates. ✓

The last rows of the GDP summary table are related to foreign trade, and they are only included if the model contains trade. The terms of trade is defined in HERCULES as

$$\begin{aligned} \text{terms of trade} &= \text{export at current prices} / \text{import price deflator} \\ &- \text{export at constant prices} \\ &= \text{export at constant prices} * \\ &\quad (\text{export price deflator} / \text{import price deflator} - 1) \end{aligned}$$

and it is a measure of how export prices have changed relative to import prices, and it is therefore only defined in constant prices -- the terms of trade in current prices would by definition be zero. A positive terms of trade indicates that export prices are higher than import prices.

Gross domestic income is defined as GDP at market prices plus terms of trade. It is often a better measure of the resources of an economy than GDP at constant prices because it includes the positive or negative benefits of changes in prices in the surrounding world.

The resource gap is defined as imports minus exports. The resource gap at constant prices is defined as imports at constant prices minus exports at constant prices plus terms of trade, i.e.

$$\begin{aligned} \text{Resource gap at constant prices} &= \text{imports at constant prices} - \\ &\quad \text{exports at constant prices} * \text{export price deflator} / \\ &\quad \text{import price deflator} \end{aligned}$$

There are several other national accounting aggregates such as Gross National Product (GNP) and Gross National Income (GNI) that can be derived from the SAM using information on account TYPES:

$$\begin{aligned} \text{GNP} &= \text{GDP} + \text{net factor payments from abroad} \\ &= \text{GDP} + \text{payments in NMF rows and ROW columns} \\ &\quad - \text{payments in ROW rows and MF and NMF columns} \end{aligned}$$

and

$$\begin{aligned} \text{GNI} &= \text{GNP} + \text{net transfers from abroad} \\ &= \text{GNP} + \text{payments in INST and INSTC rows and ROW columns} \\ &\quad - \text{payments in ROW rows and INST and INSTC columns} \end{aligned}$$

In the base case we get  $\text{GNP} = 390 + 5 - 5 = 390$  and  $\text{GNI} = 390 + 13 - 3 = 400$  starting from GDP at market prices. Both GNP and GNI include purely monetary payments to and from the rest of the world with no natural price deflators. The current price aggregates are well defined, but there are several possible definitions of the constant price aggregates, so we have chosen not to make these summary statistics a part of HERCULES.

#### 8.4 Advanced GAMS Report Features

We will regenerate the GDP summary table in Table 8.4 using GAMS statements to show some of the GAMS system's more powerful features. Before looking at the GAMS code, let us define some of the computations in words. For example, GDP at factor cost in the base case is computed as the sum of TBASE over the cells (LABOR,VAL-ADD-AG), (CAPITAL-AG,VAL-ADD-AG), (LABOR,VAL-ADD-IN), and (CAPITAL-IN,VAL-ADD-IN). Similarly, GDP at factor cost in current prices in the solution is the sum of TSOL over the same cells, and GDP at constant factor cost is the sum of QCSOL over the same cells. Exports in the base is the sum of TBASE over the cells (COM-EXP-AG,REST-WORLD) and (COM-EXP-IN,REST-WORLD), and exports in the solution in current and constant prices is computed by summing TSOL or QCSOL over the same cells.

We could implement all these sums in GAMS by writing them one at a time:

```

GDPTAB('GDP-FC', 'BASE-CASE') -
  CT('LABOR', 'VAL-ADD-AG', 'TBASE') + CT('CAPITAL-AG', 'VAL-ADD-AG', 'TBASE') +
  CT('LABOR', 'VAL-ADD-IN', 'TBASE') + CT('CAPITAL-IN', 'VAL-ADD-IN', 'TBASE');
GDPTAB('GDP-FC', 'CURRENT') -
  CT('LABOR', 'VAL-ADD-AG', 'TSOL') + CT('CAPITAL-AG', 'VAL-ADD-AG', 'TSOL') +
  CT('LABOR', 'VAL-ADD-IN', 'TSOL') + CT('CAPITAL-IN', 'VAL-ADD-IN', 'TSOL');

```

etc. This is rather lengthy, and it can be made more compact. First note that BASE-CASE is always associated with TBASE, CURRENT is always associated with TSOL, and CONSTANT is always associated with QCSOL. To take advantage of this structure, we create two sets, HD with the labels BASE-CASE, CURRENT, and CONSTANT, and CLAB with the labels TBASE, TSOL, and QCSOL. We then define a mapping or two dimensional set, HMAP, that describe the relationship between the two sets:

```

SET HD / BASE-CASE, CURRENT, CONSTANT /
CLAB / TBASE , TSOL , QCSOL /
HMAP(HD, CLAB) / BASE-CASE.TBASE
                  CURRENT .TSOL
                  CONSTANT .QCSOL /

```

The dots (.) between the labels define them to be pairs as opposed to the commas (,) that are used as separators.

With this map we can write all the equations for GDP at factor cost as one:

```

GDPTAB('GDP-FC', HD) - SUM(CLAB $ HMAP(HD, CLAB),
  CT('LABOR', 'VAL-ADD-AG', CLAB) + CT('CAPITAL-AG', 'VAL-ADD-AG', CLAB) +
  CT('LABOR', 'VAL-ADD-IN', CLAB) + CT('CAPITAL-IN', 'VAL-ADD-IN', CLAB) );

```

The left hand side is driven by the set HD which means that the assignment statement is executed once for each element in HD, i.e. for BASE-CASE, CURRENT, and CONSTANT, so we get three versions of GDP at factor cost. The trick appears with the \$- or 'such that'-operator on the right hand side

```

SUM(CLAB $ HMAP(HD, CLAB), ... ),

```

which reads as 'Sum over all elements in CLAB such that the pair of HD and CLAB elements is in the two-dimensional set HMAP'. When HD represents BASE-CASE, this condition is only satisfied by TBASE so the sum is only over this element. Similarly, when HD represents CURRENT, only TSOL is valid. So HMAP selectively picks out the relevant plane from the cell table.

There are still many aggregates, each a sum over a subset of the cells of the cell table. If the aggregates are only needed once, it may be easiest to write each of them

explicitly as shown above. If the same aggregates are needed many times, however, it may be worth while to define the aggregation once and then to use it repeatedly. This can also be done with mappings and the \$-operator. We first define the labels of the aggregates

```
SET AL / GDP-FC, NET-I-TAX, FINAL-USE, EXPORT, IMPORT /
```

and we can then define a mapping between these aggregates and the cells of the cell table

```
SET LMAP(AL,ACC,ACC) MAPPING FROM TABLE LINES TO CELLS /
  GDP-FC   .(LABOR      .VAL-ADD-AG,
             CAPITAL-AG.VAL-ADD-AG,
             LABOR      .VAL-ADD-IN,
             CAPITAL-IN.VAL-ADD-IN),
  NET-I-TAX .(INDR-TAX  .COM-DOM-AG,
             INDR-TAX  .COM-DOM-IN,
             INDR-TAX  .COM-IMP-AG,
             INDR-TAX  .COM-IMP-IN,
             INDR-TAX  .COM-EXP-AG,
             INDR-TAX  .COM-EXP-IN),
  FINAL-USE .(COM-CMP-AG.HOU-RU-CO ,
             COM-CMP-AG.HOU-UR-CO ,
             COM-CMP-AG.GOVERN-CO ,
             COM-CMP-AG.SAVING-INV,
             COM-CMP-IN.HOU-RU-CO ,
             COM-CMP-IN.HOU-UR-CO ,
             COM-CMP-IN.GOVERN-CO ,
             COM-CMP-IN.SAVING-INV),
  EXPORT   .(COM-EXP-AG.REST-WORLD,
             COM-EXP-IN.REST-WORLD),
  IMPORT   .(REST-WORLD.COM-IMP-AG,
             REST-WORLD.COM-IMP-IN) /
```

The parentheses in the definition of the three dimensional mapping are important. GDP-FC is mapped with the four pairs within the first set of parentheses, NET-I-TAX is mapped with the six pairs within the next set of parentheses, etc. Parentheses can be interpreted as 'For each element or each pair inside do ...'. Parenthesis can be nested and the mapping above can therefore be written much more compactly using the structure of the cells as follows:

```
LMAP(AL,ACC,ACC) MAPPING FROM TABLE LINES TO CELLS /
  GDP-FC   .((LABOR,CAPITAL-AG).VAL-ADD-AG,
             (LABOR,CAPITAL-IN).VAL-ADD-IN),
```

```

NET-I-TAX . INDR-TAX . (COM-DOM-AG , COM-DOM-IN ,
                        COM-IMP-AG , COM-IMP-IN ,
                        COM-EXP-AG , COM-EXP-IN) ,
FINAL-USE . (COM-CMP-AG , COM-CMP-IN) . (HOU-RU-CO , HOU-UR-CO ,
                                         GOVERN-CO , SAVING-INV) ,
EXPORT    . (COM-EXP-AG , COM-EXP-IN) . REST-WORLD ,
IMPORT    . REST-WORLD . (COM-IMP-AG , COM-IMP-IN) /

```

GDP-FC is mapped with the two rows LABOR and CAPITAL-AG and the column VAL-ADD-AG, and with the two rows LABOR and CAPITAL-IN and the column VAL-ADD-IN. Similarly, NET-I-TAX is mapped with row INDR-TAX and each of the six columns shown in the parentheses. And FINAL-USE is mapped with all eight combinations of COM-CMP-AG or COM-CMP-IN as rows and HOU-RU-CO, HOU-UR-CO, GOVERN-CO, or SAVING-INV as a column.

After defining the mapping between aggregates and cells, we can write all the aggregations in one GAMS statement

```

GDPTAB(AL,HD) = SUM( (ACC,ACCP) $ LMAP(ALINES,ACC,ACCP) ,
                    SUM(CLAB $ HMAP(HD,CLAB) , CT(ACC,ACCP,CLAB) ));

```

The left hand side is driven by both AL and HD, which means that there is one assignment for each combination of elements in AL and HD. The right hand side has two SUMs and two \$-conditions. The first or outer SUM is over two sets, (ACC,ACCP), which means that the sum is over all combinations of elements in ACC and ACCP or over all cells, conditional on ACC and ACCP matching up with the current element in AL according to LMAP. So for each GDP aggregate we only sum over the cells that belong to that aggregate. The expression we sum over is the second or inner SUM over CLAB. This sum is similar to the one we discussed first; it simply picks the plane in the cell table that matches with the element in HD.

The rest of the numbers in the GDP summary table are computed with the statements shown in line 278-293 of the GAMS listing. There is less structure in these computations than in the previous ones, with minus and division operators, so we have not attempted to make the computations more compact. The table created by these statements is shown in Table 8.7; apart from details of appearance it is equivalent to Table 8.6.

Table 8.7  
The GDP Summary Table computed from the  
solution using GAMS statements

| ---- 294 PARAMETER GOPTAB | GDP SUMMARY TABLE |           |          |            |
|---------------------------|-------------------|-----------|----------|------------|
|                           | BASE-CASE         | CURRENT   | CONSTANT | PRICE-INDX |
| GDP-FC                    | 270.000           | 285.818   | 270.000  | 1.059      |
| NET-I-TAX                 | 90.000            | 93.421    | 90.852   |            |
| PRICE-EFF                 |                   | 1.0179E-5 | -0.521   |            |
| FINAL-USE                 | 390.000           | 402.317   | 395.438  | 1.017      |
| EXPORT                    | 120.000           | 123.878   | 117.772  | 1.052      |
| IMPORT                    | 150.000           | 146.955   | 152.879  | 0.961      |
| GDP-MP                    | 360.000           | 379.239   | 360.331  | 1.052      |
| T-TRADE                   |                   |           | 11.100   |            |
| G-D-INCOME                | 360.000           | 379.239   | 371.431  |            |
| RES-GAP                   | 30.000            | 23.077    | 24.008   |            |

MISCELLANEOUS TOPICS

PART III

## CHAPTER 9

### CLOSURE RULES

#### 9.1 Introduction

The question addressed in this chapter is: "Which prices and quantities must be made exogenous in order to get a model with the same number of variables and equations?" This question is often referred to as the question of closure rules. Obviously, the number of variables is equal to the number of equations if their difference is zero. We therefore start in Section 9.2 by discussing how efficiently to count the difference between the number of variables and equations. In Section 9.3 we discuss closure rules in terms of a relatively restricted class of models, and in Section 9.4 we see how some of the restrictions may be lifted. HERCULES will complain if you do not have the same number of variables as equations. These error messages are discussed in chapter 11.

The closure rule discussions are concerned with the functioning of factor markets and with the mechanism that bring balance between savings and investment. To show how some of the concepts involved can work in practice, we implement a model in Section 9.5 and perform some experiments with alternative rules for factor markets. The model has several new HERCULES features. We will work with more than one savings and investment account, and will implement the production system in a new and more compact way using two-level CES functions. We will also introduce a few new GAMS features to do with the creation and use of dynamic sets for summary reports.

The content of this chapter and the example in section 9.5 shows one of the strong properties of HERCULES: the ability to easily experiment with alternative assumptions about the functioning of the economic system. It is customary to make sensitivity analysis by changing values of critical parameters. It is less customary to make sensitivity analysis by changing economic assumptions, simply because it is much harder to do with most modeling systems and algorithms. It turns out, however, that the sensitivity of the results to changes in closure rules often are much larger than the sensitivity to numerical parameter changes.

## 9.2 Counting Variables and Equations

All variables and equations in a HERCULES model are associated with either a cell or an account of the SAM. The best way of counting the number of variables and equations (and especially their difference) is therefore to proceed on a cell-by-cell and account-by-account basis.

We will consider that a cell is e-balanced (or equation balanced) if the number of variables associated with the cell is equal to the number of equations. And we will define the degrees of freedom of a cell as being the number of variables associated with the cell minus the number of equations. E-balanced cells have zero degrees of freedom, and unbalanced cells have positive or negative degrees of freedom. The notion of e-balance is convenient for three reasons. First, cells that are e-balanced will contribute equally to the number of variables and equations, and they can therefore be ignored in the counting process. Second, the degrees of freedom and the property of being e-balanced depend only on the specification of the cell. Third, most specifications give e-balanced cells.

Appendix A contains a description of all cell specifications recognized by HERCULES, together with a description of the variables and equations associated with each of them. All cells contain the variable TSOL, and some of them also contain the variable QCSOL. Most cells have one behavioral equation, and cells with the variable QCSOL will also have an equation that relates QCSOL to TSOL. A careful study of Appendix A will show that very few specifications give their cells positive degrees of freedom. The relevant specifications are UNSPEC, RENT, AVRG, and FAVRG, and the degrees of freedom is +1 for each of them. There is only one specification, FDIST, that gives a cell with a negative degree of freedom, in this case -1. So when counting variables and equations it is sufficient to consider cells with specifications UNSPEC, RENT, AVRG, FAVRG, and FDIST.

Next consider accounts. As with cells, an account is e-balanced if the number of variables associated with the account is equal to the number of equations, and the difference between the number of variables and equations is defined as the degrees of freedom for the account. The process of counting the degrees of freedom of an account is more cumbersome than the process for cells, but the result is equally easy to remember. Accounts with IDIST in the column have +1 degree of freedom, accounts with AVRG, FAVRG, QEXO, and VEXO or a mixture of TEXO, FEXO, and UNSPEC, have -1 degree of freedom, and all other accounts are e-balanced. These rules count all account variables, whether they are endogenous or exogenous. An exogenous PSOL, QSOL or YSOL must therefore be counted as an additional equation.

**Table 9.1**  
**Degrees of Freedom Associated with**  
**various Specifications**

|          | Plus One                        | Minus One                                |
|----------|---------------------------------|--|
| Cells    | UNSPEC<br>RENT<br>AVRG<br>FAVRG | FDIST                                    |
| Accounts | IDIST                           | QEXO<br>VEXO<br>TEXO, FEXO and/or UNSPEC |
| FIX      |                                 | P, Q, Y, or PQ                           |

### 9.3 Closure Rules in a Simple Model

We will start by examining a class of models with the following common assumptions:

- The production systems in the AC accounts are CES (or one of its special forms such as IO, CD, or SUBST), and there are sufficient substitution possibilities between factors, so that one factor will not be unemployed because of the lack of another factor, i.e. there are no bottlenecks that cannot be relieved through substitution. There may also be IMPORT and ITAX specifications in the columns of AC accounts, but no fixed prices or quantities are associated with the output of AC accounts.
- All factors have prices, i.e., they are of the type MF, and all transfers of income from factor accounts use IDIST.
- There is one combined savings/investment account for the whole economy. The cell specifications in the column are QSHR or VSHR, and total investment can be exogenous or endogenous.
- The government behaves in one of two ways: either it operates just like any other institution (see description below), or its consumption is exogenous (QEXO or VEXO), and its transfers to other institutions are exogenous (TEXO or FEXO),

except for its transfers to the consumption and savings accounts, both of which are residuals, UNSPEC.

- All other institutions save and transfer a fixed percentage of their income and consume the rest in accordance with consumption system QSHR, VSHR, or LES.
- There is only one rest of the world account. All transfers from the rest of the world to institutions is exogenous (in domestic currency, TEXO, or foreign exchange, FEXO). The only exception is transfers to the savings account, i.e. the current account deficit, that can be exogenous (TEXO or FEXO), or a residual (UNSPEC). All transfers to the rest of the world are either exogenous (TEXO or FEXO) or a fixed share of income (IDIST).

These assumptions can be translated directly into comments about e-balanced cells and accounts and degrees of freedom:

- Production system: All cells in the production columns and all production accounts are e-balanced, so the total production system adds no degrees of freedom.
- Factor accounts: All cells in the factor columns are e-balanced, and all factor accounts have one degree of freedom because they have IDIST specifications. There is therefore a total of +1 degree of freedom for each factor account.
- Savings/investment account: All cells in the investment column are e-balanced, and the account itself is e-balanced. Only if QSOL is exogenous do we have one excess equation or -1 degree of freedom associated with the account.
- Government accounts with exogenous transfer and consumption: The two cells that transfer income to the consumption and savings accounts have specification UNSPEC and they therefore have +1 degree of freedom each. All other cells are e-balanced. The government's income and consumption accounts have -1 degree of freedom each because of the mixture of TEXO and UNSPEC, and the VEXO or QEXO specifications. Taken as a whole, therefore, the government accounts have the same number of variables as equations.
- Other institutional accounts: All consumption and transfer cells are e-balanced, as are all the institutional accounts.
- Rest of the world account: All cells except possibly the current account deficit cell are e-balanced, and the account itself is also e-balanced. The current account cell has +1 degree of freedom if it is residual (UNSPEC), and it is e-balanced if it is exogenous (TEXO).

**Table 9.2**  
**Degrees of Freedom Associated with**  
**Sub-components in a Simple Model**

| Subcomponent       | Degrees of Freedom   |
|--------------------|--|
| Production         | Balanced   |
| Factors            | +1 for each account  |
| Savings/Investment | -1 if QSOL is exogenous  |
| Government         | Balanced   |
| Other Institutions | Balanced   |
| Rest-of-World      | +1 if current account is residual (UNSPEC) and balanced if current account is exogenous (TEXO or FEXO) |

Table 9.2 provides a summary of the degrees of freedom associated with each subcomponent in the simple model. From the table one can see that the closure rules will concern (1) the factor accounts and (2) some kind of balance between the savings/investment account and the rest-of-world account.

The overall model must have the same number of variables as equations, and so far the number of variables is greater than the number of equations. The extra assumptions that are added to reconcile this difference are called the closure rules.

First consider the factor accounts which are invariably unbalanced so that we must add one equation per factor. The two obvious assumptions are: (i) the quantity is exogenous or (ii) the price is exogenous. The first says that a given factor is available in a fixed quantity and that price adjusts to ensure that the factor is fully utilized. This assumption, known as the neo-classical closure rule, is the one used in all previous models. The second says that institutional constraints determines the price of a given factor, and that a sufficient quantity of the factor is available at this price to satisfy demand. This assumption is often known as the keynesian closure rule. Both closure rules represent special factor supply equations; other types of factor supply equations depending on relative prices will shortly be included in HERCULES.

The choice of closure rule to apply to individual factor markets depends on institutional arrangements in the economy, the time frame of the analysis, capacity utilization, etc, and will in general be factor specific. In the short run, trade unions will often ensure that wages in formal sectors will be above their market clearing level,

while wages in informal sectors will often be at levels that clear the market. Rental rates in sectors with excess capacity can often be modeled on a fixed rent basis while it is market determined in other sectors.

The behavior of the overall model depends crucially on the choice of closure rule for factor markets. If the quantity is fixed for all factors, then the economy is very constrained. GDP at constant factor cost will be constant under all policies, and GDP at constant market prices will only change with changes in net indirect taxes (at base period tax rates) and with price effects -- in other words, as a result of changes in the sectoral composition of supply and demand. On the other hand, if all prices are fixed we have an economy with no constraints from the factor or supply side. All prices in the model can in fact be determined independently of the level of activity, and this type of model is therefore called a fix-price model. The overall level of activity will be determined exclusively from the exogenous components of demand.

Next, we need a closure rule for savings, investment, and the current account deficit. There are two standard closure rules for balancing the rest of the model. In the first, investment is exogenous and the current account deficit is a residual that clears the savings/investment balance. This type of model is called investment driven. In the second, the current account deficit is exogenous and investment adjusts to the amount of available savings. This type of model is called savings driven.

The choice of closure rules for factor markets is not completely independent of the closure rule for savings/investment behavior. In a fix-price model we need at least one exogenous quantity to determine the overall level of activity in the economy; this exogenous quantity can only be government consumption or investment, so within the restricted set of models considered here the model must either be investment driven, or government consumption must be exogenous, or both.

The equation that fixes the numeraire is often discussed at the same time as closure rules. Recall that if a factor price (such as a wage) is exogenous, its value must be fixed relative to the numeraire. The choice of numeraire is often in turn related to the modeling of savings/investment behavior. The implicit assumption in models with residual current account deficits is that the economy is open, and in this case it makes sense to use the exchange rate as the numeraire. All prices will then be measured relative to world prices and the domestic price level appears to be based on a real foundation. But note that its overall level depends completely on the value of the numeraire.

When the current account deficit is exogenous, i.e., when the amount of foreign exchange that can be borrowed is fixed, then we will often assume that the exchange rates is floating or endogenous and that it adjusts to equilibrate the supply of foreign

exchange (mainly obtained from exports) with demand for it (mainly to pay for imports). Nevertheless, we still need a numeraire, and a domestic consumer price index is often used for this purpose. In this case we must be very careful when interpreting prices. The domestic price level is essentially exogenous, and although world prices are exogenous in foreign exchange, they become endogenous in domestic currency. The exchange rate is the ratio of the exogenous domestic price level to the exogenous world price level. Note that we could also have used the exchange rate as the numeraire in the case of the exogenous current account deficit; instead of having a floating exchange rate, we would have a floating domestic price level, measured relative to a world price level. The ultimate choice of the numeraire often depends on which other prices are fixed. If a wage rate is fixed relative to a consumer price index, then it is obviously easier to use the latter as the numeraire.

#### 9.4 Extensions to the Closure Rules

The analysis above has been based on a very simple model with restrictive assumptions. It is worth adding a few comments to the restrictions that can be relaxed without changing the essence of the closure rule discussion.

The rental rate paid to capital has hitherto been assumed to be equal to the value of its marginal product. In cases with excess capacity this model may not describe reality very well, and it may be preferable to model the rental rate as being fixed. Sometimes, however, it may be appropriate to model the rental payment to capital as a fixed percentage of the output of the relevant sector, regardless of the size of the capital stock. This is modeled in HERCULES with specification MARKUP. Since the payment is independent of the size of the capital stock, it is more realistic to leave out of consideration both the capital stock and any rental rate for it and instead to include only an income figure. This type of 'factor' is in HERCULES called a Non Market Factor and its type is NMF. The name Factor is used because it is a value added account, and Non Market indicates that there is no market or market price for the input. The term used in this book for ordinary factor accounts, i.e., MF or Market Factor, was chosen to highlight the distinction between the two types of factor accounts.

A Non Market Factor account does not need a closure decision because the number of variables associated with the account is always equal to the number of equations. In any event, NMF accounts do not have a price and quantity that can be fixed.

The assumptions about the institutional behavior in our simple model can also be relaxed. The essential assumption was the equation-balance for each group of institutions, where a group could consist of more than one account, e.g. an income and

transfer account and a consumption account. And the savings/investment account was only handled separately because the mechanism used to balance it could be different from that of other institutions.

The first relaxation of our initial assumptions in the institutional area is that one or more transfers from an institution can be made exogenous if one of the transfers for the same institution is a residual that adjusts to maintain the accounting balance.

The second relaxation relates to the number of savings and investment accounts. So far we have always had one combined savings and investment account for the whole economy, but it is possible to develop a more detailed model of the relationship between the individual components of savings and investment. This will require several additions to our simple model. First, we will need individual accounts to collect the savings from different groups of institutions, e.g. a household savings account, a company savings account, and a government savings account. We will also need several investment accounts, e.g. one for households that essentially invests in housing, one for companies that invest in productive capacity, and one for the government that invests in infrastructure, schools, etc.: each investment accounts will of course buy different goods. Finally, we will need a 'flow of funds matrix' between the savings and investment accounts, to show how savings are channeled to the different investor institutions. For this whole structure -- the savings and various investment accounts, the flow of funds matrix, and the associated investment allocation rules -- we will need consistent base case data (i.e., the data for the SAM), together with a set of behavioral assumptions.

The behaviors underlying the flow of fund matrix and the assumptions about investment determine overall savings and investment patterns, and will vary with the nature of a particular economy's capital market and institutional structure: there are no hard and fast rules for any "best" formulation. A savings account can allocate its funds in fixed ratios to investors using specification IDIST; an investment account can have an endogenous quantity, i.e., be savings driven; an investment account can have an exogenous quantity, i.e. be investment driven, and one of the savings allocations must then adjust as a residual (UNSPEC); certain components of savings allocation can be modeled as being exogenous (TEXO), and one must then adjust as a residual (UNSPEC). In general, for each account with exogenous investment we need one residual savings (UNSPEC), possibly coming from the rest of the world, and for each savings account with some exogenous allocations we also need one residual. The overall model can be completely savings driven, completely investment driven, or a mixture of the two. In practice, households' and/or companies' investments are often determined by available savings while government investment is exogenous.

## 9.5 An Example

The model related to this chapter is shown in Appendix 9.1 and it can also be found in file MODELG.GMS. It has new features in two areas. The first is related to the production system, in which the new MARKUP specification is used and the familiar CD and IO specifications for capital labor substitution and intermediate inputs are presented in a new way. The second is in connection with savings and investment, where we have introduced three savings accounts, three investment accounts, and a flow of funds matrix.

The industrial sector in the example is assumed to produce below capacity and capital is paid a fixed share of the total value of output. The specification for capital is therefore MARKUP. Labor is no longer substitutable with capital so we must introduce some other demand mechanism for labor. We will assume a fixed labor requirement per unit of output, and the specification becomes IO, just as it is for intermediate inputs. Using these model assumptions, there is no need to split production into a value added account and an activity account as we did in earlier chapters, so we only include the activity account in the model.

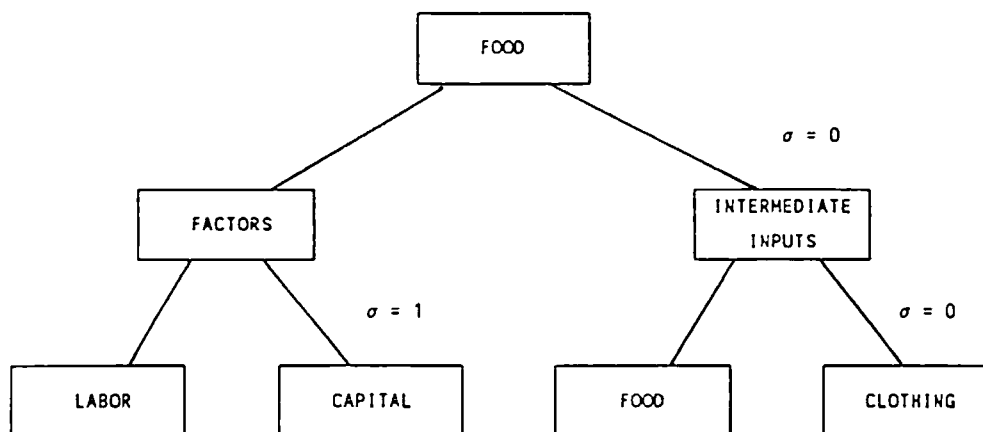
The combination of IO and MARKUP in the industrial activity column implies that the marginal cost of producing industrial goods and therefore the price of industrial goods, given fixed prices of labor and intermediate goods, is constant. The price will only change if the prices of labor or intermediate goods change.

In agriculture we keep the old formulation with a Cobb-Douglas aggregation of labor and agricultural capital, and a Leontief aggregation of intermediate inputs and the combined value added. Instead of writing this with the usual two accounts, we have used a powerful HERCULES specification, the two level CES function, CES2, to keep everything within one account.

The specification of the production function is shown in Figure 9.1 with an elasticity of substitution of zero between factors and intermediate inputs. Substitution between capital and labor is permitted ( $\sigma=1$ ) in forming the aggregate factor input, while substitution between intermediate inputs is not permitted ( $\sigma=0$ ).

The two level CES function is declared with the same CES2 specification for all inputs. The grouping of the inputs shown in Figure 9.1 is described to HERCULES through numerical elasticity values. Associated with the account is a SIGMA parameter exactly as with accounts with CES specifications. The SIGMA value in a CES2 account describes the substitution elasticity between groups of inputs, in this case between the intermediate inputs and the combined factor inputs. The value in the ACT-AGRCLT row is EPS. This value is used instead of zero (because GAMS always eliminates

Figure 9.1  
Two Level CES Production Function  
Specification in Agriculture



zeroes) to show that there is no substitution between groups.

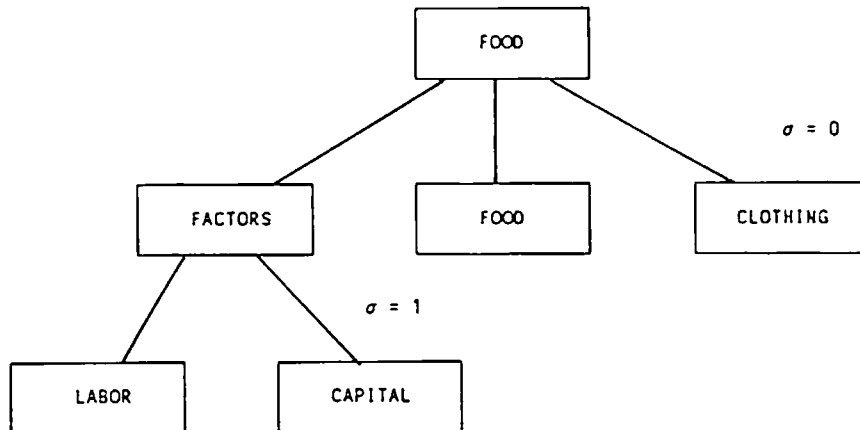
Associated with each cell is a SIGMAC parameter, which is defined in lines 208-209 and moved to the cell table in line 219. SIGMAC is 1 for the two factor inputs and EPS for the two intermediate inputs. SIGMAC contains the elasticities within the groups, and the groups are identified by having the same SIGMAC. So labor and capital belong to one group and the substitution elasticity within this group is 1, corresponding to a Cobb-Douglas formulation. Intermediate inputs have a common SIGMAC and are therefore grouped together in a second group.

It turns out, that group inputs can be considered direct inputs at the top level whenever SIGMAC is equal to SIGMA. So with the numerical values used here the two level production function can equally well be represented as in Figure 9.2.

The fixed quantity of sector specific capital in agriculture, combined with the Cobb-Douglas aggregation with labor and the Leontief aggregation with intermediate inputs, implies that additional production of agricultural goods require a more than proportionate growth of labor. The marginal cost of producing agricultural goods will therefore increase with the production level, even if the prices of labor and intermediate inputs are fixed.

The institutional part of the SAM contains three savings accounts (one for households, one for companies, and one for the government), and three investment accounts (one each for the same three groups). Savings are treated as they have been in previous

**Figure 9.2**  
**Alternative Representation of the**  
**Two Level CES Production Function**  
**Specification in Agriculture**



cases: households and companies save a fixed share of their overall income, and the government saves whatever is left after its exogenous expenditures have been paid.

The flow of funds matrix and the corresponding part of the specification matrix (the investment rows and the savings columns) show that households allocate fixed shares of their savings to the household investment account and the government's investment account (e.g. by buying government bonds). Similarly, companies allocate a fixed share of their savings to their own investment and a fixed share is used to buy government bonds. The government passes all its own savings on to its investment account, borrows parts of the savings of households and companies, and also borrows from the rest of the world. Households and companies invest the fixed share of their savings not used to buy government bonds, i.e. their investment is savings determined. The government's investments are exogenous and borrowing from the rest of the world, i.e. foreign savings, is used to balance savings with investments.

The experiment shown in the appendix investigates the effect of increasing the government's investment by 20 percent, from 65 to 78. The first round effect is to create extra demand in the economy. The marginal costs of producing extra industrial goods, given the prices of inputs, is constant, and the marginal cost of producing extra agricultural goods rises. We will therefore see a shift in labor and production towards the industrial sector, industrial capital will receive more profits, and overall GDP at factor cost will grow from 270 to 278.224 in current prices and to 272.615 in constant prices.

Since investment grows more than production, the current account deficit increases, but by less than the amount of investment.

The results above are partly explained by the scarcity of labor. Overall demand increases, but agricultural production falls. In the second experiment, we change the closure rule for the labor market. We assume that there is sufficient labor, that the wage rate is fixed, and that the quantity of labor is not fixed. This is defined in line 243 of the GAMS listing where the FIX column in the labor row of the account table is set to P. The result is that both agricultural and industrial production increase. Marginal costs in agriculture are still an increasing function of production while marginal costs in industry are constant, so industrial production still grows faster than agricultural output. The current account deficit grows more slowly than in the first experiment.

In the third experiment, we change the closure rule for agricultural capital and assume that the rental rate is fixed and the quantity of employed capital is endogenous. This is defined in line 251 where the FIX column in the agricultural capital row of the account table is set to P. Since AT("LABOR","FIX") has not been changed, labor is still assumed to have a fixed wage. All factors have fixed prices, so our model is now a fix-price model and all marginal costs are constant. Both agricultural and industrial production grow faster than before, and the current account deficit grows more slowly than in the first two experiments.

The report statements of the model show one new GAMS feature. We are interested in the final use of all consuming institutions, i.e. QSOL for all accounts of type INSTC, and in total final use, in a form like Table 9.3. For this we need a set, CON, with the relevant consuming institutions, and instead of typing it, we have created it with an assignment statement

```
CON(ACC) $ (AT(ACC,"TYPE") EQ INSTC) - YES;
```

We saw dynamic definitions of sets in Chapter 8. The statement

```
CON(ACC) - YES;
```

would add the elements of ACC to the set CON. The \$ or 'such that' operator applied to the left of the equal sign restricts the elements of ACC for which an assignment is performed, which means that CON is changed only for elements of ACC with TYPE INSTC. The following statement

```
CON(ACC) - YES $ (AT(ACC,"TYPE") EQ INSTC);
```

is also legal, but it is slightly different. An assignment is made for all elements in ACC, and the assigned value is YES (i.e., include the element) if the element has TYPE INSTC, and the assigned value is default or NO (i.e., remove the element) if the element has a different TYPE. If CON is empty to start with, then the two statements give the same result. However, the second version could remove elements that the first version would not touch.

The first dimension of the report table, RQ, is not domain checked, as shown in line 225. The reason is that we do not have a suitable set available. The domain is the union of the dynamic set CON and the label "TOTAL", which could have been defined as another dynamic set. GAMS does not allow dynamic sets as domains, however, because all domain checking is done before any of the assignment statements are evaluated.

**Table 9.3**  
**Report over Consumption for the Base Case and for**  
**the Experiment Case using three different sets**  
**of Closure Rules of the model**

| ----       | 256 PARAMETER RQ |            | CONSUMPTION LEVELS FOR ALL CONSUMING INSTITUTIONS |            |
|------------|------------------|------------|---|------------|
|            | BASE-CASE        | BASE-SOLUT | FIXED-WAGE  | FIX-WG+RNT |
| HOUSEHLD-C | 190.000          | 193.056    | 196.046   | 196.983    |
| GOVERNMT-C | 80.000           | 80.000     | 80.000  | 80.000     |
| INVEST-HH  | 20.000           | 20.330     | 20.653  | 20.735     |
| INVEST-COM | 25.000           | 25.974     | 26.218  | 26.289     |
| INVEST-GOV | 65.000           | 78.000     | 78.000  | 78.000     |
| TOTAL      | 380.000          | 397.360    | 400.917   | 402.008    |

## CHAPTER 10

### PRICE AND QUANTITY CONTROLS

#### 10.1 Introduction

In the previous chapter we looked at fixed variables in connection with factor markets and the institutional part of the model. In this chapter we will look at the additions we must make if certain variables in the activity/commodity part of the model are made exogenous, for example if the government imposes price or quantity controls.

The examples we have seen so far have been based on general equilibrium principles similar to the models described in Dervis et.al. (1982). There are in practice always disequilibria in different parts of the economic system, and analysis of the effect of these disequilibria is an important use of economy-wide models, e.g. Ahmed et.al. (1985) and Grais et.al. (1986). This chapter will introduce tools for analyzing the effect of disequilibria, and you will see that adding or removing disequilibria requires a minimal amount of work. This is, like the flexibility in changing closure rules, considered to be one of the strong properties of HERCULES.

From an economic point of view, making a price or quantity exogenous is equivalent to disrupting the supply/demand balance for the corresponding market. The material balance identity must still be satisfied, and the economic assumptions underlying the model must tell us whether supply, demand, or something else adjusts.

From a mathematical point of view, making a variable exogenous is equivalent to adding an equation to the model. In order to maintain the balance between the number of variables and equations it is therefore necessary to remove an equation, or to add another endogenous variable.

In previous chapters we have used complete models as examples. In this chapter we have several examples, but for each we will only show part of a model, since a complete model for each example would take too much space. We have, however, included files with all examples with the HERCULES system.

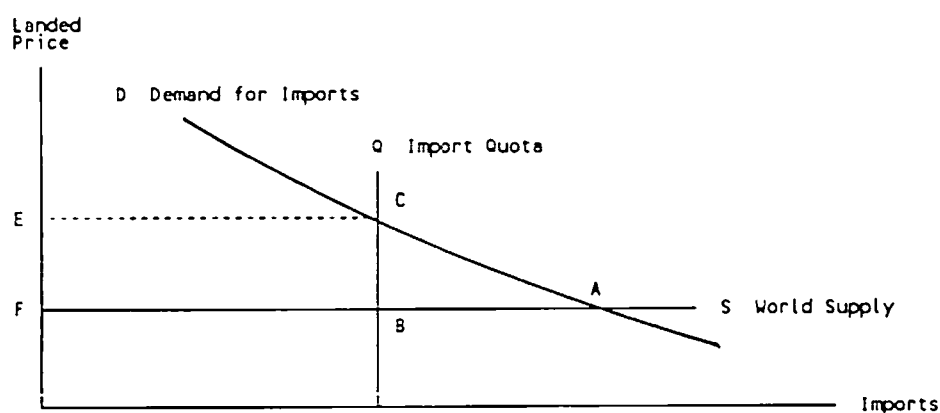
For each example of an exogenous price or quantity we will start with a model in which the variable is endogenous (usually one of the models from the previous chapters), and we will show the relevant components of the model before and after the change. In the next four sections, we will discuss (1) an example in which quantitative restrictions are imposed on imports, (2) two examples with fixed prices but different

quantity adjustment mechanisms, and (3) an example involving the pricing of a natural resource such as oil, where the price is determined independently of the production cost. Most of the assumptions in these sections on how the economic system reacts on fixed prices or quantities are based on marginal cost or marginal utility. The last section will discuss a special specification, AVRGE, that often can be used if decisions are not based on marginal costs.

## 10.2 Quantity Restrictions for Imports

In our previous models the quantity of imports into a particular sector has been determined by normal supply and demand forces, as indicated by point A in Figure 10.1. Supply from the rest of the world is perfectly elastic at the going world price, and import demand is modeled as the overall demand for goods from the sector multiplied by an import share. Overall demand is modeled indirectly, through modeling of consumption demand and the demand for intermediate inputs; the import share is modeled as a function of the landed price of imports relative to the price of comparable domestic goods through a CES aggregation.

Figure 10.1  
World Supply, Demand for Import, and Import Quota



The vertical line denoted Q in Figure 10.1 represents a quantity restriction or quota on imports. It is clear that a solution satisfying the quota Q cannot lie both on the world supply curve S and on the demand curve D. F represents the price importers can buy at, including import duties, and E represents the price the market is willing to pay, given the available quantity of import. We will assume that importers buy from the rest of the world according to point B and sell to the domestic market according to point C, and earn the 'rent' represented by the rectangle BCEF.

Tables 10.1 and 10.2 show the specification and account tables for the relevant part of the economy in a model without a quota. If a quota is imposed we need several additions. Imports need to be shown at two prices, landed prices and prices including rent, and we will therefore need an account for each. We will also need a cell for the rent payment and an account to receive it, and we will have to pass the rent on to one or more institutions. All these additions are shown in Tables 10.3 and 10.4.

Table 10.1  
Subset of the Specification Table for  
Model Without Import Quota

|               | Composite<br>good | Imported<br>good |
|---------------|-------------------|------------------|
| Domestic good | CES               |                  |
| Imported good | CES               |                  |
| Indirect tax  |                   | ITAX             |
| Rest of World |                   | IMPORT           |

Table 10.2  
Subset of the Account Table for  
Model Without Import Quota

|               | TYPE | FIX |
|---------------|------|-----|
| Imported good | AC   |     |

The account for imports excluding rent buys in Table 10.3 from the rest of the world and the specifications in that column are the same as in the imported goods column in the base model. The difference appears in the account table in Table 10.4 where FIX is set to Q in the imports excluding rent account. The row account for imports including rent in Table 10.3 appears where the imports are used, namely in the CES aggregation in the column of the composite good. One unit of imports including rent is created from one unit of imports excluding rent (specification IO) with the addition of a RENT. There is no behavioral equation in the cell with the RENT

specification. The payment is a residual, the difference between what the domestic market will pay for the imports and the foreign exchange and tariff costs of the imports.

Table 10.3  
Subset of the Specification Table for  
Model With Import Quota

|                   | Composite<br>good | Imports<br>excl rent | Imports<br>incl rent | Import<br>Rent |
|-------------------|-------------------|----------------------|----------------------|----------------|
| Domestic good     | CES               |                      |                      |                |
| Imports excl Rent |                   |                      | IO                   |                |
| Imports incl Rent | CES               |                      |                      |                |
| Import Rent       |                   |                      | RENT                 |                |
| Indirect tax      |                   | ITAX                 |                      |                |
| Rent receiver     |                   |                      |                      | IDIST          |
| Rest of World     |                   | IMPORT               |                      |                |

Table 10.4  
Subset of the Account Table for  
Model With Import Quota

|                   | TYPE | FIX |
|-------------------|------|-----|
| Imports excl rent | AC   | q   |
| Imports incl rent | AC   |     |
| Import Rent       | NMF  |     |
| Rent Receiver     | INST |     |

The RENT appears in a Non-Market Factor (NMF) row account. The rent is part of value added or GDP and it must therefore appear in a factor account. On the other hand, there is no price or quantity associated with the rent. In order to define GDP in constant prices HERCULES defines the rent payment in constant prices as the payment that would have taken place with base period prices and solution quantities. It is also equal to the solution quantity multiplied by the share the rent received in the base case. This definition implies that if a quota is imposed on a base without quota, then the constant price rent payment will be zero regardless of the size of the quota.

Factor income from rent must be passed on to an institution. The recipient institution will depend on the circumstances of the economy we model. Here we have simply included a rent receiving account and implicitly assumed that the account already existed and that its rules for further allocation of the income were unchanged.

The new model will be e-balanced (as defined in chapter 9), provided that the original model was. This can best be seen by counting changes in the number of variables and equations. The new IO and IDIST cells and the new import rent account are all e-balanced so their contributions to the change cancel out.  $Q_{SO} = \text{exogenous}$  removes one degree of freedom and the RENT cell adds one degree of freedom, giving zero on a net basis.

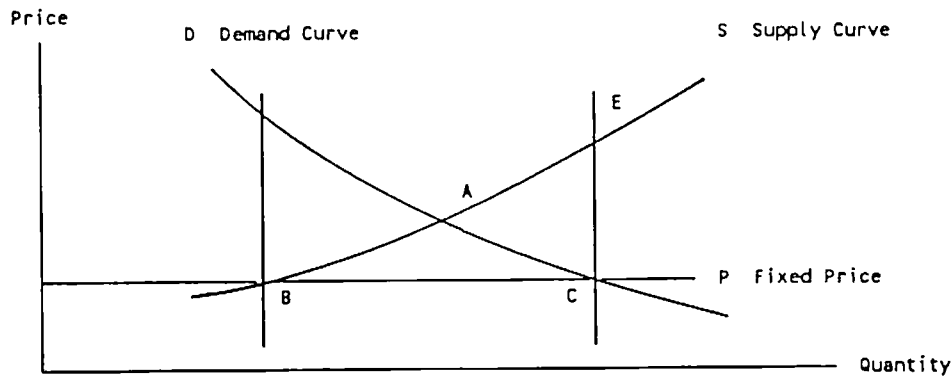
An example model with import quota based on the example in chapter 6 is available in file MODELH1.GMS. Note that if you have rents due to import quota in the base case then you must estimate the size of this rent and include it in the base SAM. This will often be rather difficult, but it is necessary for a model to give reasonable results. The model in file MODELH1.GMS is based on the SAM in chapter 6 with all rent payments in the base equal to zero. A reduced import quota in an experiment causes the rent to become positive.

### 10.3 A Fixed Price with Supply Adjusting

In this section we will look at the effect of fixing the price of a domestically produced good. In previous chapters, the price has been determined by supply and demand as indicated by A in Figure 10.2; the supply curve is defined by the marginal cost of production, and the demand curve by the behavior of the individual components of demand. The horizontal line denoted P in Figure 10.2 represents the fixed or administrated price. P intersects the supply curve S at point B and the demand curve at point C. The market is not in balance, and something must adjust.

In our first example we will assume that the good with the fixed price is a public utility, which must produce at whatever level of output is needed to satisfy demand. The marginal cost curve or the supply curve is therefore not used to determine the production level, and the equilibrium point will be C. Even if the marginal cost curve is not used to determine production it still defines the minimum cost of production at any given level of output. In the present example, this is denoted by point E in Figure 10.2, and the distance between C and E represents a subsidy to users. We will assume that this subsidy or negative rent is paid out of the profit that the producer otherwise would have made, i.e. it is paid by the account for capital.

**Figure 10.2**  
Supply and Demand Curves and Fixed Prices



**Table 10.5**  
Subset of the Specification Table for  
Model Without Fixed Prices

|              | Value-Added | Activity | Users |
|--------------|-------------|----------|-------|
| Labor        | CES         |          |       |
| Capital      | CES         |          |       |
| Value-Added  |             | 10       |       |
| Intermediate |             | 10       |       |
| Activity     |             |          | ??    |

Tables 10.5 and 10.6 shows the relevant components of the specification and account tables in a model where there is no fixed price, and Tables 10.7 and 10.8 shows the same components for a model where the price is fixed. The nature of the demand function is irrelevant for the following discussion, so the demand specifications are simply shown with question marks in the tables. In the model with the fixed price, the capital account is split up in three accounts -- one for the physical capital used in the production function, one for rent (or subsidy), and one for the total income from capital. The first two are factor accounts of type MF and NMF, respectively. The last account cannot also be a factor account, because this would lead to double counting

**Table 10.6**  
**Subset of the Account Table for**  
**Model Without Fixed Prices**

|             | TYPE | FIX |
|-------------|------|-----|
| Capital     | MF   | 0   |
| Value-Added | AC   |     |
| Activity    | AC   |     |

when estimating GDP. The collection account is therefore of type INST. The value-added and activity accounts are similar in the two models; they model production at minimum cost in both cases. The addition in the new model is the account for product-sold, to which the fixed price is applied and from which all users buy. One product-sold unit requires one activity unit as seen from the IO specification in the product-sold column. There is no connection between the prices of the activity and the product-sold, however; the two are disconnected by the RENT specification.

The new model is e-balanced, provided the old one was. This is again seen by counting the incremental number of variables and equations: the new IO cell and the two new IDIST cells are e-balanced, and so are the accounts for physical capital and rent. The exogenous price in the product-sold account adds one equation, and the RENT specification adds one variable on a net basis.

The file MODELH2.GMS contains an example with a fixed price of industrial goods and industrial production adjusting as discussed in this section. It is based on the SAM from chapter 7 with changes like the ones shown here. The fixed price is assumed to be equal to the marginal cost of production in the base case, which is equivalent to the rent being 0 in the base. In an experiment we reduce the fixed price on industrial goods, and the rent becomes positive, demand shifts toward industrial goods, etc. Try to run the model and explain the results. (Note that the consumer price index for rural households has been chosen as the numeraire, so when industrial goods become cheaper then agricultural goods must become more expensive.)

Table 10.7  
Subset of the Specification Table for  
Model With Fixed Prices

|              | Value<br>Added | Activity | Users | Product<br>sold | Capital<br>phys | Capital<br>rent |
|--------------|----------------|----------|-------|-----------------|-----------------|-----------------|
| Labor        | CES            |          |       |                 |                 |                 |
| Capital-phys | CES            |          |       |                 |                 |                 |
| Capital-rent |                |          |       | RENT            |                 |                 |
| Capital-totl |                |          |       |                 | IDIST           | IDIST           |
| Value-Added  |                | IO       |       |                 |                 |                 |
| Intermediate |                | IO       |       |                 |                 |                 |
| Activity     |                |          |       | IO              |                 |                 |
| Product-sold |                |          | ??    |                 |                 |                 |

Table 10.8  
Subset of the Account Table for  
Model With Fixed Prices

|              | TYPE | FIX |
|--------------|------|-----|
| Capital-phys | MF   | Q   |
| Capital-rent | NMF  |     |
| Capital-totl | INST |     |
| Value-Added  | AC   |     |
| Activity     | AC   |     |
| Product-sold | AC   | P   |

#### 10.4 A Fixed Price and Export Adjustment

This section continues our investigation of fixed prices, but this time we assume that the producer can decide how much to produce. An example is an agricultural producer who is free to decide his production technology, but has to turn his output over to a government marketing board at a fixed procurement price. Under these circumstances, the rational output level for the producer will be at point B in Figure 10.2, where marginal cost is equal to price.

We now need a mechanism for bringing demand in line with supply. The marketing board can of course sell to the domestic market at a higher price, e.g. at the market clearing price represented by point D, and earn a profit. Or it can impose a rationing scheme. However, we will assume that the purpose of the fixed price is to make food available cheaply, and that the marketing board therefore reduces exports of agricultural goods in order to keep enough for the domestic market.

Tables 10.9 and 10.10 again show subsets of the specification and account tables for a model without a fixed price and with exports and domestic demand competing for output; and Tables 10.11 and 10.12 show the same model components, but with fixed prices and adjusted exports.

Table 10.9  
Subset of the Specification Table for  
Model Without Fixed Prices

|              | Value Added | Activity | Domestic users | Rest of World |
|--------------|-------------|----------|----------------|---------------|
| Labor        | CES         |          |                |               |
| Capital      | CES         |          |                |               |
| Value-Added  |             | 10       |                |               |
| Intermediate |             | 10       |                |               |
| Activity     |             |          | ??             | EXPORT        |

The fixed price is associated with the account of the marketing board. The activity account sells everything to the marketing board at this fixed price, and it will continue to produce until its marginal cost is equal to the procurement price. Recall here that rows sell to columns so that the activity row sells the commodity to the market board column with the IO specification. The supply of goods is sold by the marketing board (row) to domestic users in quantities determined from the demand side, and the residual is sold to the export-goods account. The export-goods account sells the available goods to the rest of the world at a price determined from the world demand curve. Our basic rule of always having the same price for all transactions in a row implies that the SAM value in the marketing board row and export goods column reflects exports valued at the procurement price. The domestic price will most likely be different from the world price so we have included a RENT specification to collect the difference and pay it to an export-rent account. The identity of the rent recipient will depend on the institutional arrangements in the economy. In this case it is paid to the government.

Table 10.10  
Subset of the Account Table for  
Model Without Fixed Prices

|             | TYPE | FIX |
|-------------|------|-----|
| Capital     | MF   | Q   |
| Value-Added | AC   |     |
| Activity    | AC   |     |

The new model is again e-balanced, provided the original model was. Most new cells are e-balanced, and the extra equation fixing the price is canceled by the extra variable in the RENT cell.

Table 10.11  
Subset of the Specification Table for  
Model With Fixed Price and Adjusted Exports.

|              | Value<br>Added | Activity | Market<br>board | Domestic<br>users | Rest of<br>World | Export<br>good | Export<br>rent |
|--------------|----------------|----------|-----------------|-------------------|------------------|----------------|----------------|
| Labor        | CES            |          |                 |                   |                  |                |                |
| Capital      | CES            |          |                 |                   |                  |                |                |
| Value-Added  |                | IO       |                 |                   |                  |                |                |
| Intermediate |                | IO       |                 |                   |                  |                |                |
| Activity     |                |          | IO              |                   |                  |                |                |
| Market-board |                |          |                 | ??                |                  | IO             |                |
| Export-goods |                |          |                 |                   | EXPORT           |                |                |
| Export-rent  |                |          |                 |                   |                  | RENT           |                |
| Government   |                |          |                 |                   |                  |                | IDIST          |

The file MODELH3.GMS contains an example with a fixed price of agricultural goods and agricultural export adjusting as discussed in this section. It is based on the SAM from chapter 6 with changes like the ones shown here. The fixed procurement price is assumed to be equal to the world price in the base case, which is equivalent to the rent being 0 in the base. In an experiment we reduce the fixed procurement price on agricultural goods, and demand shifts toward agricultural goods, exports are reduced,

the export price increases because we move up on the world demand curve, the rent becomes positive, etc. Try to run the model and explain the results.

Table 10.12  
Subset of the Account Table for  
Model With Fixed Price and Adjusted Exports

|              | TYPE | FIX |
|--------------|------|-----|
| Capital      | MF   | Q   |
| Value-Added  | AC   |     |
| Activity     | AC   |     |
| Market-board | AC   | P   |
| Export-goods | AC   |     |
| Export-rent  | NMF  |     |

### 10.5 Pricing of Natural Resources

Monopolistic producers will generally not price their output at marginal cost (the assumption we have used in all previous models). Moreover, the price at which they sell to the domestic market may be different from the price at which they sell to the world market. A typical case might be that of an oil producer. Production levels and domestic prices will often be determined by factors not included in our model; for our purposes, therefore, we assume that they are exogenous. Tables 10.13 and 10.14 show a subset of the specification and account tables for a model with these assumptions built in.

We assume that production volume is determined outside our model so we must also assume that there is enough capacity to produce this fixed amount. The production column has therefore been specified with fixed input ratios for labor and intermediates, and capital receives the difference between income and expenditures as RENT. Output is sold as domestic oil and export oil. The single IO specification in the column for domestic oil implies that the price of the oil produced is equal to the exogenously determined price of domestic oil. The quantity of oil consumed domestically is determined by the individual components of domestic demand, given the exogenous domestic price; the quantity of oil exported is a residual. The price of the Export-Oil column is equal to the price the world market will pay, given the export oil quantity, and the difference between the fixed domestic price and this price determines the amount of RENT in the Export-Oil column. The two RENT specifications are shown in the same capital

**Table 10.13**  
**Subset of the Specification Table**  
**for a Model with Fixed Oil Production, a**  
**Fixed Domestic Price, and Residual Exports**

|                | Oil<br>production | Domestic<br>Oil | Domestic<br>users | Export<br>Oil | Rest of<br>World |
|----------------|-------------------|-----------------|-------------------|---------------|------------------|
| Labor          | IO                |                 |                   |               |                  |
| Capital        | RENT              |                 |                   | RENT          |                  |
| Intermediate   | IO                |                 |                   |               |                  |
| Oil-production |                   | IO              |                   | IO            |                  |
| Domestic-Oil   |                   |                 | ??                |               |                  |
| Export-Oil     |                   |                 |                   |               | EXPORT           |

account, indicating that the oil producer gets both of them.

**Table 10.14**  
**Subset of the Account Table**  
**for a Model With Fixed Oil Production, a**  
**Fixed Domestic Price, and Residual Exports**

|                | TYPE | FIX |
|----------------|------|-----|
| Capital        | NMF  |     |
| Oil-Production | AC   | Q   |
| Domestic-Oil   | AC   | P   |
| Export-Oil     | AC   |     |

It is possible to value oil production at world prices. In this case we get the specification table shown in Table 10.15. The only change is that one of the RENT cells has been moved from the Export-Oil column to the Domestic-Oil column. The numbers in the underlying SAM will also change; the two cells in the Oil-Production row will change to reflect the new valuation of Oil-Production, the RENT cell in the Domestic-Oil column must change so that the column total remains constant, and the RENT cell in the Oil-Production column must change so that the account remains balanced. The two models are mathematically equivalent, however, and will produce the same results, except for the few cells in which different component values are used.

**Table 10.15**  
**Subset of the Specification Table for the**  
**Model from Table 10.13 with Oil Production**  
**Valued at World Prices**

|                | Oil<br>production | Domestic<br>Oil | Domestic<br>users | Export<br>Oil | Rest of<br>World |
|----------------|-------------------|-----------------|-------------------|---------------|------------------|
| Labor          | 10                |                 |                   |               |                  |
| Capital        | RENT              | RENT            |                   |               |                  |
| Intermediate   | 10                |                 |                   |               |                  |
| Oil-production |                   | 10              |                   | 10            |                  |
| Domestic-Oil   |                   |                 | ??                |               |                  |
| Export-Oil     |                   |                 |                   |               | EXPORT           |

**Table 10.16**  
**Subset of the Specification Table for the**  
**Model from Table 10.13 with Oil Production**  
**Valued at Marginal Cost Excluding Rent**

|                | Oil<br>production | Domestic<br>Oil | Domestic<br>users | Export<br>Oil | Rest of<br>World |
|----------------|-------------------|-----------------|-------------------|---------------|------------------|
| Labor          | 10                |                 |                   |               |                  |
| Capital        |                   | RENT            |                   | RENT          |                  |
| Intermediate   | 10                |                 |                   |               |                  |
| Oil-production |                   | 10              |                   | 10            |                  |
| Domestic-Oil   |                   |                 | ??                |               |                  |
| Export-Oil     |                   |                 |                   |               | EXPORT           |

It is also possible to value oil production at marginal cost excluding rent paid to capital. The corresponding part of the specification table is shown in Table 10.16. The model is again mathematically equivalent to the previous two models and will produce the same results, except for some of the oil cells.

The segment of the model defined by all the different specification and account tables shown here is e-balanced. The extra equations from the fixed price and the fixed

quantity defined by the account table are in each case canceled out by the extra variables in the two RENT cells.

Three models corresponding to the three specification tables in this section have been included with HERCULES as MODELH4.GMS to MODELH6.GMS. The models are extensions of the one in chapter 6 with an oil sector added. The fixed domestic oil price is 25% of the world price, and the marginal production costs (labor and intermediate inputs) are around 5% of the world price, so there are large rents. The three files contain a common experiment in which the domestic oil price is reduced by 10%. Try to run the three models and compare the results, and note that all quantities, prices, and values are the same except for a few related to the oil sector.

## 10.6 The AVRГ Specification

In section 10.2 we made the assumption that the imported good was sold at a price corresponding to its 'marginal utility' derived from a CES aggregation with domestic goods, and that the importer collected the difference between this price and the world price as a rent. In this section we will look at the case where the importer is a government organization that resells the imported good without collecting a rent.

The CES aggregation is no longer a good model since the import share does not depend on prices. Instead we can use the following assumptions: Overall demand is satisfied in two stages. The cheap part of supply, imports, are first bought at the going world price up to the import quota; the remaining demand is then satisfied by domestic production. The price the market faces for the 'average' good is an average of the prices of the imported good and the domestic good. The average price assumption for all users implicitly implies, that all users of the good get the same import share, and that the two goods are identical.

These assumptions are in HERCULES implemented in the AVRГ specification. A key assumption behind AVRГ is that the two goods are identical, but that they are bought at different prices. This means that the economy is not in equilibrium, and the SAM has no information about ratio between these prices. The modeler must therefore specify these prices. This is done with parameter PBAS. PBAS defines the prices of the inputs in the base case measured in comparable units.

An example using specification AVRГ is given in file MODELH7.GMS. It is based on the same SAM as MODELH1.GMS and the only difference is the specification of the substitution between imports and domestic goods in agriculture. We assume that the imported agricultural goods are 25% cheaper than comparable domestic goods to begin with, and we make an experiment in which we reduce the import quota by 10%. Try to

run the model and look at the results. The price index for the composite agricultural good grows faster than the price index for the domestic agricultural good. The reason is, that the lower import quota increases the share of the more expensive domestic goods.

## CHAPTER 11

### MODEL ERRORS

#### 11.1 Introduction

This chapter will review many of the types of problems and errors that can be encountered while building a model using GAMS/HERCULES. The discussion is organized according to the order in which the problems or errors can appear: GAMS compiler and execution errors, HERCULES input errors, errors in the SAM, non uniqueness of the overall model in the base case, and finally problems related to variables becoming negative or moving off to infinity during the solution process. The chapter will show some of the different types of error messages and explain the structure of the error messages in general. We will also use the opportunity to explain many of the pieces of output provided by GAMS and HERCULES that can be useful for tracing errors.

In order to keep the size of this chapter and the corresponding appendices within reasonable limits we will in the following refer to several models that show some of the errors. The models are all distributed with HERCULES, and their names are HERCER\*.GMS. You are encouraged to run these models and inspect the output.

#### 11.2 GAMS Compiler Errors

The first types of errors messages you will encounter are printed by the GAMS compiler. The compiler marks the errors by including one or more message lines just below each offending line in the source listing file. All message lines start with '\*\*\*\*'. The position within the line where the error is recognized is indicated with an \$ followed by an error number as shown in the example in Figure 11.1. The actual error messages can be found after the source listing in a section labeled 'ERROR MESSAGES'. The error messages should be self explanatory. The most common errors are connected with missing semicolons (;) at the end of GAMS statements, misspelled identifiers or labels, undeclared identifiers, or numbers that are not entered below their column heading. Remember that all identifiers, including acronyms, must be declared before they are used; if they are not you will get an error message about 'UNKNOWN SYMBOL', or 'REAL NUMBER EXPECTED', depending on where the identifier is used. Figure 11.1 and 11.2 gives examples of these error messages. These two small error files are included with HERCULES with the names HERCER1.GMS and HERCER2.GMS.

Figure 11.1

GAMS compile errors are identified by '\*\*\*\*' and an error number. The error message is given in an 'ERROR MESSAGES' section after the source listing

```
COMPI LATION
 1 SET A / A1,A2 /;
 2 PARAMETER B(A);
 3 B(A) = ACR;
****          $140

ERROR MESSAGES

140 UNKNOWN SYMBOL, ENTERED AS PARAMETER
```

### 11.3 Domain Checking

GAMS has a facility for assuring that the labels or sets used in a particular index position always belong to or a subset of a particular set, the domain of that index position. This is referred to as domain checking. Domain checking is in general highly recommended because it reduces the chance of undetected typing errors in labels as in Figure 11.2 or undetected logical errors, for example due to reversed indices. All examples shown so far have used domain checking when the domain is the set of accounts, i.e. for the first dimension of the account table, and for the first two dimensions of the cell table. However, we have on purpose never used domain checking for the last dimension of these two parameters, the attribute set; we have always used a '\*' in the domain list indicating that any label can be used. The reason is that HERCULES returns numerical values in the account and cell tables using labels defined by HERCULES. If the modeler uses domain checking on the last dimension and do not include all the labels that HERCULES uses, then HERCULES cannot report the solution back. So HERCULES users are encouraged to use domain checking for the account set and not to use domain checking for the attribute set.

Figure 11.2

The content of tables must be real numbers or identifiers already declared as acronyms (error #1), the numbers must appear under the column label (error #225), and for domain checked parameters, labels must belong to the proper domains (error #170)

```

C O M P I L A T I O N

  1 SET A / A1,A2 /;
  2 TABLE B(A,A)
  3           A1      A2
  4 A1           1.0   ACR
****                               $1
  5 A3                               1
**** $170                          $225

ERROR MESSAGES

  1 REAL NUMBER EXPECTED
 170 DOMAIN VIOLATION FOR ELEMENT
 225 FLOATING ENTRY IGNORED

```

#### 11.4 GAMS Execution Errors

Once GAMS compiles without errors it will execute the GAMS assignment statements, and it will execute without logical errors. There can still be numerical execution error such as division by zero, square roots of negative numbers, or arithmetic operations on acronyms. All illegal numerical operations are caught by GAMS and identified by error messages that pinpoint the line in which the error occurred and the cause of the error. The execution error messages start with '\*\*\*\*' like all other error messages, and they should again be self explanatory. An example is shown in Figure 11.3 (and the corresponding file named HERCER3.GMS is distributed with HERCULES). If you have trouble finding the error then display the left hand side of the assignment with the error and look for 'UNDF', the GAMS value for undefined. All arithmetic errors gives the result UNDF, and operations on UNDF gives UNDF, so errors will not cancel out in later computations.

A common error is division by the GAMS default value of zero as shown by the example in Figure 11.3; B("A3") is not explicitly defined and is therefore zero, and C("A3") becomes UNDF. If you only want to do the computation in cases where the denominator is non-zero then you can use the following construct:

```
C(A) $ B(A) = 1./ B(A);
```

that limits the execution of the statement to cases where B(A) exists, i.e. is nonzero.

If there is an execution error before a SOLVE statement GAMS will stop at the SOLVE statement. We will from now on assume that the GAMS program compiles correctly and executes without errors down to the SOLVE statement, and we will concentrate on errors that are detected by the HERCULES solution system.

Figure 11.3

Execution errors are also flagged with '\*\*\*\*' followed by an error message. The result of arithmetic errors is always UNDF

```

C O M P I L A T I O N

  1 SET A / A1,A2,A3 /;
  2 PARAMETER B(A) / A1 1.0, A2 2.0 /, C(A);
  3 C(A) = 1./B(A); DISPLAY C;

E X E C U T I N G

**** EXECUTION ERROR AT LINE 3 .. DIVISION BY ZERO

----      3 PARAMETER C

A1 1.000,   A2 0.500,   A3 UNDF

```

## 11.5 Account and Cell Specific Tests

GAMS communicates with solution systems like HERCULES through a set of files. GAMS writes the set of accounts and the account and cell tables as well as the acronyms on some files that HERCULES reads, and HERCULES writes the solution and error messages on a set of files that GAMS reads.

HERCULES first reads the complete account set defined through the first identifier in the MODEL statement. It then reads the account table. Only the rows that corresponds to accounts defined in the account set, and columns that corresponds to known attributes like TYPE, FIX, and SIGMA are kept; all other elements in the table are ignored. Warning messages, again flagged with '\*\*\*\*', are issued for unknown columns but there are no warnings for rows that do not belong to the set of accounts. The treatment of the cell table is similar: only cells with row and column labels correspon-

ding to labels in the account set and planes corresponding to known attributes like TBASE and SPECS are kept, and warnings are only issued for unknown planes. The reason for this asymmetric treatment of accounts and attributes with respect to error messages is related to our encouragement to use domain checking for the account sets and not to use it for the attribute sets.

Figure 11.4

The account type in account A2, i.e. parameter AT(A2,TYPE), is not one of the legal acronyms for this position

```

C O M P I L A T I O N

 1 SET ACC / A1, A2, A3 /;
 2
 3 ACRONYMS MF, ACT, INST, Q, IDIST, VSHR, CD;
 4
 5 TABLE AT(ACC,*) ACCOUNT TABLE
 6     TYPE      FIX
 7  A1      MF      Q
 8  A2      ACT
 9  A3      INST
10
11 TABLE CT(ACC,ACC,*) CELL TABLE
12     TBASE     SPECS
13 A1.A3      80     IDIST
14 A3.A2      80     VSHR
15 A2.A1      80     CD
16
17 MODEL ERR / ACC,AT,CT /; SOLVE ERR USING HERCULES;

SOLUTION REPORT      SOLVE ERR USING HERCULES FROM LINE 17

**** THE FOLLOWING PARAMETER HAS AN ILLEGAL VALUE
      (IT SHOULD BE A LEGAL TYPE ACRONYM):
      PARAMETER AT(A2,TYPE)

**** THE MODEL DOES NOT HAVE A NUMERAIRE.
      USE ACCOUNT TYPE NP, NY, OR NPQ.

```

Acronyms are interpreted when the account and cell tables are read, and unknown or misplaced acronyms are reported as errors. You can declare any identifier as an acronym in GAMS, but only the ones mentioned in this book are understood by HERCULES. See the appendices for a complete list. Furthermore, a TYPE acronym cannot be used in the FIX column of the account table, SPECS acronyms cannot be used in the account table, etc. The error messages should be quite explicit. An error of

this type is shown in Figure 11.4. Note that the error message is preceded by the usual '\*\*\*\*'. The file corresponding to Figure 11.4 is named HERCER4.GMS.

Before we continue with other error checks it can be useful to look at line 11-15 in Figure 11.4. They show an alternative way of entering the complete cell table directly and not through individual two dimensional tables and assignment statements. Notice that the three dimensional table is entered in exactly the same format as the GAMS DISPLAY statement produces it. Display output can usually be used directly as input again, which can be convenient for cleaning up messy tables in a model: the tables are displayed and the output produced by GAMS is used to replace the old tables. The pagesize and pagewidth parameters of the GAMS call can be used to prevent page breaks in the tables and to change their width.

Table 11.1

Legal combinations of account TYPE and FIX are shown with 'X'. 'blank' is used to denote that FIX is equal to the GAMS default value of zero, which by HERCULES is considered as not defined

|              | <u>FIX:</u> |   |   |    |    |    |     |       |
|--------------|-------------|---|---|----|----|----|-----|-------|
|              | P           | Q | Y | PQ | NP | NY | NPQ | blank |
| <u>TYPE:</u> |             |   |   |    |    |    |     |       |
| MF           | x           | x |   |    | x  |    | x   | x     |
| NMF          |             |   |   |    |    |    |     | x     |
| INST         |             |   | x |    |    | x  |     | x     |
| INSTC        |             | x | x |    | x  | x  |     | x     |
| TAX          |             |   |   |    |    |    |     | x     |
| AC           | x           | x |   | x  | x  |    | x   | x     |
| TNMF         |             |   |   |    |    |    |     | x     |
| ROW          | x           |   |   |    | x  |    |     | x     |

After the individual input data are analysed the system checks many combinations of information within each account or each cell:

First, the TYPE and FIX acronyms must be consistent. The legal combinations of TYPE and FIX are shown in table 11.1. An example with an error can be found in the file HERCERX.GMS.

Second, all cells with a nonzero TBASE value must also have a value in the SPECS plane, i.e. all payments must be explained (but cells with a SPECS value may have a zero TBASE).

Third, the SAM must be balanced, i.e. the income in each account must be equal to the expenditure within a small tolerance. This check is necessary to ensure that the calibrated parameter values are reasonable, that the final model does have a solution and that comparisons of solution values with the base case makes sense. A companion system for balancing SAMs is under development, see Zenios (1988) for details.

Fourth, if the SPECS acronym indicates a behavior that by definition must be non-negative, then TBASE must be non-negative.

Figure 11.5a  
Definition of a model with illegal combinations of specifications

```

C O M P I L A T I O N

 1 SET ACC / A1, A2, A3 /;
 2
 3 ACRONYMS MF, AC, INSTC, Q, NP, IDIST, CD;
 4
 5 TABLE AT(ACC,*) ACCOUNT TABLE
 6     TYPE    FIX
 7 A1     MF    Q
 8 A2     AC
 9 A3     INSTC NP
10
11 TABLE CT(ACC,ACC,*) CELL TABLE
12     TBASE   SPECS
13 A3.A1    80    IDIST
14 A1.A2    80    CD
15 A2.A3    80    IDIST
16
17 MODEL ERR / ACC,AT,CT /; SOLVE ERR USING HERCULES;

```

Fifth, the SPECS value in cell must be consistent with the TYPE of both the row and the column, and the combination of SPECS values in a row or column must also make sense for that particular TYPE of account. The legal combinations are defined verbally together with the specifications in appendix A. If the model contains a mistake it can have two reasons: the specifications are correct but the TYPE of the account is wrong, or the TYPE is correct but one or more of the specifications are illegal. HERCULES will therefore list all legal combinations for the particular account type and also mention if the combination you have used is legal for a different account type. An example in which a specification is inconsistent with the TYPE of the column, in this

Figure 11.5b  
Output from the model with illegal combinations of specifications

```

SOLUTION REPORT      SOLVE ERR USING HERCULES FROM LINE 17

**** THE 'TYPE' OF THE ROW AND THE 'SPEC' OF THE
      CELL ARE INCONSISTENT FOR THE FOLLOWING CELL(S):
      PARAMETER CT(A2,A3,SPECS)

      THE FOLLOWING COMBINATIONS OF 'TYPE' FOR THE ROW
      AND 'SPECS' FOR THE CELL ARE POSSIBLE:

```

|        | MF | NMF | INST | INSTC | TAX | AC | TNMF | ROW |
|--------|----|-----|------|-------|-----|----|------|-----|
| AVRG   | *  |     |      |       |     | *  |      |     |
| CD     | *  |     |      |       |     | *  |      |     |
| CES    | *  |     |      |       |     | *  |      |     |
| CES2   | *  |     |      |       |     | *  |      |     |
| CET    |    |     |      |       |     | *  |      |     |
| CETINF |    |     |      |       |     | *  |      |     |
| CETO   |    |     |      |       |     | *  |      |     |
| DQEXO  |    |     |      |       |     | *  |      |     |
| DTAX   |    |     |      |       | *   |    |      |     |
| DVEXO  |    |     |      |       |     | *  |      |     |
| EXPORT |    |     |      |       |     | *  |      |     |
| -----  |    |     |      |       |     |    |      |     |
| IDIST  |    |     | *    | *     |     |    |      | *   |
| IMPORT |    |     |      |       |     |    |      | *   |
| IO     | *  |     |      |       |     | *  |      |     |
| ITAX   |    |     |      |       | *   |    |      |     |
| LES    |    |     |      |       |     | *  |      |     |
| MARKUP |    | *   |      |       |     |    | *    |     |
| QEXO   |    |     |      |       |     | *  |      |     |
| QSHR   |    |     |      |       |     | *  |      |     |
| RENT   |    | *   |      |       |     |    | *    |     |
| SUBST  | *  |     |      |       |     | *  |      |     |
| UNSPEC |    | *   | *    | *     |     |    | *    | *   |
| TEXO   |    |     | *    | *     |     |    |      | *   |
| TRANSF |    | *   |      |       |     |    |      |     |
| VEXO   |    |     |      |       |     | *  |      |     |
| VSHR   |    |     |      |       |     | *  |      |     |

Figure 11.6

Definition of model with illegal combination of specifications in an account.

```

C O M P I L A T I O N

 1 SET ACC / A1, A2, A3, A4 /;
 2
 3 ACRONYMS MF, AC, INSTC, Q, NP, IDIST, CD, VSHR, QSHR;
 4
 5 TABLE AT(ACC,*) ACCOUNT TABLE
 6         TYPE    FIX
 7 A1      MF      Q
 8 A2      AC
 9 A3      AC
10 A4      INSTC  NP
11
12 TABLE CT(ACC,ACC,*) CELL TABLE
13         TBASE   SPECS
14 A4.A1    80     IDIST
15 A1.A2    30     CD
16 A1.A3    50     CD
17 A2.A4    30     VSHR
18 A3.A4    50     QSHR
19
20 MODEL ERR / ACC,AT,CT /; SOLVE ERR USING HERCULES;

SOLUTION REPORT      SOLVE ERR USING HERCULES FROM LINE 20

**** ILLEGAL COMBINATION OF SPECIFICATIONS IN THE COLUMN OF
SET ACC(A4)
THE ACCOUNT TYPE IS INSTC

THE FOLLOWING COMBINATIONS ARE LEGAL FOR ACCOUNT TYPE INST
 1 DQEXO (ONE OR MORE )
 2 DVEXO (ONE OR MORE )
 3 LES   (ONE OR MORE )
 4 QEXO  (ONE OR MORE )
 5 QSHR  (ONE OR MORE )
 6 VEXO  (ONE OR MORE )
 7 VSHR  (ONE OR MORE )

```

case a VSHR specification in an INSTC column is shown in Figure 11.5a and b (corresponding to file HERCER6.GMS), and an example where QSHR and VSHR are mixed in a column is shown in Figure 11.6a and b (corresponding to file HERCER7.GMS).

Sixth, parameter values specified by the modeler must satisfy certain constraints, for example be positive or add to one in a column or row. A more detailed description of all the parameters, their default values, and the constraints they must satisfy is given in appendix C.

Seventh, parameters that cannot be calibrated from the SAM, such as elasticities, must be supplied by the modeler. Remember that GAMS throws away zeros, so always use the special GAMS value EPS instead of zero for parameter values that must be defined and for parameter values where you want to overwrite a calibrated value with a zero.

Finally, the model must have exactly one numeraire defined with an account FIX of NP, NY, or NPQ.

## 11.6 Economywide and Modelwide Tests

The checks mentioned in the previous subsection were all local in nature in the sense that they looked at one account at a time or at one cell and the accounts in which it appear. If the local checks are passed, HERCULES starts to check consistency at a more global level.

First, the system checks that the economy is connected. An economy is defined as connected if money can flow, directly or indirectly, between any pair of accounts in the economy. If the economy is not connected it can be broken down into two or more connected economies, and an independent model should be solved for each economy. An example of a disconnected economy can be found in the file HERCER8.GMS distributed with HERCULES.

If the connectedness check is satisfied HERCULES starts to generate the variables and equations of the model. During this phase HERCULES does many things for the modeler. He does not have to enter complicated mathematical equations explicitly; he can refer to an equation by entering CES without being concerned with the mathematical details. Furthermore, the system creates many equations such as linkage equations and identities implicitly, and it estimates most parameters. Unfortunately, all this convenience has a price. It can be difficult for a modeler to find errors that are caused by inconsistencies between equations entered explicitly for example through cell specifications and equations that were created without notice. This is probably the most difficult

aspect of using HERCULES so we will show several examples of wrong models and the corresponding error messages.

There are several types of possible errors or inconsistencies and we will review them in order of increasing complexity:

- a) The overall model is over- or underdetermined or unbalanced, i.e. the number of variables is not equal to the number of equations.
- b) The overall model is balanced but there exist over- and underdetermined submodels, i.e. submodels with more equations than variables and other submodels with more variables than equations. The term 'submodel' used here should not be confused with the term 'submodel for an agent' used in chapter 1. A 'submodel' as used here is a subset of the equations of the model and the variables that appear in these equations.
- c) There are no unbalanced submodels, but numerical parameter values are such that the model does not have a locally unique solution. This case is in the following referred to as numerical singularity.

## 11.7 Unbalanced Models

The model in Appendix 11.1 (and in file HERCER9.GMS) shows an example of an unbalanced model. The model is quite standard with the exception that we have fixed the price of an activity without including a RENT specification or some other mechanism for making supply and demand equal for this activity.

The first part of the output, on the page with GAMS page number 3, shows some statistics that always are printed by HERCULES. The first set of statistics show the size of the SAM as entered by the modeler (before expansion), and as used internally by HERCULES (after expansion). The implied expansion of the SAM takes place if the model contains specification CES2 or LES, or certain combinations of ITAX and RENT, and the way the expansion is done is explained in appendix A under the specifications that cause the expansion. All results, however, refer to the original SAM, so the modeler should usually not be concerned with expansion. But many modelers still find it interesting to see the size of the SAM they are ultimately working with, and the number of variables counted in the next section refers to the expanded SAM.

The second set of statistics show the size of the model. This model has 21 variables, namely 4 PSOL, 4 QSOL, 4 YSOL, 5 TSOL, 3 QCSOL, and one residual variable that removes the linear dependence in the model due to Walras law (see chapter 1). These

numbers are shown in the column labeled TOTAL. The model also has 22 equations, namely 4 equations that guarantee that the rows balance, 2 equations that guarantee that the columns balance, 4 price time quantity identities for accounts, 5 cell equations, 3 price time quantity identities for cells, 3 equations that defines variables as exogenous, and one equation that fixes the numeraire. The cell equations associated with each specification and the account equations associated with different combinations of specifications are defined in appendix A, and the account equations themselves are defined in appendix B. The model that is actually solved is smaller with 12 variables and 13 equations as shown in the column labeled EXPLICIT. Exogenous variables and their corresponding equations are not included explicitly, and endogenous QSOL and QCSOL variables are computed from their definitional equations after the rest of the model has been solved.

Table 11.2

The cell variables and cell equations are grouped as belonging to Factors (FAC), Institutions (INS), Activities/Commodities/Rest of world (ACR), or Indirect taxes (TAX) according to the TYPE of the row and column in which they appear

|       | MF  | NMF | INST | INSTC | TAX | AC  | TNMF | ROW |
|-------|-----|-----|------|-------|-----|-----|------|-----|
| MF    |     |     |      |       |     | ACR |      |     |
| NMF   |     |     |      |       |     | ACR | ACR  | ACR |
| INST  | FAC | FAC | INS  |       | TAX |     |      | INS |
| INSTC | FAC | FAC | INS  |       | TAX |     |      | INS |
| TAX   | FAC | FAC | INS  |       |     | ACR | ACR  |     |
| AC    |     |     |      | INS   |     | ACR |      | ACR |
| TNMF  |     |     |      |       |     | ACR |      |     |
| ROW   | FAC | FAC | INS  |       |     | ACR |      | ACR |

In this case we have an unbalanced model because we have more equations than variables. The third set of statistics can often be useful for finding which part of the model the error comes from. The variables and equations are grouped according to the types of the accounts, with cell variables and equations counted as shown in table 11.2. The statistics shows that there is an imbalance in the activity/commodity/rest of world part, so the error is most likely in this part. It is possible to have positive and negative imbalances in different components of a correct model, so these statistics should only be used as a first guide.

If a model is created by HERCULES, but it cannot be solved, then HERCULES returns a column in the account table and a plane in the cell table both labeled 'NEQ-NVAR' with the number of equations minus the number of variables associated with each account and each cell. The equation and variable counting procedure discussed in chapter 10 can then be applied to these numbers to help identify the error. We extract this information into the parameters NENVA and NENVC and display it in line 37-42 in the GAMS program. The result is shown on page 5 of the output; there is only one unbalanced account, namely the ACTIVITY account, and no unbalanced cells, so it is clear that an investigation of the ACTIVITY account will be useful. The remaining output identifies both unbalanced models and unbalanced submodels and it will therefore be explained in connection with unbalanced submodels in the next section.

### 11.8 Unbalanced Submodels

The problem with the fixed price in the ACTIVITY account has been corrected in the model in Appendix 11.2 (and in file HERCER10.GMS). The output shows that the model now has the same number of variables as equations, namely 21, and that the subsets grouped by account type are balanced. However, the model has at least one overdetermined submodel and one underdetermined submodel. HERCULES tries to identify both over- and underdetermined submodels, and displays all small offending submodels. In this case there are two. The smaller of these, the overdetermined submodel, is identified at the ennd of page 4 of the output.

The way over- and underdetermined submodels are identified needs a little explanation. The submodel has 3 variables identified as

```
PARAMETER AT(ACTIVITY,QSOL),  
PARAMETER CT(LABOR,ACTVITY,QCSOL), and  
PARAMETER CT(CAPITAL,ACTIVITY,QCSOL).
```

GAMS always reports variables and equations in terms of the identifiers passed between GAMS and HERCULES, i.e. the account set, the account table, and the cell table. Using the same notation as in Appendix A, B, and C we can also identify the variables as

```
QSOL(ACTIVITY),  
QCSOL(LABOR,ACTIVITY), and  
QCSOL(CAPITAL,ACTIVITY).
```

The submodel also has 4 equations. Two of the equations are identified as the R5 equations for set ACC(LABOR) and set ACC(CAPITAL). Account equations can be found in appendix B. R5 is the equation

$$QSOL(i) = \text{SUM}(j, QCSOL(i,j))$$

that guarantees balance in the row. With  $i$  equal to LABOR and CAPITAL, respectively, and  $j$  representing the nonzero elements in the row, the R5 equations specialize to

$$QSOL(LABOR) = QCSOL(LABOR,ACTIVITY)$$

and

$$QSOL(CAPITAL) = QCSOL(CAPITAL,ACTIVITY)$$

Note that  $QSOL(LABOR)$  and  $QSOL(CAPITAL)$  were not included as variables in the list of variables in the submodel because they are exogenous. The two equations effectively fix  $QCSOL(LABOR,ACTIVITY)$  and  $QCSOL(CAPITAL,ACTIVITY)$ . The next two equations are identified as the QSHR equations associated with  $CT(LABOR,ACTIVITY,SPECS)$  and  $CT(CAPITAL,ACTIVITY,SPECS)$ . These are the cell equations. The name QSHR is the name of the cell equation used internally in HERCULES and it is only needed to distinguish individual equations for the few specifications like FDIST that have more than one equation associated with each cell. The cell equations and their internal names are described in connection with the individual specifications in appendix A, and since the specifications are IO we have the equations

$$QCSOL(LABOR,ACTIVITY) = a * QSOL(ACTIVITY)$$

and

$$QCSOL(CAPITAL,ACTIVITY) = a * QSOL(ACTIVITY)$$

that both independently define  $QSOL(ACTIVITY)$  from the previously defined  $QCSOL$  values.

The economics of the problem should by now be clear. The account ACTIVITY is producing using a production technology without any substitution possibility and simultaneously trying to exhaust two fixed factors. There must be some substitution possibilities before the two factors both can be used fully, and replacing the IO specifications with CES and a nonzero SIGMA value will create a valid model.

### 11.9 Numerically Singular Models

Most model errors are caught as unbalanced models or submodels, but a few models that pass these logical or structural tests do not have a locally unique solution. The Jacobian of the model, i.e. the matrix of derivatives of the model equations with respect to the endogenous variables, is numerically singular. The theoretical basis for analyzing which part of the model the numerical singularity comes from is not very solid, and the error messages are at this point not very informative. The output in appendix 11.3, created by the file HERCER11.GMS, shows an example of a numerically singular model. The error message on page 4 tells that the Jacobian is singular, but the reason is not related to any particular part of the model.

The problem in the particular model arises from the fact that the two activities ACTIV-1 and ACTIV-2 uses exactly the same production technology. The consumer price is fixed as the numeraire; but since the two activity prices must be the same they are both equal to the numeraire. However, the price of ACTIV-2 is fixed independently, causing the numerical singularity. If the two activities have different input shares then the model would be unique and it would solve correctly.

This example shows that numerical singularity can be associated with identical production technologies in different sectors, combined with inconsistently fixed prices. Identical production technologies can sometimes be hidden in the use of intermediate inputs. If the intermediate inputs are 'substituted out' the total use of inputs may become identical. It is not currently known if numerical singularity can appear for other reasons than identical production technology or identical consumption systems linked with fixed prices.

### 11.10 Problems and Errors During Model Solution

This section will give a short explanation of the algorithm used to solve the model, and it will at the same time outline some of the problems and errors that may be encountered.

The mathematical structure of our model can be viewed as

$$F(X,Y,Z) = 0$$

where

$F(.,.,.)$  is a  $n$ -dimensional nonlinear twice differentiable function representing the equations of the model,

$X$  is an  $n$ -dimensional vector of endogenous variables ( $P, Q, Y$ , and  $T$ ),

$Y$  is a vector of exogenous variables and parameters that can be calibrated from the base SAM, and

$Z$  is a vector of parameters that cannot be calibrated from the base SAM.

Initialize  $X(0)$  to represent base prices of 1 and the payments and account totals of the base SAM, and initialize  $Y(0)$  to the parameter values calibrated from the base SAM. The checks and calibrations performed so far ensures that  $X(0)$ ,  $Y(0)$ , and  $Z$  satisfy the model, i.e.  $F(X(0), Y(0), Z) = 0$ , and that the Jacobian of the model can be inverted. We call this point the base point. We also have a set of target values for the exogenous variables  $Y$  that we will call  $Y(1)$ . The implicit function theorem now tells us that  $X$  can be defined as a locally unique and continuously differentiable function of  $Y$  and  $Z$  in a neighbourhood of the base point, i.e.

$$X = G(Y, Z)$$

with

$$X(0) = G(Y(0), Z)$$

$X$  can in this neighbourhood be computed from  $F(.,.,.)$  and the Jacobian in the base point, using an approximation of Newtons method. We are not directly interested in the local behavior of  $G(.,.)$  but rather in

$$X(1) = G(Y(1), Z)$$

We therefore proceed by defining

$$Y(t) = Y(0) + t * (Y(1) - Y(0)) \quad \text{for } 0 \leq t \leq 1$$

and

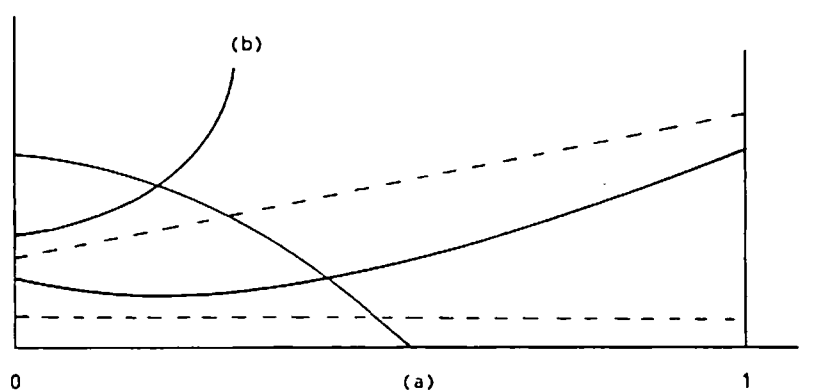
$$X(t) = G(Y(t), Z) \quad \text{for } 0 \leq t \leq 1$$

and we attempt to 'trace'  $X(t)$  from  $X(0)$  to  $X(1)$  as shown in Figure 11.7. Initially, a small value of  $t$  is chosen and  $X(t)$  is computed using Newtons method. If the iterations do not converge we decrease  $t$  and try again; when  $t$  is sufficiently small we are guaranteed convergence. After one point,  $X(t)$ , has been found  $t$  is gradually increased and the starting point for the next set of Newton iterations is computed using a linear or quadratic extrapolation from previous point(s) and Newtons method is applied again. If the new points  $X(t)$  are computed using few Newton iterations then  $t$  is increased fast,

and if many Newton iterations are needed then  $t$  is increased slowly. The solution output from HERCULES shows the number of 'time steps', i.e. the number of different  $t$  values, and the number of Newton iterations used during the whole solution process.

Figure 11.7

The exogenous variables, shown by the dotted lines, are gradually moved from their base value to their target value along straight lines. The endogenous variables, shown by full lines, move along curves in response to the exogenous variables. Endogenous variable (a) and (b) show examples of unnormal behavior.



During this process several problems can be encountered:

- a. An endogenous variable, that should be positive, becomes zero. This is not an algorithmic problem but rather a problem with the model and the exogenous variables. HERCULES stops, prints the value of  $t$ , identifies the variable that becomes zero, and returns  $X(t)$  and  $Y(t)$  to GAMS. Both  $X(t)$  and  $Y(t)$  are returned in the usual columns and planes of the account and cell tables (PSO-L, QSOL, etc. and A-USED, B-USED, etc). Note, that  $X(t)$ ,  $Y(t)$ , and  $Z$  satisfies all the equations of the model so the resulting SAM is balanced and all economic relations are satisfied for the given set of exogenous variables. An inspection of the relationships around the variable that becomes zero will in most cases pinpoint the problem.

Some algorithms based on linearizations, the so-called Johansen Approach, Johansen (1960), the cannot handle negativity and some will not even check for negativity. Other systems, such as Rutherford's MPS/GE, Rutherford (1988), can handle the problem above by switching regime. When a price becomes zero

then the corresponding activity stops producing. Similarly, if a previously unprofitable activity becomes profitable, then it can start producing. It is not clear how this type of switching behaviour can be fitted into a SAM-based system like HERCULES. An activity that suddenly starts producing corresponds to an account suddenly being added to the SAM, or a previously empty account becoming active. How are the parameters calibrated? How are the switching constraints defined? etc.)

- b. An endogenous variable approaches infinity or grows very rapidly. This indicates a rapid structural change in the economy and is also a problem with the model and the exogenous variables. Again, HERCULES stops, identifies the problem and the variable that caused it with a message like 'RATE OF CHANGE GREATER THAN ... IN ...' or 'ACCELERATION GREATER THAN ... IN ...', prints the value of  $t$ , and returns  $X(t)$ ,  $Y(t)$ , and the rate of change of the endogenous variables,  $dX(t)/dt$ . The rate of change is returned in the account and cell tables under the column and plane names of DPSOL, DQSOL, DYSOL, DTSOL, and DQCSOL. An inspection of the rates of change will in most cases pinpoint the problem.

- c. The Newton iterations do not converge or they repeatedly converge slowly. HERCULES will in this case recompute and reinvert the Jacobian in the latest feasible point, and progress will in most cases pick up again. Unfortunately, the Jacobian may also have become numerically singular. This is a case that is even less likely than having a numerically singular Jacobian in the base point, and it has the same problems of not being theoretically well understood. The few cases seen so far were all caused by two production technologies (or their first order approximations) becoming identical. The error messages are in this case similar those one discussed in section 11.7.

Appendix 11.4, created from file HERCER12.GMS, shows selected parts of the computer output of an example of a model in which a variable is going off to infinity. The behavior is created by changing the availability of the factor CAPITAL to zero (EPS). The corresponding price shoots up, and  $dP/dt$  becomes 227 when  $t$  reaches 0.953. The final  $t$  is shown on page 4 of the output and  $dP/dt$  can be seen as the new column DPSOL in the solution summary on page 5.

## CHAPTER 12

### PRODUCT DIFFERENTIATION

#### 12.1 Introduction

The specifications in the HERCULES approach can be thought of as building blocks for model building. They can be combined in a large number of different ways, and only imagination sets the limit. The previous chapters have described the specifications that are used most frequently and given detailed examples of their use. This chapter and the following will go through most of the remaining specifications, but without many details. We will, however, touch on a few modeling areas not covered so far. The interested reader is referred to the appendices for more detailed definitions of the economics and mathematics of the remaining specifications.

So far we have assumed that each activity produces one good only, and that it is sold at the same price to whoever wants it. In section 12.2 we will show how to remove this assumption and replace it by the assumption that one activity produces several (similar) products. The different outputs can have different prices, and the producer can change the output mix as a reaction to changes in prices. In section 12.3 we continue the discussion of multiple outputs and discuss the Make matrix from Input/Output analysis.

We have also assumed that different activities produce different goods, and we will show how to remove this assumption and replace it by the assumption that several activities produce the same or similar goods. The prices of and demand for these goods will then have to be related, and we will show how this can be done in section 12.4.

In section 12.5 we will finally look at several issues around the modeling of labor markets. It is included in this chapter because labor market issues usually are related to differences in labor, e.g. due to difference in skill or sector.

#### 12.2 The CET Specification

In chapter 6 we made the assumption that each sector produces one good, and this good is sold both on the domestic market and on the export market. In addition we assumed that the producer demands the same price from each market (before sales taxes), but otherwise is indifferent about who to sell to. In this chapter we will make an alternative assumption: Each sector produces two types of goods, goods for domestic

consumption and goods for exports, and the export share will depend on the export price relative to the domestic price. This type of assumption can also be used to model subsectors, for example an agricultural production activity that allocates its gross production on grain, vegetables, and other agricultural goods.

**Table 12.1**  
**The Original Specifications for the Sales**  
**from the Activity Accounts**

|            | COM-DOM-AG | COM-DOM-IN | COM-EXP-AG | COM-EXP-IN |
|------------|------------|------------|------------|------------|
| ACT-AGRCLT | 10         |            | 10         |            |
| ACT-INDSTR |            | 10         |            | 10         |
| INDR-TAX   | ITAX       | ITAX       | ITAX       | ITAX       |

The relevant component of the specification table for the model from chapter 6 is repeated here in Table 12.1. The IO specifications defines that one unit of COM-DOM-AG requires one unit of ACT-AGRCLT and one unit of COM-EXP-AG also requires one unit of ACT-AGRCLT. Indirectly, the IO specifications also imply that the two buyers get the same good at the same price (remember the basic HERCULES rule that the price is constant across a row of the SAM), and that the producer is indifferent whether he sells on the domestic market or the export market. There is no direct export supply function but rather a function for total supply. The overall quantity of export depends largely on the demand for exports from the rest of the world as a function of the marginal cost of production.

Let us now change the assumptions slightly. The exported goods are of a different quality from the domestic goods, and the prices can therefore be different. Also, the production equipment is such that the producer cannot shift completely from one output to another; there is a certain transformation possibility. We will assume that the producer will determine the production schedule to maximize profits subject to the relative prices and the transformation possibilities.

A convenient formulation of the transformation possibilities is a CET (Constant Elasticity of Transformation) function, and the corresponding allocation rule is available in HERCULES with specification CET. CET is similar to CES with a few differences: CET describes outputs where CES describes inputs, and the CET specifications that belong together are therefore in a row while the CES specifications are in a column. When prices change CES will substitute away from the more expensive inputs while CET will transform to produce more of the expensive output.

CET depend on a transformation elasticity that in HERCULES is called SIGMAR. Its value must be less than or equal to zero. Zero is equivalent to no transformation, which means that the outputs are produced in fixed (quantity) shares, independent of prices. And minus infinity (-INF) is equivalent to perfect transformation, which means that the outputs essentially are the same, i.e. IO the assumption we used in chapter 6.

**Table 12.2**  
**The Revised Specifications for the Sales**  
**from the Activity Accounts**

|            | COM-DOM-AG | COM-DOM-IN | COM-EXP-AG | COM-EXP-IN |
|------------|------------|------------|------------|------------|
| ACT-AGRCLT | CET        |            | CET        |            |
| ACT-INDSTR |            | CET        |            | CET        |
| INDR-TAX   | ITAX       | ITAX       | ITAX       | ITAX       |

**Table 12.3**  
**Parameter values of Agricultural Transformation Elasticity, SIGMAR,**  
**and Agricultural Export Demand Elasticity, ETA,**  
**for four cases**

| CASE    | SIGMAR | ETA |
|---------|--------|-----|
| INF/INF | -INF   | INF |
| INF/3   | -INF   | 3.0 |
| 3/INF   | -3.0   | INF |
| 3/3     | -3.0   | 3.0 |

Using CET we can reformulate the activity to commodity sales part of the model onto the form shown in Table 12.2. Note that the definition of CET is such that the transformation between domestic and export supply is based on the prices that the producer receives, i.e. the prices before taxes, and not directly in the prices of the columns with specification CET. The formulation with CET has been implemented in MODEL11.GMS, that is similar to MODEL.D.GMS with the only change being the one from Table 12.1 to 12.2. The initial value of SIGMAR is -INF in both sectors, i.e. the model is equivalent to the one in MODEL.D.GMS. The experiment is also the same as

in chapter 6, namely an increase in agricultural world prices of 10% and a decrease in industrial world prices of 10%.

To test the sensitivity of the model results to changes in transformation and export demand supply parameter values, the model is run with the four sets of parameters shown in Table 12.3 and the results are collected in the reports shown in Table 12.4 and 12.5. When SIGMAR = -INF the domestic price of agricultural goods is equal to the export price, and when SIGMAR = -3 it is less. SIGMAR = -INF therefore reduces the domestic demand and allows the export to grow relative to the case when SIGMAR = 3. Similarly, the increased world price has full impact on the export price when ETA = INF, and only partial impact when ETA = 3, and the quantity effects are therefore also larger when ETA = INF.

**Table 12.4**  
**Quantities of all Commodities for different values of**  
**Transformation Elasticity and Export Demand Elasticity**

| ----        | 236 PARAMETER REPORTQ |         | QUANTITIES OF ALL COMMODITIES |         |         |
|-------------|-----------------------|---------|-------------------------------|---------|---------|
|             | BASE-CASE             | INF/INF | INF/3                         | 3/INF   | 3/3     |
| COM-DOM-AG  | 160.000               | 162.028 | 165.652                       | 165.930 | 166.998 |
| COM-DOM-IND | 195.000               | 192.719 | 192.973                       | 192.917 | 193.032 |
| COM-IMP-AG  | 70.000                | 70.887  | 65.041                        | 66.846  | 64.087  |
| COM-IMP-IND | 120.000               | 130.071 | 128.033                       | 129.070 | 127.899 |
| COM-CMP-AG  | 230.000               | 232.915 | 230.600                       | 232.721 | 230.949 |
| COM-CMP-IND | 315.000               | 322.142 | 320.578                       | 321.456 | 320.518 |
| COM-EXP-AG  | 75.000                | 90.257  | 83.569                        | 84.468  | 81.812  |
| COM-EXP-IND | 55.000                | 41.690  | 43.886                        | 42.799  | 44.064  |

### 12.3 Make Matrices

In detailed Input/Output analysis we will often see that most firms produce a main product, namely the product used to classify the firm into a sector, and several by-products. The output of one sector will not be one product but rather several products. This multiplicity of outputs can be described with the so-called Make Matrix, that is completely analogous to the Input/Output matrix describing the multiplicity of inputs. Please note, that the make matrix does not have to be square; the number of commodities can in principle be different from the number of activities.

The standard way to model the make matrix in Input/Output theory is to assume that the outputs are proportional. This can in HERCULES be modeled with specification CET with SIGMAR = 0, or with the equivalent specification CET0. It is of course also possible to model the make matrix with specification CET with non-zero

Table 12.5  
Prices of all Commodities for different values of  
Transformation Elasticity and Export Demand Elasticity

| --- 236 PARAMETER REPORT  |         |       |       |       |       |
|---------------------------|---------|-------|-------|-------|-------|
| PRICES OF ALL COMMODITIES |         |       |       |       |       |
|                           | INF/INF | INF/3 | 3/INF | 3/3   |       |
| COM-DM-AG                 | 1.000   | 1.100 | 1.061 | 1.070 | 1.053 |
| COM-DM-IN                 | 1.000   | 1.083 | 1.046 | 1.064 | 1.043 |
| COM-IMP-AG                | 1.000   | 1.100 | 1.100 | 1.100 | 1.100 |
| COM-IMP-IN                | 1.000   | 0.900 | 0.900 | 0.900 | 0.900 |
| COM-CMP-AG                | 1.000   | 1.100 | 1.072 | 1.079 | 1.067 |
| COM-CMP-IN                | 1.000   | 1.011 | 0.989 | 1.000 | 0.987 |
| COM-EXP-AG                | 1.000   | 1.100 | 1.061 | 1.100 | 1.069 |
| COM-EXP-IN                | 1.000   | 1.083 | 1.046 | 1.064 | 1.043 |

transformation elasticity SIGMAR in which case the technical coefficients in the make matrix becomes price responsive.

Please note that our usual price and homogeneity rules are reversed in connection with specification CET. The activities or the rows sell to the commodities or the columns so the transactions in one row describe the different outputs produced by the activity. The prices associated with the transactions are the prices of the columns, and they will in general be different. On the other hand, the transactions in a column describe the different source of the commodity, and these commodities are assumed to be homogenous, and all transactions in the column are therefore assumed to take place at the same price.

The reversal of the role of rows and columns has an implication of the definition of the constant price SAM in QCSOL and the RESIDUALS. The price used to deflate cells with specification CET is the price of the column. The sum of QCSOL in the row can therefore be different from QOL, and this difference is recorded in the RESIDUAL column in the account table. On the other hand, the sum of QCSOL in the column will be equal to QOL. Note, that the RESIDUAL in connection with CET always is positive, i.e. the transformation possibility gives rise to a gain in constant

prices, while the RESIDUAL in connection with CES is negative, i.e. the substitution gives rise to a loss.

MODEL2.GMS gives an example of a model with a make matrix. It is based on MODEL1 with the only exception that the submatrix of the SAM that describe how commodities buy from activities is changed from a diagonal matrix into a full matrix. The specification matrix for this component is shown in Table 12.6. Note that the commodity columns buy from several activities and pay a sales tax. Internally in HERCULES this is split into two stages: the commodities buy from the activities at pre-tax prices in stage one, so the activities use the pre-tax prices as a basis for their product transformation decisions. The sales tax is paid in stage two, and the final price of the commodity account is the post-tax price.

Table 12.6  
The Specification Matrix for Activities selling to Commodities  
when each Activity produces Several Outputs.

|            |            |            |
|------------|------------|------------|
|            | COM-AGRCLT | COM-INDSTR |
| INDR-TAX   | ITAX       | ITAX       |
| ACT-AGRCLT | CET        | CET        |
| ACT-INDSTR | CET        | CET        |

#### 12.4 Multiple Producers and Similar Goods

In reality, a 'sector' will almost always consist of many producers that produce similar goods. However, we must limit the size and data requirement of our models and we will therefore aggregate the producers and the product they produce into groups that are as homogenous as possible and describe the part of reality we want to model as accurately as possible. The homogeneity test is not straight forward, and in certain cases you will have two or more groups of producers that produce very similar products, but otherwise are quite different e.g. in the technology they use (modern vs. traditional or capital intensive vs. labor intensive), in their ownership (private vs. public), or in they way the operate (marginal cost pricing in private companies and fixed prices in public companies).

It these cases you may need two or more producers for the sector in question in the production part of the model, but you may still only want one commodity in the demand part of the model. The standard way to model this situation is to model the

Table 12.7  
 Results of a Model with Two Agricultural Producers.  
 SIGMA represents the Elasticity of Substitution  
 between the Agricultural Products

| ---- | 164 PARAMETER COMM | SUMMARY TABLE OF QUANTITIES<br>AFTER CHANGING TAX RATES |           |           |           |
|------|--------------------|---|-----------|-----------|-----------|
|      |                    | BASE-CASE   | SIGMA=INF | SIGMA=2.0 | SIGMA=0.0 |
|      | ACT-AG-FOR         | 80.000  | 76.043    | 78.963    | 79.278    |
|      | ACT-AG-INF         | 120.000   | 121.571   | 119.175   | 118.916   |
|      | ACT-INDSTR         | 235.000   | 236.401   | 236.786   | 236.828   |
|      | COM-AGRCLT         | 230.000   | 227.256   | 227.858   | 227.923   |
|      | COM-INDSTR         | 245.000   | 246.461   | 246.862   | 246.905   |

producers as we have done before and then introduce a trader that buys the output from the producers and creates a uniform product to be sold on the market. The creation of this uniform product can very conveniently be modeled the same way as we modeled composite commodities based on domestically produced and imported goods, namely with CES substitution specifications (see the Armington approach in chapter 6). If the products are perfect substitutes we just define the elasticity of substitution, SIGMA, to be infinite (INF), or we use the special case of specification CES, SUBST.

MODEL13.GMS contains an example of this. The model is created by splitting the agricultural sector in MODEL1.GMS from chapter 5 into two sectors, a labor intensive informal sector and a capital intensive formal sector. The agricultural commodity is in the base version of the model created from the output of the two agricultural sectors using specification SUBST, i.e. the outputs are considered to be the same. In two alternative versions of the model we change the specifications to CES with SIGMA = 2 and SIGMA = 0. (Note that it is easy to change model specifications by just changing entries in the Cell Table).

The results of increasing the agricultural tax by a factor 1.1 in these three model versions can be seen in Table 12.7. The increased tax reduces demand for agricultural goods in all cases. When SIGMA = 0 there is no substitution possibilities between the two agricultural sectors, and the decrease in demand is split equally between them. However, when SIGMA = INF we see a very different picture. Production in the informal sector grows while production in the formal sector must take all the decrease. The reason is, that the increase in agricultural tax favors industry; labor becomes

cheaper and capital becomes more expensive, and agricultural production shifts towards the labor intensive informal sector.

A warning about specification SUBST may be useful. SUBST indicates that two or more goods are perfect substitutes, which implies that their prices must be the same. However, there is no longer a rule for splitting the overall demand on the different goods; the split must be defined from the supply side, e.g. from the marginal costs of production. You will therefore run into trouble if two suppliers both have constant prices. This is for example the case if one supplier is an importer facing fixed world prices and the other supplier is a public sector with regulated supply prices.

## 12.5 Models of the Labor Market

In the models we have seen so far there has only been one homogenous, perfectly mobile type of labor.

The first extension that comes to mind is one in which there are several labor categories, e.g. agricultural, unskilled, skilled, and managerial labor. The producers will buy varying amounts of each labor type, and the natural model is one in which labor is combined using some kind of aggregation function like CES, possibly with  $SIGMA = 0$ . If each labor category can be assumed to be homogenous then the relevant part of the specification matrix will look like Table 12.8.

Table 12.8  
The Labor-Activity part of the Specification Matrix  
for a model with Multiple Labor Categories

|         | ACTIVITY-1 | ACTIVITY-2 | ACTIVITY-3 | ACTIVITY-4 |
|---------|------------|------------|------------|------------|
| LABOR-1 | CES        | CES        | CES        | CES        |
| LABOR-2 | CES        | CES        | CES        | CES        |
| LABOR-3 | CES        | CES        | CES        | CES        |

The assumption of homogenous labor is not always a good description of reality. In many countries we observe fixed wage differentials between sectors, even for similar labor categories. Or in other words, if a worker moves from one activity to another he is paid a different wage. In HERCULES you can model this by using the MARKUP specification: The labor used in a particular sector is 'created' from a basic type of labor with the addition of a fixed percentage markup. The modeling of the overall wage level can be done independently. The overall supply of the basic labor type can be fixed and

the wage level can adjust to ensure full employment, i.e. all wages move proportionate, or the overall wage level, i.e. the wage level for the basic labor, can be fixed and total employment will vary. The relevant subset of the specification matrix for this type of model is shown in Table 12.9. Note that the corresponding base SAM must be constructed properly. The numbers in the BASE-LABOR row with specification IO must correspond to employment valued at a basic wage rate, and the numbers in the EXTR-LABOR row with specification MARKUP must correspond to the additional wage. The account type for BASE-LABOR is MF, and the account type for EXTR-LABOR is NMF.

**Table 12.9**  
**The Labor Component of a Model with**  
**Fixed Wage Differentials**

|            | BASE-LABOR | EXTR-LABOR | LABOR-AG | LABOR-IND | LABOR-SER | AGRICULT | INDUSTRY | SERVICES |
|------------|------------|------------|----------|-----------|-----------|----------|----------|----------|
| BASE-LABOR |            |            | IO       | IO        | IO        |          |          |          |
| EXTR-LABOR |            |            | MARKUP   | MARKUP    | MARKUP    |          |          |          |
| LABOR-AG   |            |            |          |           |           | CES      |          |          |
| LABOR-IND  |            |            |          |           |           |          | CES      |          |
| LABOR-SER  |            |            |          |           |           |          |          | CES      |
| HOUSEHOLDS | IDIST      | IDIST      |          |           |           |          |          |          |

In developing countries we will often observe a formal sector with an organized labor market with wages that are fixed in the medium term, and an informal sector in which wages are best described as market clearing. In HERCULES you will model this with two independent types of labor, formal labor with a fixed wage (P) and varying demand, and informal with a fixed quantity (Q) and varying wage. The formal sectors buy formal labor and the informal sector buys informal labor.

You may extend this model by assuming that the two labor categories are not independent. The workers that do not get a job in the formal sector will seek employment in the informal sector. From a modeling point of view this means, that there is really only one type of labor, but some are lucky to get employment in the formal sector at a higher wage, i.e. they collect a rent that depend on the endogenous wage differential. The relevant subset of the specification matrix for a model based on this story is shown in Table 12.10. The account type for LABOR-INF is MF and it has a fixed quantity (Q), the account type for LABOR-FOR is AC and it has a fixed wage (P), and the account type for LABOR-RENT is NMF. LABOR-FOR is of type AC and not MF because it is a transformed factor, namely LABOR-INF with an added RENT.

If LABOR-FOR had been counted as a factor account then we would double count the labor income in GDP at factor cost.

**Table 12.10**  
**The Specification Matrix for a Formal-Informal**  
**Labor Market with Spillover**

|            | LABOR-INF | LABOR-FOR | LABOR-RENT | ACTIV1-FOR | ACTIV2-FOR | ACTIV1-INF | ACTIV2-INF |
|------------|-----------|-----------|------------|------------|------------|------------|------------|
| LABOR-INF  |           | IO        |            |            |            | CES        | CES        |
| LABOR-FOR  |           |           |            | CES        | CES        |            |            |
| LABOR-RENT |           |           | RENT       |            |            |            |            |
| HOUSEHOLDS | IDIST     |           |            | IDIST      |            |            |            |

## CHAPTER 13

### MULTIPLE REST OF WORLD ACCOUNTS

#### 13.1 Introduction

This final chapter will make a quick tour through most of the specifications that can be used to model foreign exchange transactions. HERCULES was originally developed in the World Bank and used for models of developing countries, and at that time many developing countries had all kinds of regulations on the use of foreign exchange. The number of different specifications in this area is therefore quite large.

The exchange rate is in HERCULES interpreted as a price, the price of foreign currency measured in domestic currency. The exchange rate is, as all other prices, an index normalized to one in the base case, so it measures the number of foreign currency units per domestic currency unit in the solution case divided by the number of foreign currency units per domestic currency unit in the base case. An exchange rate greater than one will therefore mean that the domestic currency has been devalued.

There can be many reasons for having more than one Rest-Of-World (ROW) account in a SAM-based model. The most common reasons are:

- The country has several markets for the same export good and we want to model the export markets differently, e.g. with different growth prospects and/or different price elasticities.
- The country has a multiple exchange rate system in which certain transactions take place at one exchange rate and other transactions take place at another exchange rate. The relative exchange rates may be fixed or determined by supply and demand. And the foreign exchange may or may not be freely available at the prevailing exchange rate.
- The country allocates foreign exchange between different types of imports. The exchange rate that the different groups of importers pay may and may not be the same.

Table 13.1 shows the general structure of the ROW accounts of a HERCULES model with multiple ROW accounts. Transfers between our economy and the rest of the world has not been included. They were discussed in chapter 8 and will only be mentioned in this chapter when necessary. We will shortly discuss the ordinary trade

payments related to imports and exports in section 13.2 followed by a discussion of money exchanges and other transactions between ROW accounts, those labeled EXCHANGE and EX.PRF. in Table 13.1, in section 13.3.

**Table 13.1**  
The structure of the Rest Of World  
accounts in a HERCULES model

|               | IMPORT<br>GOOD 1 | IMPORT<br>GOOD 2 | IMPORT<br>GOOD 3 | ROW<br>1 | ROW<br>2 | ROW<br>3 |
|---------------|------------------|------------------|------------------|----------|----------|----------|
| NATIONAL BANK |                  |                  |                  | EX.PRF   | EX.PRF   |          |
| EXPORT GOOD A |                  |                  |                  | EXPORT   | EXPORT   |          |
| EXPORT GOOD B |                  |                  |                  |          | EXPORT   | EXPORT   |
| EXPORT GOOD C |                  |                  |                  | EXPORT   |          |          |
| INDIRECT TAX  | ITAX             | ITAX             | ITAX             |          |          |          |
| ROW 1         | IMPORT           |                  |                  |          |          |          |
| ROW 2         |                  | IMPORT           | IMPORT           |          |          |          |
| ROW 3         |                  |                  |                  | EXCHANGE | EXCHANGE |          |

## 13.2 Import and Export

The supply of imports from the rest of the world, modeled with the IMPORT specification, is in HERCULES assumed to be perfectly elastic and we can therefore only have one IMPORT specification in each column. Imports from different geographical areas must have separate accounts and the demand for each component of import must be modeled as part of the domestic economy, e.g. using the Armington approach discussed in chapter 6.

Export demand, modeled with the EXPORT specification, is in general a function of the price at which it is supplied, i.e. the price in the row, and we can therefore have several EXPORT specifications in the same row. The different demand functions will determine the different components of exports.

It is possible to model an export market with infinite demand elasticity by combining the EXPORT specification with the parameter value  $\text{ETA} = \text{INF}$  or by using specification EXPINF. Export is in this case the difference between the domestic production and other demands, and you can therefore only have one EXPORT with infinite elasticity in each row.

### 13.3 Transfers between Rest of World Accounts

A ROW account describes flow of foreign exchange and the account must by definition balance in foreign exchange; any changes in reserves must be included in the accounts. However, the flows in the SAM must all be reported in the same unit, usually in domestic currency. The foreign exchange figures in the ROW accounts must therefore be converted to domestic currency. When all transactions in the account, both in the row and in the column, are converted using the same exchange rate then the ROW account will automatically balance. The conversion with the same exchange rate can therefore only create problems when one ROW account transfers foreign exchange to another ROW account with a different exchange rate. An example of this complication is shown by the example in Table 13.2 and 13.3.

Table 13.2  
Flows measured in Foreign Currency Units (FCU)

|         | IMPORT1 | IMPORT2 | ROW1 | ROW2 | TOTAL |
|---------|---------|---------|------|------|-------|
| EXPORT1 |         |         | 10   |      | ..    |
| EXPORT2 |         |         |      | 25   | ..    |
| ROW1    |         | 20      |      |      | 20    |
| ROW2    | 15      |         | 10   |      | 25    |
| TOTAL   | ..      | ..      | 20   | 25   |       |

Table 13.2 shows the transactions in foreign exchange. Note, that the two ROW accounts balance. Exports at the ROW2 exchange rate are larger than imports, and ROW1 buys the extra foreign exchange from ROW2. This is shown with 10 (FCU) in row ROW1 and column ROW2.

The National Bank is assumed to buy the foreign exchange earned by exporters, and to sell the necessary foreign exchange to importers. The exchange rate for transactions in ROW1 is 15 Domestic Currency Units (DCU) per Foreign Currency Unit (FCU), and the exchange rate for ROW2 is 10 DCU per FCU. Table 13.3 shows the transactions converted into domestic currency using these exchange rates. Note how the transfer of foreign exchange from one ROW account to another ROW account with a different exchange rate is recorded. The National Bank buys 10 FCU net from the exporters at the 10 DCU per FCU exchange rate for a total of 100 DCU. This number is reported in row ROW2 and column ROW1. It then sells the same 10 FCU to importers at the 15 DCU per FCU exchange rate for a total of 150 DCU and it earns

the difference of 50 DCU. This earning is reported in row NAT-BANK and column ROW1. This is an 'exchange profit' and it was included in Table 13.1 in the cells labeled 'EX.PRF.'.

Table 13.3  
Flows measured in Domestic Currency Units (DCU)

|          | IMPORT1 | IMPORT2 | ROW1 | ROW2 | TOTAL |
|----------|---------|---------|------|------|-------|
| NAT-BANK |         |         | 50   |      | ..    |
| EXPORT1  |         |         | 150  |      | ..    |
| EXPORT2  |         |         |      | 250  | ..    |
| ROW1     |         | 300     |      |      | 300   |
| ROW2     | 150     |         | 100  |      | 250   |
| TOTAL    | ..      | ..      | 300  | 250  |       |

When foreign exchange is freely mobile between ROW accounts we cannot model the gross transfers between any pairs of ROW accounts with the specifications currently available in HERCULES. We will only be able to model net transfers, and a sequence of transfers (ROWa -> ROWb -> ROWc) cannot be distinguished from a single transfer (ROWa -> ROWc) and vice versa. To guarantee that the HERCULES model is unique we therefore require that each ROW account has a maximum of one transfer to another ROW account, i.e. that there is no more than one transfer per column in the ROW by ROW block of the SAM. To prevent circular flows we also require that at least one of the ROW columns does not have any transfers to other ROW accounts. Uniqueness can be guaranteed under more general conditions, but this condition is easy to verify, both for the modeler and for HERCULES.

It will usually be convenient to net all transfers out by channeling them through one base account. The structure of the ROW by ROW block will in this case be as shown in Table 13.4 where ROW<sub>i</sub> is the base account, for example the account with the official exchange rate.

The ROW by ROW block will usually have exactly  $n-1$  nonzero cells and all foreign exchange markets will be connected. Nonzero is here as everywhere else in HERCULES used to denote "potentially nonzero", i.e. cells in which payments can take place for some parameter values. In rare cases there will only be  $n-j$ ,  $j > 1$ , nonzeros in the block and then there will be  $j$  disconnected foreign exchange markets, where disconnected means, that foreign exchange bought on one market cannot be used in the other market. An example with two disconnected markets is shown in Table 13.5. ROW<sub>i</sub> is

Table 13.4  
The Net Payments in the ROW by ROW part of the SAM

|      |             |
|------|-------------|
|      | R R R R     |
|      | 0 0 . 0 . 0 |
|      | W W W W     |
|      | 1 2 i n     |
| ROW1 |             |
| ROW2 |             |
| ..   |             |
| ROWi | X X 0 X     |
| ..   |             |
| ROWn |             |

Table 13.5  
The ROW by ROW block of a SAM with two Disconnected Foreign Exchange Markets

|        |                     |
|--------|---------------------|
|        | R R R R R R         |
|        | 0 . 0 . 0 0 . 0 . 0 |
|        | W W W W W W         |
|        | 1 i k k+ j n        |
|        | 1                   |
| ROW1   |                     |
| ..     |                     |
| ROWi   | X 0 X               |
| ..     |                     |
| ROWk   |                     |
| ROWk+1 |                     |
| ..     |                     |
| ROWj   | X 0 X               |
| ..     |                     |
| ROWn   |                     |

the base account for the first group of ROW accounts and ROW<sub>j</sub> is the base account for the second group.

The specification for an unconstrained flow between ROW accounts with the same exchange rate is FEXCHE where the last E represent the Equal exchange rates. The cell equation ensures that the two exchange rates are equal. The term unconstrained flow is used here to indicate that this particular part of the model does not impose any constraints on the movement of foreign exchange, i.e. there is no equation involving TSOL. However, there can be constraints elsewhere like for example quantity restrictions on imports.

The specification for an unconstrained flow between ROW accounts with different exchange rates is FEXCHD where the last D represent the Different exchange rates. When ROW<sub>i</sub> and ROW<sub>j</sub> have different exchange rates and there is a transfer between row ROW<sub>i</sub> and column ROW<sub>j</sub> modeled with specification FEXCHD then an exchange agent must earn (or loose) on the transaction. The HERCULES conventions require that this exchange earning is reported in a single cell in column ROW<sub>j</sub>. The specification in this cell is FTAX if the two exchange rates are proportionate, and FRENT if the ratio of the exchange rates is endogenous.

Many countries have schemes for allocating available foreign exchange for different purposes. In HERCULES you can allocate foreign exchange in fixed proportions to accounts with the same exchange rate with the FDIST specification. The foreign exchange is earned in the column of the SAM and distributed to two or more FDIST cells in the row. FDIST is an unusual specification in HERCULES because it gives two cell equations: one that defines the payment as the correct share of the total, and one that defines the exchange rate of the row to be equal to the exchange rate of the column.

Other allocation schemes may have shares that depend on the relative exchange rates. Workers working abroad can sometimes send money home via an official or an unofficial market, and the share send via the unofficial market may depend on the exchange premium. The FALOC specification, that is similar to specification CET used for commodity allocations, can be used to model this type of allocation.

The allocation schemes for 'cheap' foreign exchange will in many cases not give enough foreign exchange to cover demand. The remaining demand can then be covered from a free market. The marginal cost of foreign exchange is of course the cost on the free market. If imports are priced at this marginal cost then the importer collects a foreign exchange rent. This can be modeled by moving the cheap foreign exchange to an account with the exchange rate of the free market using specification FEXCHD combined with FRENT. An alternative formulation will describe some government con-

trolled importers. They use the average cost of foreign exchange to calculate the sales price of imports, and the exchange rate for this average foreign exchange can in HERCULES be modeled with specification FAVRG, that is similar to specification AVRGR used for commodities.

All these specifications can be put together in numerous ways, depending on how the exchange rate system works. We will cover some of them by reviewing four hypothetical cases. We will for each verbally explain the exchange rate system and show the relevant part of the specification table.

CASE 1: The model uses two ROW accounts. The two accounts represent different parts of the world, e.g. close neighbors vs. distant trading partners, or OECD countries vs. developing countries. The reason for the disaggregation is different behaviours of the export markets and/or different tax rates for imports. There is no difference between the foreign exchange from the two markets, and foreign exchange earned in one market can freely be spent in the other market.

A HERCULES model describing this type of environment will have specifications like those shown Table 13.6. The common exchange rate can be either endogenous or exogenous. But note, that you can only fix one of the exchange rates; the other one is then determined by the equation associated with the FEXCHE cell.

Table 13.6  
The Specification for Case 1

|                   | IMPORT<br>GOOD 1 | IMPORT<br>GOOD 2 | ROW<br>1 | ROW<br>2 |
|-------------------|------------------|------------------|----------|----------|
| SAVING/INVESTMENT |                  |                  | UNSPEC   |          |
| EXPORT GOOD A     |                  |                  | EXPORT   | EXPORT   |
| EXPORT GOOD B     |                  |                  | EXPORT   |          |
| INDIRECT TAX      | ITAX             | ITAX             |          |          |
| ROW 1             | IMPORT           |                  |          |          |
| ROW 2             |                  | IMPORT           | FEXCHE   |          |

The foreign saving is in the example modeled with the UNSPEC specification, i.e. foreign saving is assumed to clear the savings-investment balance. Only one of the ROW accounts can have an UNSPEC specification; there is nothing in the model that can choose where to borrow. Note, that a model with no transfer between the two ROW accounts, i.e. no FEXCHE specification, and instead one UNSPEC specification

in each of the two ROW accounts is a different model; the two foreign exchange markets will be disconnected and the model will not have a relationship between the two exchange rates.

CASE 2: This model uses two ROW accounts representing two exchange rates that are kept in fixed proportion. All imported and exported commodities are classified according to which exchange rate they are traded at. The specifications for this model are shown in Table 13.7.

Table 13.7  
The Specifications for Case 2

|                   | IMPORT<br>GOOD 1 | IMPORT<br>GOOD 2 | ROW<br>1 | ROW<br>2 |
|-------------------|------------------|------------------|----------|----------|
| SAVING/INVESTMENT |                  |                  | UNSPEC   |          |
| NATIONAL BANK     |                  |                  | FTAX     |          |
| EXPORT GOOD A     |                  |                  | EXPORT   |          |
| EXPORT GOOD B     |                  |                  |          | EXPORT   |
| INDIRECT TAX      | ITAX             | ITAX             |          |          |
| ROW 1             | IMPORT           |                  |          |          |
| ROW 2             |                  | IMPORT           | FEXCHD   |          |

Note that there is only one EXPORT specification per row in contrast to case 1. Also note the presence of the FEXCHD and FTAX specifications.

As in case 1, both exchange rates can be endogenous, or one of them can be fixed effectively fixing the other. The comment from case 1 about modeling foreign savings will also apply.

CASE 3: ROW1 is in this case a Rest-Of-World account with an 'official' exchange rate. All export earnings must be exchanged in the National Bank at this exchange rate. A few 'essential' imports can be payed with foreign exchange bought at the official exchange rate. The foreign exchange that is left after these imports have been payed is sold by the National Bank in an open market at a price that will equate demand and supply, and all 'non-essential' imports must be payed with foreign exchange bought in this open market. The HERCULES specifications for this type of model are shown in Table 13.8. Note the combination of FEXCHD and FRENT that defines the floating

relationship between the two exchange rates. The exchange rate in the ROW2 account must be endogenous. The official exchange rate in ROW1 can be exogenous or endogenous.

Table 13.8  
The Specifications for Case 3

|               | IMPORT<br>ESSENT | IMPORT<br>NON-ESS | ROW<br>1 | ROW<br>2 |
|---------------|------------------|-------------------|----------|----------|
| NATIONAL BANK |                  |                   |          | FRENT    |
| EXPORT GOOD A |                  |                   | EXPORT   |          |
| EXPORT GOOD B |                  |                   | EXPORT   |          |
| INDIRECT TAX  | ITAX             | ITAX              |          |          |
| ROW 1         | IMPORT           |                   |          | FEXCHD   |
| ROW 2         |                  | IMPORT            |          |          |

CASE 4: All exports in this model takes place at an 'official' exchange rate, and all export earnings are exchanged in the National Bank at this exchange rate. The foreign exchange is then allocated in fixed proportions to three types of imports after certain fixed payments, e.g. interest payments, have been made. The importers are assumed to use all the foreign exchange they are allocated and to sell the imported goods at the market clearing price, thereby collecting a rent.

The HERCULES specifications for this type of model is shown in Table 13.9. Note the difference in who receives the rent between this case and the previous one.

Table 13.9  
The Specifications for Case 4

|                  | INTEREST | IMPORT | IMPORT | IMPORT | ROW    | ROW  | ROW   | ROW   | ROW   |
|------------------|----------|--------|--------|--------|--------|------|-------|-------|-------|
|                  |          | AGR    | IND    | SERV   | TOTAL  | DISP | AGR   | IND   | SERV  |
| EXPORT GOOD A    |          |        |        |        | EXPORT |      |       |       |       |
| EXPORT GOOD B    |          |        |        |        | EXPORT |      |       |       |       |
| IMPORT RENT AGR  |          | RENT   |        |        |        |      |       |       |       |
| IMPORT RENT IND  |          |        | RENT   |        |        |      |       |       |       |
| IMPORT RENT SERV |          |        |        | RENT   |        |      |       |       |       |
| INDIRECT TAX     |          | ITAX   | ITAX   | ITAX   |        |      |       |       |       |
| ROW TOTAL        | FEXO     |        |        |        |        | EXCH |       |       |       |
| ROW DISPOSABLE   |          |        |        |        |        |      | FDIST | FDIST | FDIST |
| ROW AGRICULT     |          | IMPORT |        |        |        |      |       |       |       |
| ROW INDUSTRY     |          |        | IMPORT |        |        |      |       |       |       |
| ROW SERVICES     |          |        |        | IMPORT |        |      |       |       |       |

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APPENDICES

VOLUME II



## APPENDIX A

### CELL SPECIFICATIONS

#### Introduction

This appendix describes the cell specifications that form the building blocks for defining SAM based models with HERCULES. The possible specification acronyms are shown in alphabetical order in Table A.1 and are classified according to their use in Table A.2; their descriptions follow in alphabetical order. The description of each specification follows the same pattern. The first item is a description of the underlying economic assumptions including a definition of where in the model the specification can be used and which other specifications it can be combined with. If there is no comment about legal combinations in the row or the column, then this specification does not restrict the combinations.

The second item is a list of the mathematical equations in the cell. Unless explicitly stated, there are two variables, TSOL and QCSOL, in each cell; in most cases there are also two equations, a behavioral equation defining TSOL or QCSOL, and an identity relating TSOL and QCSOL. Many special cases exist, however. QCSOL need not exist; there can be two behavioral equations or none; and the behavioral equation(s) can relate the price of the row to the price of the column instead of defining TSOL or QCSOL. In this last case, TSOL is a 'residual', defined implicitly from the balancing equations of the SAM. The third item consists of references to the account equations that link the cells within the account. The account equations themselves can be found in appendix B. Fourth comes a list of restrictions on the variables used in the cell itself and in the corresponding row and column accounts; these usually specify that certain variables must be positive. Note that if variable TSOL must be positive then both the variable QCSOL and the base period value of TSOL, TBASE, must be positive as well. Prices are not mentioned explicitly in this section since there is a global condition that all prices must be positive. If there are interesting relationships between specifications or if specifications have special features, this will be mentioned in a comment section.

The final item is a list of internal equation names for the behavioral equations. These names are only needed for tracing model errors. If a structural error is identified (as will be described in chapter 11) then the equation names used in the error report will be these internal names.

We use the following notation for variables and parameters. ABC(i) (with one index) refers to column "ABC" in the account table, i.e., to AT(i,"ABC") where AT

Table A.1:  
Alphabetic List of Specification Acronyms.

|        |   |  |
|--------|---|--|
| AVRG   | - | perfect substitution with AVerAge price              |
| CD     | - | Cobb-Douglas   |
| CES    | - | Constant Elasticity of Substitution                  |
| CES2   | - | Constant Elasticity of Substitution, 2 levels        |
| CET    | - | Constant Elasticity of Transformation                |
| CETINF | - | CET with infinite elasticity                         |
| CETO   | - | CET with zero elasticity                             |
| DQEXO  | - | change in (Delta) EXOgenous Quantity                 |
| DTAX   | - | Direct TAX   |
| DVEXO  | - | change in (Delta) EXOgenous Value                    |
| EXPORT | - | EXPORT demand function from the rest of the world    |
| EXPINF | - | EXPORT with infinite elasticity of demand            |
| EXP1   | - | EXPORT with elasticity of demand equal to 1          |
| EXPO   | - | EXPORT with elasticity of demand equal to 0          |
| FALOC  | - | Foreign exchange ALLOCation                          |
| FALOCI | - | Foreign exchange ALLOCation with Infinite elasticity |
| FALOCO | - | Foreign exchange ALLOCation with 0 elasticity        |
| FAVRG  | - | Foreign exchange with AVerAge exchange rate          |
| FDIST  | - | Foreign exchange DISTribution                        |
| FEXCHD | - | Foreign EXCHange with Different exchange rate        |
| FEXCHE | - | Foreign EXCHange with Equal exchange rate            |
| FEXO   | - | EXOgenous in Foreign exchange                        |
| FEXDF  | - | EXOgenous in Foreign exchange received From abroad   |
| FEXOT  | - | EXOgenous in Foreign exchange paid To abroad         |
| FRENT  | - | RENT on Foreign exchange                             |
| FTAX   | - | TAX on Foreign exchange                              |
| IDIST  | - | Income DISTribution                                  |
| IMPORT | - | Import of goods and services                         |
| IO     | - | Input/Output   |
| ITAX   | - | Indirect TAX   |
| MARKUP | - | MARKUP over and above costs                          |
| QEXO   | - | EXOgenous Quantity                                   |
| QSHR   | - | fixed Quantity SHaRes                                |
| SUBST  | - | perfect SUBSTitutes                                  |
| TEXO   | - | EXOgenous Tsol                                       |
| TRANSF | - | TRANSfer   |
| UNSPEC | - | UNSPECified (residual)                               |
| VEXO   | - | EXOgenous Value                                      |
| VSHR   | - | fixed Value SHaRes                                   |

represents the account table and ABC represents some legal column label. DEF(i,j) (with two indices) refers to plane "DEF" in the cell table, i.e., to CT(i,j,"DEF") where CT represents the cell table and DEF represents some legal plane label. A zero at the end of a name denotes a base case value, so A0(i,j) is the value of A(i,j) in the base case.

A special convention is used for taxes and markup rates. THETA(i,j) (with two indices) denotes the tax or markup rate associated with cell (i,j), and THETA(j) (with one index) is the total tax and markup rate in column j, i.e.,

$$\text{THETA}(j) = \text{SUM}(i, \text{THETA}(i, j))$$

A more detailed description of the parameters can be found in Appendix C.

**Table A.2:**  
Specification Acronyms by Primary Use.

|  |        |        |        |        |        |        |
|--|--------|--------|--------|--------|--------|--------|
| Consumption, Investment, and other Final Use:                                    |        |        |        |        |        |        |
| DOEXO  | DVEXO  | LES    | QEXO   | QSHR   | VEXO   | VSHR   |
| Production, Product Transformation, Payments to Factors and Intermediate Inputs: |        |        |        |        |        |        |
| AVRG   | CD     | CES    | CES2   | CET    | IO     | MARKUP |
| RENT   | SUBST  | TRANSF |        |        |        |        |
| Income Distribution and Transfer:  |        |        |        |        |        |        |
| IDIST  | TEXO   | UNSPEC |        |        |        |        |
| Taxation:  |        |        |        |        |        |        |
| DTAX   | FTAX   | ITAX   |        |        |        |        |
| Foreign Exchange Transactions:   |        |        |        |        |        |        |
| EXPORT   | FALOCO | FALOCI | FAVRG  | FDIST  | FEXCHD | FEXCHE |
| FEXO   | FRENT  | FTAX   | IMPORT | UNSPEC |        |        |

**SPECS: AVRGE** - perfect substitution with AVerAge price.

**ECONOMICS:** Specification AVRGE is used to model a situation in which perfectly substitutable goods are bought from different sources at different prices, and the total amount is sold at the average price. The amounts of all of the inputs except one must be determined from restrictions elsewhere, e.g., from availability, and the last input must be freely available to balance supply and demand.

The specification can only be used in AC columns and in MF and AC rows. If one AVRGE is used in a column, then all other specifications in the column must also be AVRGE.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:** There is no behavioral cell equation. QCSOL is defined in the cell with the usual equation:

$$QCSOL(i,j) = TSOL(i,j) / PSOL(i)$$

**ACCOUNT EQUATIONS:** There are two column equations in columns with specification AVRGE. C1 guarantees that the solution SAM balances and C5 defines the price index for the column. There is one row equation, R5.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be greater than or equal to zero.

**COMMENTS:** The specification requires the parameter PBAS(i) that defines the 'absolute' price in the base case. PBAS(i) is used to derive the price index for the column. Note that the price index, PSOL(j), can be different from one, even though the prices of all inputs, PSOL(i), are one, provided that all PBAS(i) are not the same and the allocation pattern in the column changes.

There are two variables and one equation in the cell, so the cell introduces one degree of freedom. On the other hand, there are two column equations so on balance there are (d-1) degrees of freedom in the column, where d is the number of AVRGE specifications. These (d-1) degrees of freedom must be absorbed by (d-1) constraints on the availability of goods introduced by other parts of the model.

**INTERNAL EQUATION NAME:** None.

**REFERENCE:** Chapter 10.

SPECS: CD - Cobb-Douglas.

ECONOMICS: The specification indicates that the account produces an output by combining several inputs according to a Cobb-Douglas production function. It is assumed that the producer is a profit maximizer, i.e., that the producer minimizes output costs, given the current market prices of inputs and given the production function. The cell equations are simply the first order optimality conditions for the producer's optimization problem.

The specification can only be used in AC columns and in MF and AC rows. If one CD is used in a column, then all other specifications in the column must also be CD, ITAX and/or MARKUP (with the exception that the column can contain one RENT specification; the equations generated if a RENT specification is present are explained under RENT).

CELL VARIABLES:  $TSOL(i,j)$  and  $QCSOL(i,j)$ .

CELL EQUATIONS:

$$TSOL(i,j) - A(i,j) * YSOL(j) * (1+THETA(j)) / (1+THETA(j))$$

$$QCSOL(i,j) - TSOL(i,j) / PSOL(i)$$

If there is no tax or markup in the column, we have the simpler equation:

$$TSOL(i,j) - A(i,j) * YSOL(j)$$

ACCOUNT EQUATIONS: There is one column equation, C3, in columns with specification CD (and without RENT), and there is one row equation, R5.

RESTRICTIONS:  $TSOL(i,j)$  and  $YSOL(j)$  must be greater than or equal to zero.

COMMENTS: CD is equivalent to CES with  $SIGMA(j) = 1$ . Although the cell equation does not show any efficiency parameters, E and EC, they are still there. They are used in the column equation that relates all prices of inputs to the price of output.

INTERNAL EQUATION NAME: VSHR.

REFERENCE: Chapter 2 and 3.

**SPECS: CES - Constant Elasticity of Substitution.**

**ECONOMICS:** The specification indicates that the account produces an output by combining several inputs according to a CES or Constant Elasticity of Substitution production function. It is assumed that the producer is a profit maximizer, i.e., that the producer minimizes output costs, given the production function and the current market prices of inputs. The cell equations are simply the first order optimality conditions for the producer's optimization problem.

The specification can only be used in AC columns and in MF and AC rows. If one CES is used in a column, then all other specifications in the column must also be CES, ITAX or MARKUP (with the exception that the column can contain one RENT specification; the equations generated if a RENT specification is present are explained under RENT).

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:** General case:

$$\begin{aligned} \text{TSOL}(i,j) &= A(i,j) * \text{YSOL}(j) * \\ &\quad ((1+\text{THETA}(j))/(1+\text{THETA0}(j))) ** (-\text{SIGMA}(j)) * \\ &\quad ((\text{PSOL}(i)/(1+\text{EC}(i,j))) / (\text{PSOL}(j)/(1+\text{E}(j)))) ** (1-\text{SIGMA}(j)) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

If there is no tax or markup and there are no efficiency gains, the TSOL equation reduces to the simpler equation:

$$\text{TSOL}(i,j) = A(i,j) * \text{YSOL}(j) * (\text{PSOL}(i)/\text{PSOL}(j)) ** (1-\text{SIGMA}(j))$$

Special Cases:

$\text{SIGMA}(j) = 0$  (corresponds to specification IO)

$$\begin{aligned} \text{TSOL}(i,j) &= A(i,j) * \text{YSOL}(j) * (\text{PSOL}(i)/(1+\text{EC}(i,j))) / (\text{PSOL}(j)/(1+\text{E}(j))) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

$\text{SIGMA}(j) = 1$  (corresponds to specification CD)

$$\begin{aligned} \text{TSOL}(i,j) &= A(i,j) * \text{YSOL}(j) * (1+\text{THETA0}(j))/(1+\text{THETA}(j)) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

$\text{SIGMA}(j) = \text{INF}$  (corresponding to specification SUBST)

$$\frac{PSOL(i)}{(1+EC(1,j)) * (1+THETA(j))} - \frac{PSOL(j)}{(1+E(j)) * (1+THETA0(j))}$$
$$QCSOL(1,j) - TSOL(1,j) / PSOL(i)$$

**ACCOUNT EQUATIONS:** One row equation, R5, is always associated with the row of the cell, and one column equation is associated with the column of the cell. In the general case the column equation is C4, if SIGMA(j) = 0 it is C2, if SIGMA(j) = 1 it is C3, and if SIGMA(j) = INF it is C1.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be greater than or equal to zero.

**INTERNAL EQUATION NAME:** CES (general case), QSHR (SIGMA(j)=0), VSHR (SIGMA(j) = 1), or SUBST (SIGMA(j) = INF).

**REFERENCE:** Chapter 6.

**SPECS: CES2** - Constant Elasticity of Substitution, 2 levels.

**ECONOMICS:** The specification indicates that the account produces an output by combining several inputs according to a two level CES or Constant Elasticity of Substitution production function. It is assumed that the producer is a profit maximizer, i.e., that the producer minimizes output costs, given the production function and the current market prices of inputs.

The inputs to the lower levels of the CES2 production function are represented by the elements in the column of the SAM. The top level, or final output, corresponds to the column itself. The intermediate levels are defined implicitly through a SIGMAC parameter in the Cell Table. All cells in the column with the same SIGMAC value are defined to belong to the same intermediate level, and the elasticity of substitution between the inputs to that intermediate level is their common SIGMAC. The elasticity of substitution at the top level is defined by the SIGMA parameter in the Account Table in the same way that elasticities are defined for ordinary CES specifications. Cells that have the same SIGMAC as the SIGMA for the account are treated as inputs directly at the upper level.

The CES2 specification uses a cell specific share parameter,  $A$ , a cell specific efficiency parameter,  $EC$ , and an account specific efficiency parameter,  $E$ , just like the CES specification. The  $A$  parameters must as usual add to one in the column; the shares of the various inputs to a specific intermediate level are derived by scaling the corresponding  $A$ s. The cell specific efficiencies apply to the inputs at the lowest level, and the account specific efficiency applies to the overall output. There are no share or efficiency parameters for the use of the output of the intermediate levels in the production of the top level. The second example below will clarify the use of the parameter values.

The CES2 specification can only be used in AC columns and in MF and AC rows. If one CES2 is used in a column, then all other specifications in the column must be CES2, ITAX, and/or MARKUP, with the exception that the column can contain one RENT specification. The additional complications introduced by a RENT specification are explained under RENT.

**IMPLEMENTATION:** The implementation of CES2 is done internally by disaggregating the SAM to include the intermediate level. The following examples of a column account with the CES2 specification show how the disaggregation is performed. Note that the output of the GAMS-HERCULES system does not report on any of the intermediate values; they are aggregated away or ignored. Users interested in these items must perform the disaggregation explicitly.

## Example 1:

The first example shows the basic principles, and therefore does not use any share or efficiency parameters. The user represents the CES2 column, labeled ACC below, as follows:

## Cell Table for column ACC:

|        | TBASE | SPECS | SIGMAC |
|--------|-------|-------|--------|
| Rows   |       |       |        |
| INPUT1 | 10    | CES2  | 0.1    |
| INPUT2 | 20    | CES2  | 1.5    |
| INPUT3 | 10    | CES2  | 0.5    |
| INPUT4 | 30    | CES2  | 0.1    |
| INPUT5 | 10    | CES2  | 0.5    |
| INPUT6 | 40    | CES2  | 0.5    |
| INPUT7 | 60    | CES2  | 1.5    |

## Account Table for account ACC:

|     | TYPE | FIX | SIGMA | PFIX | QFIX |
|-----|------|-----|-------|------|------|
| ACC | AC   | f   | 0.1   | p    | q    |

where f, p, and q represent arbitrary values.

The corresponding disaggregated representation of the Cell Table after addition of two new accounts, NEW1 and NEW2, is:

|        | Column ACC |       | Column NEW1 |       | Column NEW2 |       |
|--------|------------|-------|-------------|-------|-------------|-------|
|        | TBASE      | SPECS | TBASE       | SPECS | TBASE       | SPECS |
| Rows   |            |       |             |       |             |       |
| INPUT1 | 10         | CES   |             |       |             |       |
| INPUT2 |            |       | 20          | CES   |             |       |
| INPUT3 |            |       |             |       | 10          | CES   |
| INPUT4 | 30         | CES   |             |       |             |       |
| INPUT5 |            |       |             |       | 10          | CES   |
| INPUT6 |            |       |             |       | 40          | CES   |
| INPUT7 |            |       | 60          | CES   |             |       |
| NEW1   | 80         | CES   |             |       |             |       |
| NEW2   | 60         | CES   |             |       |             |       |

And the Account Table for the accounts ACC, NEW1, and NEW2 is:

|      | TYPE | FIX | SIGMA | PFIX | QFIX |
|------|------|-----|-------|------|------|
| ACC  | AC   | f   | 0.1   | p    | q    |
| NEW1 | AC   |     | 1.5   |      |      |
| NEW2 | AC   |     | 0.5   |      |      |

Note that the cell specific SIGMACs have disappeared and are replaced by more SIGMAs in the Account Table. Also note that the account FIX for ACC remains unchanged, and that the new accounts have FIX = 0 (or blank), i.e., prices, quantities, and values are all free endogenous variables.

Before the GAMS-HERCULES system reports the solution back as part of the Cell Table, it aggregates the three accounts ACC, NEW1, and NEW2 into one account, ACC, and eliminates the internal payments from ACC to NEW1 and from ACC to NEW2. It also eliminates all information about account NEW1 and NEW2 from the Account Table.

Example 2:

The second example is an expansion of the previous one with the addition of share and efficiency parameters.

Cell Table for column ACC contains:

|        | TBASE | SPECS | SIGMAC | A    | E    |
|--------|-------|-------|--------|------|------|
| Rows   |       |       |        |      |      |
| INPUT1 | 10    | CES2  | 0.1    | 0.05 | 0.01 |
| INPUT2 | 20    | CES2  | 1.5    | 0.10 | 0.03 |
| INPUT3 | 10    | CES2  | 0.5    | 0.05 |      |
| INPUT4 | 30    | CES2  | 0.1    | 0.20 | 0.01 |
| INPUT5 | 10    | CES2  | 0.5    | 0.05 |      |
| INPUT6 | 40    | CES2  | 0.5    | 0.25 |      |
| INPUT7 | 60    | CES2  | 1.5    | 0.30 |      |

and the Account Table for account ACC is:

|     | TYPE | FIX | SIGMA | E    | PFIX | QFIX |
|-----|------|-----|-------|------|------|------|
| ACC | AC   | f   | 0.1   | 0.04 | p    | q    |

The corresponding disaggregated representation of the Cell Table after addition of two new accounts, NEW1 and NEW2, is:

| Rows   | column ACC |       |      |      | column NEW1 |       |      |      | column NEW2 |       |      |   |
|--------|------------|-------|------|------|-------------|-------|------|------|-------------|-------|------|---|
|        | TBASE      | SPECS | A    | E    | TBASE       | SPECS | A    | E    | TBASE       | SPECS | A    | E |
| INPUT1 | 10         | CES   | 0.05 | 0.01 |             |       |      |      |             |       |      |   |
| INPUT2 |            |       |      |      | 20          | CES   | 0.25 | 0.03 |             |       |      |   |
| INPUT3 |            |       |      |      |             |       |      |      | 10          | CES   | 0.14 |   |
| INPUT4 | 30         | CES   | 0.20 | 0.01 |             |       |      |      |             |       |      |   |
| INPUT5 |            |       |      |      |             |       |      |      | 10          | CES   | 0.14 |   |
| INPUT6 |            |       |      |      |             |       |      |      | 40          | CES   | 0.72 |   |
| INPUT7 |            |       |      |      | 60          | CES   | 0.75 |      |             |       |      |   |
| NEW1   | 80         | CES   | 0.40 |      |             |       |      |      |             |       |      |   |
| NEW2   | 60         | CES   | 0.35 |      |             |       |      |      |             |       |      |   |

And the Account Table for the accounts ACC, NEW1, and NEW2 is:

|      | TYPE | FIX | SIGMA | E    | PFIX | QFIX |
|------|------|-----|-------|------|------|------|
| ACC  | AC   | f   | 0.1   | 0.04 | p    | q    |
| NEW1 | AC   |     | 1.5   |      |      |      |
| NEW2 | AC   |     | 0.5   |      |      |      |

Note that the cell efficiencies, EC, have been moved unchanged with the cells, that new cells have no EC, that the shares, A, in the original account are computed by aggregating the original As, and that the shares in the new accounts are derived from the original As by scaling them so they add to one in each of the new columns.

**CELL AND ACCOUNT EQUATIONS:** The equations are not defined or implemented directly but implicitly through the disaggregation described above.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be greater than or equal to zero.

**COMMENTS:** The lower levels are grouped using unique values of SIGMAC. This means that the system cannot distinguish two lower level groups with the same SIGMAC. However, a little mathematical manipulation shows that the two lower level groups can be combined without affecting the production function or its dual cost function, so the limitation in formulation is more apparent than real.

**REFERENCE:** Chapter 9.

SPECS: CET - Constant Elasticity of Transformation.  
 CETINF - CET with infinite elasticity.  
 CET0 - CET with zero elasticity.

ECONOMICS: The specification indicates that the gross output produced by the activity is transformed into different output commodities according to a Constant Elasticity of Transformation function. It is assumed that the distributor maximizes profit, i.e., maximizes the total value of output, given the transformation function and the current market prices of individual outputs. The cell equations are simply the first order optimality conditions for the distributor's optimization problem.

The specification can only be used where AC rows intersect with other AC rows. If one CET, CETINF, or CET0 is used in a row, then all other specifications in the row must be the same. And if CET, CETINF, or CET0 are used in a column, then all other specifications in the column must be CET, CETINF, CET0, ITAX, and/or MARKUP. If one or more ITAX or MARKUP specifications are in the column, then the price used in the allocation is the price before tax or markup, i.e., the price the distributor receives.

CELL VARIABLES: TSOL(i,j) and QCSOL(i,j).

CELL EQUATIONS: General Case for CET:

$$\begin{aligned} \text{TSOL}(i,j) &= B(i,j) * \text{YSOL}(i) * \\ &\quad (\text{PSOL}(i)/\text{PSOL}(j) * (1+\text{THETA}(j))/(1+\text{THETA0}(j))) ** (\text{SIGMAR}(i)-1) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(j) \end{aligned}$$

If there is no tax or markup we have the simpler equation:

$$\text{TSOL}(i,j) = B(i,j) * \text{YSOL}(i) * (\text{PSOL}(i)/\text{PSOL}(j)) ** (\text{SIGMAR}(i)-1)$$

Special Cases:

SIGMAR(i) = -INF (the outputs are physically the same or perfect substitutes):

$$\begin{aligned} \text{PSOL}(j) * (1+\text{THETA0}(j)) &= \text{PSOL}(i) * (1+\text{THETA}(j)) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(j) \end{aligned}$$

This special case can be specified as CETINF, in which case SIGMAR is defined implicitly.

SIGMAR(i) = 0 (the quantities are in fixed proportions):

$$\begin{aligned} \text{TSOL}(i,j) &= \text{B}(i,j) * \text{YSOL}(i) * \text{PSOL}(j) / \text{PSOL}(i) * (1 + \text{THETA0}(j)) / (1 + \text{THETA}(j)) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(j) \end{aligned}$$

This special case can be specified as CET0, in which case SIGMAR is defined implicitly.

**ACCOUNT EQUATIONS:** One column equation, C1, is always associated with the column of the cell, and one row equation is associated with the row of the cell. In the general case the row equation is R2, if SIGMAR(i) = -INF it is R1, and if SIGMAR(i) = 0 it is R3.

**RESTRICTIONS:** TSOL(i,j) and YSOL(i) must be greater than or equal to zero.

**INTERNAL EQUATION NAMES:** CET (general case), CET0 (SIGMAR(i) = 0), and SUBST (SIGMAR(i) = -INF).

**REFERENCE:** Chapter 12.

**SPECS: DQEXO** - change in (Delta) EXogenous Quantity.

**ECONOMICS:** The specification describes a consumption system; it indicates that the transaction covers a prespecified quantity, independent of the price of the good. The specification has been included to model an account for changes in stocks and the quantity can therefore be negative. The column will have a price index, but the value will be the GAMS value UNDF (undefined) unless the sum of the exogenous quantities is significantly different from zero.

The specification can only be used in INSTC columns and AC rows, and if a column contains one DQEXO specification then all specifications in the column must be DQEXO.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$QCSOL(1,j) - FV(1,j)$$

$$TSOL(1,j) - QCSOL(1,j)*PSOL(1)$$

**ACCOUNT EQUATIONS:** There is one column equation, C9, in columns with specification DQEXO, and one row equation, R5.

**RESTRICTIONS:** None.

**COMMENTS:** The specification is similar to QEXO, except that the exogenous quantity can be negative and that the quantity parameter is FV instead of FQ.

**INTERNAL EQUATION NAME:** QEXO.

**SPECS: DTAX - Direct TAX.**

**ECONOMICS:** The specification describes a direct tax payment as part of an income distribution column. The specification can only appear in a TAX row and in MF, NMF, and INST columns, and it can only be combined with other DTAX and with IDIST specifications. The specification assumes that a percentage of the income is paid as tax, and that the remaining income is split in fixed proportions. The tax rate, THETA(i,j), is defined as a percentage of post-tax income in order to make this tax specification similar to the other tax specifications (FTAX and ITAX).

**CELL VARIABLES: TSOL(i,j).**

**CELL EQUATIONS:**

$$TSOL(i,j) = THETA(i,j) / (1+THETA(i,j)) * YSOL(j)$$

**ACCOUNT EQUATIONS:** There is one row equation, R1, associated with the row of the specification, and no column equation.

**RESTRICTIONS:** None.

**COMMENTS:** The direct tax can also, of course, be modeled with only IDIST specifications and proper choice of share parameters. The advantage of the DTAX-IDIST combination is that the IDIST shares will be calibrated automatically from the base SAM even if the tax rates change, unless the modeler defines other shares.

**SPECS:** DVEXO - change in (Delta) EXOgenous Value.

**ECONOMICS:** The specification describes a consumption system; it indicates that the transaction covers a prespecified value, independent of the price of the good. The specification has been included to model an account for changes in stocks and the value can therefore be negative. The column will have a price index, but the value will be the GAMS value UNDF (undefined) unless the sum of the quantities is significantly different from zero.

The specification can only be used in INSTC columns and AC rows, and if a column contains one DVEXO specification then all specifications in the column must be DVEXO.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} \text{TSOL}(i,j) &= \text{FV}(i,j) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

**ACCOUNT EQUATIONS:** There is one column equation, C9, in columns with specification DVEXO, and one row equation, R5.

**RESTRICTIONS:** None.

**COMMENTS:** The specification is similar to VEXO, except that the exogenous value can be negative and the value parameter is FV instead of FQ.

**INTERNAL EQUATION NAME:** VEXO.

**SPECS:** EXPORT - EXPORT demand function from the rest of the world.  
 EXPINF - EXPORT with infinite elasticity of demand.  
 EXP1 - EXPORT with elasticity of demand equal to 1.  
 EXP0 - EXPORT with elasticity of demand equal to 0.

**ECONOMICS:** The specification indicates that the cell describes the demand from the rest of the world for exports from the country. The specification assumes that demand depends on the price of the goods to be exported, PSOL(i), relative to the price of comparable products on the world market, properly converted to domestic currency, WP(i,j)\*PSOL(j). Note that PSOL(j) is the exchange rate, and that the world price, WP(i,j), is measured in foreign exchange. The elasticity of demand is ETA(i,j). The transaction takes place at the price PSOL(i), measured in domestic currency, independent of the world price, WP(i,j). Supply is assumed to be specified implicitly through the rest of the model; all goods sold in row i are homogenous, and supply therefore depends on the transactions that supply account i with goods.

The specification can only be used in the column of a ROW account and in the row of an AC account.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:** General Case for EXPORT:

$$\begin{aligned} \text{QCSOL}(i,j) &= \text{FQ}(i,j) * ( \text{WP}(i,j) * \text{PSOL}(j) / \text{PSOL}(i) ) ** \text{ETA}(i,j) \\ \text{TSOL}(i,j) &= \text{QCSOL}(i,j) * \text{PSOL}(i) \end{aligned}$$

Special Cases:

$$\text{ETA}(i,j) = \text{INF:}$$

$$\begin{aligned} \text{PSOL}(i) &= \text{WP}(i,j) * \text{PSOL}(j) \\ \text{TSOL}(i,j) &= \text{QCSOL}(i,j) * \text{PSOL}(i) \end{aligned}$$

This special case can be specified with EXPINF, in which case ETA is not needed.

$$\text{ETA}(i,j) = 1:$$

$$\begin{aligned} \text{TSOL}(i,j) &= \text{FQ}(i,j) * \text{WP}(i,j) * \text{PSOL}(j) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

This special case can be defined with EXP1, in which case ETA is not needed.

$ETA(i,j) = 0$ :

$QCSOL(1,j) = FQ(1,j)$

$TSOL(1,j) = QCSOL(1,j) * PSOL(i)$

This special case can be defined with EXP0, in which case ETA is not needed.

**ACCOUNT EQUATIONS:** In all cases, one row equation, R5, and one column equation, C1, are associated with the row and column of the cell respectively.

**RESTRICTIONS:**  $TSOL(i,j)$  must be greater than or equal to zero. Only one EXPINF or EXPORT with  $ETA(i,j) = INF$  can be present in one row of the SAM.

**INTERNAL EQUATION NAME:** EXPORT (general case), EXPINF ( $ETA(i,j) = INF$ ), EXP1 ( $ETA(i,j) = 1$ ), and EXP0 ( $ETA(i,j) = 0$ ).

**REFERENCE:** Chapter 6 and 13.

SPECS: FALOC - Foreign exchange ALLOCAtion.

FALOC0 - Foreign exchange ALLOCAtion with 0 elasticity.

FALOCI - Foreign exchange ALLOCAtion with Infinite elasticity.

ECONOMICS: The specification models allocation of foreign exchange. The total amount of foreign exchange earned in the column of the account is allocated to other ROW accounts in the row. The receiving accounts will in general have different exchange rates, and the allocation shares depend on the relative exchange rates with some elasticity of transformation, SIGMAR.

Specification FALOC with elasticity of transformation zero (or FALOC0) can be used to model government regulations that require a certain proportion of foreign exchange to be exchanged in the National Bank at a fixed low exchange rate, while the rest can be exchanged on a free market with a higher exchange rate. The case with a nonzero elasticity can be used in cases where, e.g., the free market is illegal and the share that goes to this market depends on the premium.

The specification can only appear in the ROW \* ROW block of the model, and all specifications in the row must be FALOC.

CELL VARIABLES: TSOL(i,j).

CELL EQUATIONS:

The cell equation is derived from

$$TSOL(i,j)/EXCH(j) - C(i,j) * YSOL(i)/EXCH(i) \\ *(PSOL(j)/PSOL(i)) ** (-SIGMAR(i))$$

where EXCH(\*) is the absolute exchange rate and PSOL(\*) is the exchange rate index. The allocation shares measured in foreign exchange are

$$C(i,j) * (PSOL(j)/PSOL(i)) ** (-SIGMAR(i))$$

consisting of the share C(i,j) and the price adjustment term. Introducing EXCH(j) = PBAS(j)\*PSOL(j), where PBAS(j) is the exchange rate in the base case, defined through the Account Table, and EXCH(i) = PBAS(i)\*PSOL(i), where PBAS(i) is an average exchange rate in the base case computed internally as  $PBAS(i) = \text{SUM}(j, C0(i,j) * PBAS(j))$ , we get the cell equation

$$TSOL(i,j) - C(i,j) * YSOL(i) * PBAS(j)/PBAS(i) \\ * (PSOL(j)/PSOL(i)) ** (1-SIGMAR(i))$$

INTERNAL EQUATION NAME: FALOC (general case), FALOC0 (SIGMAR(i)=0), and SUBST (SIGMAR(i)=-INF).

COMMENT: Note that the average exchange rate index, PSOL(i), can be different from 1 even if all PSOL(j)=1, provided that PBAS(j) is not constant and the allocation shares C(i,j) are different from the base shares, C0(i,j). In other words, the exchange rate the exporter receives can change, even if the exchange rates are fixed provided the forced exchange requirements, i.e. C(i,j), change.

RESTRICTIONS: TSOL(i,j) and YSOL(i) must be greater than or equal to zero.

ACCOUNT EQUATIONS: One column equation, C1, is always associated with the column of the cell, and there is one row equation. The row equation is R2 in the general case, R3 if SIGMAR(i) = 0, and R1 if SIGMAR(i) = -INF.

This special case can be specified as FALOC1, in which case SIGMAR is defined implicitly. PBAS(j) is not needed since all exchange rates are assumed to be the same, both in the solution and in the base case.

$$PSOL(i) - PSOL(j)$$

SIGMAR(i) = -INF (The allocation is very flexible, and the shares adjust to equilibrate the exchange rates of the row and column)

This special case can be specified as FALOC0 in which case SIGMAR is defined implicitly.

$$TSOL(i,j) - C(i,j) * YSOL(i) * PBAS(j)/PBAS(i) * (PSOL(j)/PSOL(i))$$

SIGMAR(i) = 0 (The allocation is in fixed shares measured in foreign exchange, independent of the exchange rates)

Special cases:

which is similar to specification CET. However, the computed B values need not add to 1 in the row as they must for CET.

$$TSOL(i,j) - B(i,j) * YSOL(i) * (PSOL(j)/PSOL(i)) ** (1-SIGMAR(i))$$

Internally, C(i,j)\*PBAS(j)/PBAS(i) is replaced by B(i,j) and the equation becomes

**SPECS:** FAVRG - Foreign exchange with AVerAge exchange rate.

**ECONOMICS:** Specification FAVRG is used to model a case where foreign exchange is available from different sources at different exchange rates, but the total amount is treated as having the average exchange rate. The amounts bought from different sources are determined from restrictions elsewhere, e.g., from availability, so there is no cell equation.

The specification can only be used in ROW columns and ROW rows. If one FAVRG is used in a column, then all other specifications in the column must also be FAVRG.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:** There is no behavioral cell equation.

**ACCOUNT EQUATIONS:** There are two column equations in columns with specification FAVRG. C1 guarantees that the solution SAM balances and C5 defines the average exchange rate for the column.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be greater than or equal to zero.

**COMMENTS:** The specification requires the parameter PBAS(i) that defines the 'absolute' exchange rate in the base case. PBAS(i) is used to derive the index for the average exchange rate in the column. Note that PSOL(j) can be different from one even though all PSOL(i) are one, provided that all PBAS(i) are not the same and the allocation pattern in the column changes.

There is one variable and no equation in the cell so the cell introduces one degree of freedom. On the other hand, there are two column equations, so on balance there are (d-1) degrees of freedom in the column where d is the number of FAVRG specifications. These (d-1) degrees of freedom must be absorbed by (d-1) constraints elsewhere in the model.

**INTERNAL EQUATION NAME:** None.

**REFERENCE:** Chapter 13.

REFERENCE: Chapter 13.

equation).

INTERNAL EQUATION NAME: FALOCO (TSOL equation) and SUBST (PSOL

COMMENTS: The specification has two cell equations and only one cell variable, so there is one additional equation for each FDIST cell that must be offset by a RENT, AVRG, or FAVRG specification elsewhere.

RESTRICTIONS: YSOL(i) and TSOL(i,j) must be positive.

ACCOUNT EQUATIONS: No row equations and one column equation, C1, are associated with the row and column of the cell.

$$TSOL(i,j) - B(i,j) * YSOL(i) \\ PSOL(i) - PSOL(j)$$

CELL EQUATIONS:

CELL VARIABLES: TSOL(i,j).

The specification can only be used in ROW rows and ROW columns, and if one FDIST is used in a row then all other specifications in the row must also be FDIST.

ECONOMICS: The specification indicates that the foreign exchange earned in the column of the account is distributed in the row of the account in fixed proportions for different purposes. The receiving accounts all have the same exchange rate as the paying account. Foreign exchange is rationed with FDIST; meanwhile, the exchange rate is not free to vary, so there must be a mechanism such as a RENT, AVRG, or FAVRG specification elsewhere that equilibrates foreign exchange supply and demand for individual purposes.

SPECS: FDIST - Foreign exchange DISTRIBUTION.

**SPECS:** FEXCHD - Foreign EXCHange with Different exchange rate.

**ECONOMICS:** The specification describes internal transactions between ROW (Rest of the World) accounts that have different exchange rates. It specifies that foreign exchange can be moved freely from one account to another, but since the exchange rate of the row is different from the exchange rate of the column, the valuation of foreign exchange in the two accounts will differ. The difference can be considered a 'tax' or a 'rent' depending on whether the relative exchange rates are exogenous or endogenous. The 'tax' or 'rent' must appear in the same column as the FEXCHD specification, and they are represented by the FTAX and FRENT specifications, respectively. FTAX must appear in the row of a TAX account, and FRENT must appear in the row of a NMF account.

The specification can only appear in a ROW row and a ROW column. There must be exactly two specifications in the column, and the other must be either an FTAX or FRENT specification.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:**

$$TSOL(i,j) = A0(i,j) * YSOL(j) * PSOL(i) / PSOL(j)$$

where  $A0(i,j)$  is calibrated as  $TBASE(i,j) / YBASE(j)$

**ACCOUNT EQUATIONS:** One row equation and one column equation are associated with the row and column of the cell. The row equation is always R1, and the column equation is C2 for the FEXCHD-FTAX combination and C1 for the FEXCHD-FRENT combination.

**RESTRICTIONS:** None.

**INTERNAL EQUATION NAME:** QSHR.

**REFERENCE:** Chapter 13.

**SPECS:** FEXCHE - Foreign EXCHange with Equal exchange rate.

**ECONOMICS:** The specification describes internal transactions between ROW (Rest of the World) accounts. It specifies that foreign exchange can be moved freely from one account to another and the flow equilibrates the exchange rates of the two accounts.

**CELL VARIABLES:**  $TSOL(i,j)$ .

**CELL EQUATIONS:** The specification can only appear in a ROW row and ROW column.

$PSOL(j) - PSOL(1)$

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation, C1, are associated with the row and column of the cell.

**RESTRICTIONS:** None.

**INTERNAL EQUATION NAME:** SUBST.

**REFERENCE:** Chapter 13.

**SPECS:** FEXO - EXOgenous in Foreign exchange.  
FEXOF - EXOgenous in Foreign exchange received From abroad.  
FEXOT - EXOgenous in Foreign exchange paid To abroad.

**ECONOMICS:** The specification indicates a payment that is exogenous measured in foreign exchange. FEXO and FEXOF can be used in the column of a ROW account and the row of a NMF, INST, or INSTC account, and FEXO and FEXOT can be used in the row of a ROW account and the column of an INST or ROW account.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:**

Case 1: The column is a ROW account and the row a NMF, INST, or INSTC account.

$$TSOL(i,j) - FV(i,j) * PSOL(j)$$

This case is equivalent to specification FEXOF.

Case 2: The row is a ROW account and the column an INST or ROW account.

$$TSOL(i,j) - FV(i,j) * PSOL(i)$$

This case is equivalent to specification FEXOT. Note that if FEXO appears in both a ROW row and a ROW column, then the exchange rate of the row is used.

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation, C1, are associated with the row and column of the cell.

**RESTRICTIONS:** None.

**INTERNAL EQUATION NAME:** EXP1 (FEXOF) or EXP0 (FEXOT).

**REFERENCE:** Chapter 8.

**SPECS:** FRENT - RENT on Foreign exchange.

**ECONOMICS:** The specification describes a payment earned by converting foreign exchange from one exchange rate to another in a case where relative exchange rates are endogenous and are determined by supply and demand.

FRENT can only appear in a ROW column and in a NMF row, and it must be combined with exactly one FEXCHD specification in the column.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:** There is no equation in the cell.

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation, C1, are associated with the row and column of the cell.

**RESTRICTIONS:** None.

**COMMENT:** Since the cell contains one variable and no equation, it introduces one degree of freedom that must be absorbed by constraints elsewhere, e.g., by fixing the exchange rate.

**INTERNAL EQUATION NAME:** None.

**REFERENCE:** Chapter 13.

**SPECS:** FTAX - TAX on Foreign exchange.

**ECONOMICS:** The specification describes a payment earned by converting foreign exchange from one exchange rate to another in a case where the relative exchange rate is fixed exogenously. The relative exchange rate is implicitly determined by the tax rate, THETA(i,j), in the cell.

FTAX can only appear in a ROW column and in a TAX row, and it must be combined with exactly one FEXCHD specification in the column.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:**

$$TSOL(i,j) - YSOL(j) * THETA(i,j) / (1+THETA(i,j))$$

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation, C2, are associated with the row and column of the cell.

**RESTRICTIONS:** None.

**INTERNAL EQUATION NAME:** ITAX.

**REFERENCE:** Chapter 13.

**SPECS:** IDIST - Income DISTRibution.

**ECONOMICS:** The specification describes a distribution of income whereby the total income of an account is distributed to the cells in the column in fixed shares. Together with DTAX, it is the only specification that can be used in the columns of MF, NMF, and TAX accounts. It can also appear in the column of an INST account, again only combined with DTAX. It can only appear in INST, INSTC, and ROW rows.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:**

$$TSOL(i,j) = A(i,j) * YSOL(j) * (1+THETA0(j))/(1+THETA(j))$$

If there is no direct tax (DTAX) in the column, then we get the simpler equation:

$$TSOL(i,j) = A(i,j) * YSOL(j)$$

**ACCOUNT EQUATIONS:** One row equation, R1, is associated with the row of the cell; there is no column equation.

**RESTRICTIONS:** None.

**INTERNAL EQUATION NAME:** VSHR.

**REFERENCE:** Chapter 2 and 3.

**SPECS: IMPORT - IMPORT** of goods and services.

**ECONOMICS:** The cell describes payments to the rest of the world for imported goods. The specification assumes that there is an infinitely elastic supply from the world market at a fixed world price,  $WP(i,j)$ , measured in foreign exchange. The payment included in the SAM is, of course, denominated in the units used for all other entries in the SAM, usually domestic currency.

The specification can only be used in a ROW row and an AC column. The only other specifications allowed in the column are ITAX and RENT.

**CELL VARIABLES:**  $TSOL(i,j)$  and  $QCSOL(i,j)$ .

**CELL EQUATIONS:**

$$TSOL(i,j) = (WP(i,j)*PSOL(i)) * (YSOL(j)/PSOL(j)) * AO(i,j)$$

$$QCSOL(i,j) = TSOL(i,j) / (WP(i,j)*PSOL(i))$$

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation are associated with the row and column of the cell. The column equation is C1 if the column contains a RENT specification and C6 otherwise.

**RESTRICTIONS:**  $TSOL(i,j)$  and  $YSOL(j)$  must be non negative.

**COMMENTS:** The factor  $(YSOL(j)/PSOL(j))$  is the quantity of imports. However, it is measured in a quantity unit based on prices of imports inclusive of import duties. The factor  $AO(i,j)$  converts the quantity unit to a unit based on prices of imports exclusive of import duties.  $AO(i,j)$  is automatically calibrated to  $TBASE(i,j)/YBASE(j)$  and the modeler cannot change this value.

Although we have assumed an infinitely elastic world supply, the government can still restrict imports, e.g., by fixing their total quantity.

**INTERNAL EQUATION NAME:** IMPORT.

**REFERENCE:** Chapter 6 and 13.

**SPECS:** IO - Input/Output.

**ECONOMICS:** The specification indicates that the account produces an output by combining one or more inputs in a fixed ratio. This corresponds to a Leontief or Input/Output production function.

Specification IO can only be used in AC columns and in MF and AC rows. It can be combined with other zero or more ITAX and/or MARKUP specifications and with zero or one RENT specification in the column. If IO is the only specification in the column, then the column is a true Input/Output production activity, and the price of output will be directly related to the prices of inputs. The addition of one or more ITAX or MARKUP specifications simply means that the final product is taxed or marked up with one or more taxes or markup rates that are all based on its production cost, and that the price of output is correspondingly higher. IO can also be combined with RENT. The implications of this combination is explained under RENT.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} \text{TSOL}(i,j) &= A(i,j) * \text{YSOL}(j) * (\text{PSOL}(i)/(1+\text{EC}(i,j))) / (\text{PSOL}(j)/(1+\text{E}(j))) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

If there are no efficiency gains we have the simpler equation:

$$\text{TSOL}(i,j) = A(i,j) * \text{YSOL}(j) * \text{PSOL}(i)/\text{PSOL}(j)$$

**ACCOUNT EQUATIONS:** One row equation, R5, and one column equation are associated with the row and column of the cell. The column equation is C1 if the column contains a RENT specification, and C2 otherwise.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be non negative.

**COMMENTS:** In the case where there are only IO specifications in the column the specification is equivalent to a CES specification with SIGMA = 0 in the column.

**INTERNAL EQUATION NAME:** QSHR.

**REFERENCE:** Chapter 4 and 5.

## SPECS: ITAX - Indirect TAX.

**ECONOMICS:** The specification indicates that the payment in the cell is an indirect tax with an exogenously specified tax rate. The tax rate,  $\text{THETA}(i,j)$ , is levied on all non-ITAX and non-MARKUP payments in the column. It is possible to have several tax payments in the same column. The specification can only be used in the row of a TAX account and the column of an AC account, and the other specifications in the column must be IMPORT, or one or more CD, CES, CES2, IO, MARKUP, SUBST, and zero or one RENT.

The column is expanded if ITAX is combined with RENT and/or CES2. When ITAX is combined with RENT and other specifications, the column is expanded as shown under the description of RENT. When ITAX is combined with CES2, the column is expanded as shown by the following example:

Column ACC of the Cell Table before expansion is

|    | TBASE | SPECS | A   | SIGMAC |
|----|-------|-------|-----|--------|
| R1 | 10    | ITAX  |     |        |
| R2 | 30    | CES2  | 0.4 | 2.0    |
| R3 | 20    | CES2  | 0.2 | 2.0    |
| R4 | 40    | CES2  | 0.3 | 0.5    |

and account ACC of the Account Table is

|     | TYPE | FIX | PFIX | QFIX | SIGMA |
|-----|------|-----|------|------|-------|
| ACC | AC   | f   | p    | q    | 0.5   |

where f, p, and q represent arbitrary values.

The expansion adds the account labeled NEW, and column ACC and NEW of the Cell Table after expansion is

|      | ACC   |       |   |        | NEW1  |       |       |        |
|------|-------|-------|---|--------|-------|-------|-------|--------|
|      | TBASE | SPECS | A | SIGMAC | TBASE | SPECS | A     | SIGMAC |
| R1   | 10    | ITAX  |   |        |       |       |       |        |
| R2   |       |       |   |        | 30    | CES2  | 0.445 | 2.0    |
| R3   |       |       |   |        | 20    | CES2  | 0.222 | 2.0    |
| R4   |       |       |   |        | 40    | CES2  | 0.333 | 0.5    |
| NEW1 | 90    | IO    |   |        |       |       |       |        |

and account ACC and NEW1 of the Account Table is

|      | TYPE | FIX | PFIX | QFIX | SIGMA |
|------|------|-----|------|------|-------|
| ACC  | AC   | f   | p    | q    |       |
| NEW1 | AC   |     |      |      | 0.5   |

Note two things about the share parameter A. First, in the original Cell Table an implied A-value of  $10/(10+30+60) = 0.1$  is associated with the ITAX cell, so the As add to 1 in the column. (See Appendix C for further details). Second, the As are scaled with 100/90 in account NEW so they still add to 1. Efficiency parameters are not shown. If they are present, they always move unchanged with the CES2 specification. Also note that account NEW will be subject to the expansions described under CES2.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\text{TSOL}(i,j) = \text{YSOL}(j) * \text{THETA}(i,j)/(1+\text{THETA}(j))$$

$$\text{QCSOL}(i,j) = \text{QSOL}(j) * \text{THETA0}(i,j)/(1+\text{THETA0}(j))$$

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation are associated with the row and column of the cell. The column equation depends on the other specifications in the column.

**RESTRICTIONS:** YSOL(j) must be greater than or equal to zero.

**COMMENTS:** Note that a constant price payment (QCSOL) is defined for the cell. It is defined as the payment that would have been paid with base period tax rates and base period prices.

**INTERNAL EQUATION NAME:** ITAX.

**REFERENCE:** Chapter 4.

**SPECS: LES - Linear Expenditure System.**

**ECONOMICS:** The specification describes a consumption system. The composition of consumption is assumed to follow Stone's Linear Expenditure System, i.e. consumption is composed of a committed quantity component independent of income, ALPHA(i,j), and a discretionary expenditure component proportional to the remaining income with proportionality factor BETA(i,j). The two components have own price elasticities of zero and minus one, respectively, and income elasticities of zero and one, respectively. The aggregate elasticities will depend on ALPHA relative to QCSOL.

The expenditure system can be derived by assuming the consumer has a utility function of the form

$$\text{SUM}(i, \text{BETA}(i,j) * \log( \text{QCSOL}(i,j) - \text{ALPHA}(i,j) ) )$$

and is maximizing utility for given income and prices.

The specification can only be used in INSTC columns and AC rows, and if a column contains one LES specification then all specifications in the column must be LES.

**IMPLEMENTATION:** If the parameter ALPHA is zero (its default value) in the whole column, then LES is equivalent to VSHR and will be translated into VSHR. In this case, the cell and account equations are those of the VSHR specification.

If one or more ALPHA values are nonzero, then the column is expanded into two. The original column keeps the committed expenditures modeled with specification DQEXO and gets one new cell in the row of the new account to take the remaining income. The specification in the new cell is an internal specification, 1LES. The new account receives the remaining income from the 1LES cell and spends it as discretionary expenditures modeled with another internal specification, 2LES, that has most of the properties of VSHR or CD.

The following example shows this expansion. The original account is labeled ACC and the new account is labeled NEW.

The ACC column of the Cell Table before expansion is:

|       | TBASE | SPECS | ALPHA | BETA |
|-------|-------|-------|-------|------|
| GOOD1 | 20    | LES   | 10    | 0.10 |
| GOOD2 | 40    | LES   | 0     | 0.20 |
| GOOD3 | 60    | LES   | 30    | 0.40 |
| GOOD4 | 60    | LES   | 20    | 0.30 |

and the ACC and NEW columns after expansion are:

|       | ACC   |       |    | NEW   |       |      |
|-------|-------|-------|----|-------|-------|------|
|       | TBASE | SPECS | FV | TBASE | SPECS | A    |
| GOOD1 | 10    | DQEXO | 10 | 10    | 2LES  | 0.10 |
| GOOD2 |       |       |    | 40    | 2LES  | 0.20 |
| GOOD3 | 30    | DQEXO | 30 | 30    | 2LES  | 0.40 |
| GOOD4 | 20    | DQEXO | 20 | 40    | 2LES  | 0.30 |
| NEW   | 120   | 1LES  |    |       |       |      |

Similarly, the Account Table before expansion is:

|     | TYPE  | FIX | PFIX |
|-----|-------|-----|------|
| ACC | INSTC | f   | p    |

where f and p represent arbitrary values, and the Account Table after expansion is:

|     | TYPE  | FIX | PFIX |
|-----|-------|-----|------|
| ACC | INSTC | f   | p    |
| NEW | AC    |     |      |

Before GAMS-HERCULES reports the solution back as part of the Account and Cell Tables, it aggregates account NEW with ACC and removes account NEW as well as the internal payment between the two accounts.

CELL VARIABLES: TSOL(i,j) and QCSOL(i,j).

CELL EQUATIONS: The cell equations for DQEXO are shown under DQEXO. The cell equations for 2LES are the same as for VSHR, and the only cell equation for 1LES is:

$$QCSOL(i,j) = TSOL(i,j)/PSOL(i)$$

Overall, this gives the following combined cell equations (but note that these are not the equations actually solved):

$$TSOL(i,j) = ALPHA(i,j)*PSOL(i) + BETA(i,j)*(Y(j) - SUM(k, ALPHA(k,j)*P(k)))$$

$$QCSOL(i,j) = TSOL(i,j)/PSOL(i)$$

**ACCOUNT EQUATIONS:** The account equations for the new account are the same as for an ordinary VSHR account. For the original account, one row equation, R5, is associated with each of the rows, and two column equations are associated with the column -- C1, which guarantees that the solution SAM is balanced, and CA, which defines the consumer price index.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be non negative.

**INTERNAL EQUATION NAMES:** DQEXO has the internal name QEXO, 2LES has the name VSHR, and 1LES does not have any behavioral equation.

**SPECS: MARKUP - MARKUP** over and above costs.

**ECONOMICS:** The specification indicates that the payment in the cell is a factor payment that is not determined from the marginal product of a factor but rather as a fixed markup over and above other costs. The markup rate, THETA(j), is levied on all non-ITAX and non-MARKUP payments in the column. It is possible to have several MARKUP payments in the same column; this simply means that each of several factors gets a markup over other costs. The specification can only be used in the row of a NMF account and the column of an AC account. It must be combined in the column with at least one of the production functions CD, CES, CES2, CET, IO, and SUBST, or with IMPORT; it may also be combined with one or more ITAX and/or zero or one RENT. If MARKUP is combined with the RENT and/or CES2 specifications, then the column is expanded in accordance with the principles explained under specification ITAX.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} \text{TSOL}(i, j) &= \text{YSOL}(j) * \text{THETA}(i, j) / (1 + \text{THETA}(j)) \\ \text{QCSOL}(i, j) &= \text{QSOL}(j) * \text{THETA0}(i, j) / (1 + \text{THETA0}(j)) \end{aligned}$$

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation are associated with the row and column of the cell. The column equation depends on the other specifications in the column.

**RESTRICTIONS:** YSOL(j) must be greater than or equal to zero.

**COMMENTS:** Note that a constant price payment (QCSOL) is defined for the cell. It is defined as the payment that would have been paid with base period markup rates and base period prices.

**INTERNAL EQUATION NAME:** ITAX.

**REFERENCE:** Chapter 9.

**SPECS:** QEXO - EXOgenous Quantity.

**ECONOMICS:** The specification describes a consumption system; it indicates that the transaction covers a prespecified quantity, independent of the price of the good. The specification can be used to model a government budget or an investment plan with a fixed set of activities.

The specification can only be used in INSTC columns and AC rows, and if a column contains one QEXO specification then all specifications in the column must be QEXO.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$QCSOL(i,j) - FQ(i,j)$$

$$TSOL(i,j) - QCSOL(i,j)*PSOL(i)$$

**ACCOUNT EQUATIONS:** One row equation, R5, is associated with the row of the cell. Two column equations are associated with the column of the cell: C1 guarantees that the solution SAM balances, and C7 defines a consumer price index for the column.

**RESTRICTIONS:** TSOL(i,j) must be greater than or equal to zero.

**INTERNAL EQUATION NAME:** QEXO.

**REFERENCE:** Chapter 6.

**SPECS:** QSHR - fixed Quantity SHaRes.

**ECONOMICS:** The specification describes a consumption system. The composition of consumption is assumed to be in fixed shares, measured in quantity units, independent of prices. The overall scale depends on the available income, which can be either exogenous or endogenous.

The specification can only be used in INSTC columns and AC rows, and if a column contains one QSHR specification then all specifications in the column must be QSHR.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$QCSOL(i,j) = A(i,j) * YSOL(j) / PSOL(j)$$

$$TSOL(i,j) = QCSOL(i,j) * PSOL(i)$$

**ACCOUNT EQUATIONS:** One row equation, R5, and one column equation, C2, are associated with the row and column of the cell.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be non negative.

**INTERNAL EQUATION NAME:** QSHR.

**REFERENCE:** Chapter 6.

**SPECS: RENT.**

**ECONOMICS:** The specification is used for a cell that collects rents that accrue to a production or trading activity because of market rigidities, capacity constraints, or prices or quantities that are fixed by the government. If rent accrues to capital owners, it is always measured over and above the marginal value of capital, and it must be paid to a separate account.

The specification can only appear in an AC column and in an NMF row, and only one RENT specification can be used in each column. There must be at least one other specification in the column, chosen from CD, CES, CES2, IO, IMPORT, and SUBST. The column may also contain ITAX and/or MARKUP specifications.

If the RENT specification is combined with IO specifications only, then it will be implemented with the equations mentioned below. All other combinations are decomposed into simpler combinations, one of which is RENT and IO, by expanding the SAM as shown by the examples below.

We use the following common notation in the examples. The original account is labeled ACC and new accounts are labeled NEW1, NEW2, etc. The specifications CD, CES, CES2, IMPORT, and SUBST behave in the same way; for convenience they are treated together under the name PROD. Similarly, ITAX and MARKUP behave in the same way and only ITAX is mentioned.

Example 1: RENT combined with PROD. This case creates one new account, NEW1.

Column ACC of the Cell Table before expansion is

|    | TBASE | SPECS | A   |
|----|-------|-------|-----|
| R1 | 10    | RENT  |     |
| R2 | 30    | PROD  | 0.4 |
| R3 | 60    | PROD  | 0.5 |

and account ACC of the Account Table is

|     | TYPE | FIX | PFIX | QFIX |
|-----|------|-----|------|------|
| ACC | AC   | f   | p    | q    |

where f, p, and q represent arbitrary values.

Account ACC and NEW1 of the Cell Table after expansion is

|      | TBASE | ACC<br>SPECS | A | TBASE | NEW1<br>SPECS | A     |
|------|-------|--------------|---|-------|---------------|-------|
| R1   | 10    | RENT         |   |       |               |       |
| R2   |       |              |   | 30    | PROD          | 0.444 |
| R3   |       |              |   | 60    | PROD          | 0.556 |
| NEW1 | 90    | IO           |   |       |               |       |

and account ACC and NEW1 of the Account Table is

|      | TYPE | FIX | PFIX | QFIX |
|------|------|-----|------|------|
| ACC  | AC   | f   | p    | q    |
| NEW1 | AC   |     |      |      |

Note two things about the share parameter A. First, in the original Cell Table there is an implied A-value of  $10/(10+30+60) = 0.1$  associated with the RENT cell so the As add to 1 in the column. (See Appendix C for further details.) Second, the As are scaled with  $100/90$  in account NEW1 so they still add to 1.

Elasticity and efficiency parameters are not shown in this or any of the other examples. If they are present in a model they always move unchanged with the PROD specification. Also, if PROD represents CES2 then the new account with the PROD specification will be subject to the expansions described under CES2.

Example 2: RENT combined with IO and ITAX. This case also creates one new account, NEW1.

Column ACC of the Cell Table before expansion is

|    | TBASE | SPECS | A   |
|----|-------|-------|-----|
| R1 | 15    | RENT  |     |
| R2 | 30    | IO    | 0.5 |
| R3 | 50    | IO    | 0.3 |
| R4 | 5     | ITAX  |     |

and account ACC of the Account Table is

|     | TYPE | FIX | PFIX | QFIX |
|-----|------|-----|------|------|
| ACC | AC   | f   | p    | q    |

Account ACC and NEW1 of the Cell Table after expansion is

|      | ACC   |       |   | NEW1  |       |       |
|------|-------|-------|---|-------|-------|-------|
|      | TBASE | SPECS | A | TBASE | SPECS | A     |
| R1   |       |       |   | 15    | RENT  |       |
| R2   |       |       |   | 30    | IO    | 0.526 |
| R3   |       |       |   | 50    | IO    | 0.316 |
| R4   | 5     | ITAX  |   |       |       |       |
| NEW1 | 95    | IO    |   |       |       |       |

and account ACC and NEW1 of the Account Table is

|      | TYPE | FIX | PFIX | QFIX |
|------|------|-----|------|------|
| ACC  | AC   | f   | p    | q    |
| NEW1 | AC   |     |      |      |

The disaggregation shows that ITAX is applied last (to the total), reflecting the convention that tax and markup is paid on all non-tax and non-markup payments, i.e. also on RENTs.

Example 3: RENT combined with PROD and ITAX. This case creates two new accounts, NEW1 and NEW2.

Column ACC of the Cell Table before expansion is

|    | TBASE | SPECS | A   |
|----|-------|-------|-----|
| R1 | 15    | RENT  |     |
| R2 | 30    | PROD  | 0.5 |
| R3 | 50    | PROD  | 0.3 |
| R4 | 5     | ITAX  |     |

and account ACC of the Account Table is

|     | TYPE | FIX | PFIX | QFIX |
|-----|------|-----|------|------|
| ACC | AC   | f   | p    | q    |

Account ACC, NEW1, and NEW2 of the Cell Table after expansion is

|      | ACC   |       |   | NEW1  |       |   | NEW2  |       |       |
|------|-------|-------|---|-------|-------|---|-------|-------|-------|
|      | TBASE | SPECS | A | TBASE | SPECS | A | TBASE | SPECS | A     |
| R1   |       |       |   | 15    | RENT  |   |       |       |       |
| R2   |       |       |   |       |       |   | 30    | PROD  | 0.625 |
| R3   |       |       |   |       |       |   | 50    | PROD  | 0.375 |
| R4   | 5     | ITAX  |   |       |       |   |       |       |       |
| NEW1 | 95    | IO    |   |       |       |   |       |       |       |
| NEW2 |       |       |   | 80    | IO    |   |       |       |       |

and account ACC, NEW1, and NEW2 of the Account Table is

|      | TYPE | FIX | PFIX | QFIX |
|------|------|-----|------|------|
| ACC  | AC   | f   | p    | q    |
| NEW1 | AC   |     |      |      |
| NEW2 | AC   |     |      |      |

**WARNING:** The disaggregation again shows that ITAX is applied to the total. In the production cases, i.e. when PROD represents CD, CES, CES2, or SUBST, this is reasonable since most taxes are paid on the pre-tax sales price regardless of whether this price is based on the costs of intermediates or has a pure profit (RENT) component. If RENT is combined with IMPORT and ITAX, however, then the tax in the model will be based on both the IMPORT and the RENT cells, i.e., both on the value paid to the rest of the world and the additional profit. If the tax is in fact only paid on the value paid to the rest of the world, then it is necessary to include explicitly one account for the IMPORT and ITAX specifications and another account that buys the landed good and adds the RENT.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$QCSOL(i,j) - TBASE(i,j) / YBASE(j) * QSOL(j)$$

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation, C1, are associated with the row and column of the cell.

**RESTRICTIONS:** None.

**COMMENTS:** Note that a constant price payment (QCSOL) is defined for the cell. It is defined as the rent payment that would exist with base period prices and with the solution quantities. The definition is similar to the one for ITAX and MARKUP. The specification has one cell equation and two cell variables so there is one additional variable in the cell. To get a balanced model, it is necessary to add an equation somewhere by, e.g., fixing a price or a quantity.

**REFERENCE:** Chapter 10.

**SPECS:** SUBST - perfect SUBSTITutes.

**ECONOMICS:** The specification indicates that several inputs are combined to create one output in a process where the former are essentially perfect substitutes. The specification can be used when a single composite good is made up of essentially identical inputs which are nevertheless of different origin (i.e., produced by different technologies, or derived from domestic and imported sources). The demand side of the model does not distinguish between the goods; it only has to describe demand for the composite good.

The specification can only be used in AC columns and MF or AC rows.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} & \text{PSOL}(i)/(1+\text{EC}(i,j)) * (1+\text{THETA}(j)) - \text{PSOL}(j)/(1+\text{E}(j)) * (1+\text{THETA0}(j)) \\ & \text{QCSOL}(i,j) - \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

If there is no tax and no efficiency gains in the column then the behavioral equation takes the simpler form

$$\text{PSOL}(i) - \text{PSOL}(j)$$

**ACCOUNT EQUATIONS:** One row equation, R5, and one column equation, C1, are associated with the row and column of the cell.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be greater than or equal to zero.

**COMMENTS:** SUBST is equivalent to CES with SIGMA = INF in the column.

**INTERNAL EQUATION NAME:** SUBST.

**REFERENCE:** Chapter 12.

SPECS: TEXO - EXOgenous Tsol.

ECONOMICS: The specification indicates that the transaction covers a prespecified value (in domestic currency).

The specification can be used in the column of INST accounts where it pays to other INST, INSTC, or ROW accounts, and it can be used in the column of ROW accounts where it pays to NMF, INST, and INSTC accounts.

CELL VARIABLES: TSOL(i,j).

CELL EQUATIONS:

TSOL(i,j) - FV(i,j)

ACCOUNT EQUATIONS: One row equation, R1, and one column equation, C1, are associated with the row and column of the cell.

RESTRICTIONS: None.

INTERNAL EQUATION NAME: TEXO.

REFERENCE: Chapter 8.

**SPECS:** TRANSF - TRANSFer.

**ECONOMICS:** The specification describes a transfer of income from an TNMF account to a NMF account. The total income of the account is distributed to the cells in the column according to exogenous shares. The specification can only be used in the column of an TNMF account and in the row of a NMF account, and it can only be combined with ITAX specifications in the column.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} \text{TSOL}(i,j) &= A(i,j) * \text{YSOL}(j) * (1+\text{THETA0}(j))/(1+\text{THETA}(j)) \\ \text{QCSOL}(i,j) &= \text{TBASE}(i,j) / \text{YBASE}(j) * \text{sum}(k, \text{QCSOL}(j,k)) \end{aligned}$$

**ACCOUNT EQUATIONS:** One row equation, R1, is associated with the row of the cell, and there is no column equation.

**RESTRICTIONS:** None.

**INTERNAL EQUATION NAME:** VSHR.

**SPECS:** UNSPEC - UNSPECified (residual).

**ECONOMICS:** The specification indicates that there is no direct equation defining the payment. The payment is a residual after all income has been received and all other expenditures have been paid.

The specification can only be used in INST accounts where they intersect with other INST or INSTC accounts, i.e., in the income transfer matrix or where they intersect with ROW accounts. More than one UNSPEC specification can appear in one row or column, provided that each is uniquely determined.

**CELL VARIABLES:** TSOL(i,j).

**CELL EQUATIONS:** None.

**ACCOUNT EQUATIONS:** One row equation, R1, and one column equation, C1, are associated with the row and column of the cell.

**RESTRICTIONS:** None.

**COMMENTS:** The cell contains one variable and no equations, so it introduces one degree of freedom that must be absorbed by constraints elsewhere, e.g., by fixing Q in a receiving account.

**INTERNAL EQUATION NAME:** None.

**REFERENCE:** Chapter 6.

**SPECS:** VEXO - EXOgenous Value.

**ECONOMICS:** The specification describes a consumption system; it indicates that the transaction covers a prespecified value, independent of the price of the good. The specification can be used to model a government budget that is fixed in nominal terms.

The specification can only be used in INSTC columns and AC rows, and if a column contains one VEXO specification then all specifications in the column must be VEXO.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} \text{TSOL}(i, j) &= \text{FQ}(i, j) \\ \text{QCSOL}(i, j) &= \text{TSOL}(i, j) / \text{PSOL}(i) \end{aligned}$$

**ACCOUNT EQUATIONS:** One row equation, R5, is associated with the row of the cell, and two column equations are associated with the column of the cell: C1 guarantees that the solution SAM balances, and C8 defines a consumer price index for the column.

**RESTRICTIONS:** TSOL(i,j) must be greater than or equal to zero.

**INTERNAL EQUATION NAME:** TEXO.

**SPECS:** VSHR - fixed Value SHaRes.

**ECONOMICS:** The specification describes a consumption system. The composition of consumption is assumed to be in fixed shares, measured in monetary units, and independent of prices. The overall scale depends on available income, which can be either exogenous or endogenous.

The specification can only be used in INSTC columns and AC rows, and if a column contains one VSHR specification then all specifications in the column must be VSHR.

**CELL VARIABLES:** TSOL(i,j) and QCSOL(i,j).

**CELL EQUATIONS:**

$$\begin{aligned} \text{TSOL}(i,j) &= A(i,j) * \text{YSOL}(j) \\ \text{QCSOL}(i,j) &= \text{TSOL}(i,j) / \text{PSOL}(i) \end{aligned}$$

**ACCOUNT EQUATIONS:** One row equation, R5, and one column equation, C3, are associated with the row and column of the cell.

**RESTRICTIONS:** TSOL(i,j) and YSOL(j) must be non negative.

**INTERNAL EQUATION NAME:** VSHR.

**REFERENCE:** Chapter 2 and 3.

## APPENDIX B

### ACCOUNT RELATED EQUATIONS

This appendix describes the equations that link the individual payments in each account. Some specifications will automatically give cell values that satisfy the accounting identity for the column or row in which they appear. Others require an explicit equation to ensure that the final SAM is balanced. Many of the equations that guarantee this outcome are equivalent to ones that define price indices, but explicit price index equations are necessary in other cases.

In this appendix, the account related equations are broken down into equations that guarantee balance in the row or relate prices in the row (labeled R1, R2, etc.), equations that guarantee balance in the column or relate prices in the column (labeled C1, C2, etc.), and the equation that relates price and quantity to total value, labeled Q1. Throughout the presentation we will use  $i$  and  $j$  to denote the row and column, respectively, and we will implicitly assume that all summations are only over the nonzero cells of the SAM.

The notation is as explained in Appendix A.

#### R1-EQUATION:

$$YSOL(i) - \text{SUM}(j, TSOL(i, j) )$$

**COMMENTS:** The equation simply states that the total income of the account is the sum of the individual payment to the accounts. The payments have no special structure, and the equation is used in most rows without a special pattern of specifications.

#### R2-EQUATION:

$$PSOL(i) - \text{SUM}(j, B(i, j) * (PSOL(j) * (1 + THETA(j)) / (1 + THETA(j)))) ** (1 - SIGMAR(i))) \\ ** (1 / (1 - SIGMAR(i)))$$

**COMMENTS:** The equation is only used in rows where all specifications are CET or FALOC (with  $B(i, j)$  defined from  $C(i, j)$  as described under FALOC). CET specifications relate the price of gross production,  $PSOL(i)$ , to the prices of the individual outputs,  $PSOL(j)$ , adjusted for changes in the rate of tax on output. It is mathematically

equivalent to equation R1, but it demonstrates the relationship between the prices more explicitly. If there are no taxes on output then the equation assumes the simpler form:

$$PSOL(i) = \sum(j, B(i,j) * PSOL(j) ** (1 - SIGMAR(i))) ** (1 / (1 - SIGMAR(i)))$$

In the case of FALOC specifications the equation defines an index for the average exchange rate. Expanding B(i,j) we get the equation

$$PSOL(i) = \frac{\sum(j, C(i,j) * PBAS(j) * PSOL(j) ** (1 - SIGMAR(i))) ** (1 / (1 - SIGMAR(i)))}{\sum(j, C0(i,j) * PBAS(j))},$$

which shows that PSOL(i) can be different from 1 even if PSOL(j) is 1, provided that the values of PBAS vary and the target shares, C, are different from the base shares, C0.

**SPECIAL VERSIONS:** When SIGMAR(i) is 0, i.e. when CET is converted into CET0 and the goods are produced in fixed quantity ratios with ratio B, then the equation becomes R3 below. When SIGMAR(i) is -INF (minus infinity), i.e., when CET is converted into CETINF and the outputs are perfect substitutes, then R2 is replaced by R1; the cell equations will relate the prices to each other and R1 will guarantee balance in the SAM.

### R3-EQUATION:

$$PSOL(i) = \sum(j, B(i,j) * PSOL(j))$$

**COMMENTS:** The equation is only used in rows where all specifications are CET0 or FALOC0 (with B(i,j) defined from C(i,j) as described under FALOC0 in Appendix A). It is similar to R2, but is applied to the transformation function with elasticity of transformation equal to zero.

CET0 specifications are mathematically equivalent to equation R1, but demonstrate the relationship between the prices more explicitly.

FALOC0 specifications define an index for the average exchange rate. Expanding B(i,j) we get the equation

$$PSOL(i) = \frac{\sum(j, C(i,j) * PBAS(j) * PSOL(j))}{\sum(j, C0(i,j) * PBAS(j))}$$

which shows that PSOL(i) can be different from 1 even if PSOL(j) is 1, provided that the values of PBAS vary and the target shares, C, are different from the base shares, C0.

#### R5-EQUATION:

$$QSOL(I) - \text{SUM}(j, QCSOL(i, j) )$$

COMMENTS: The equation is similar to R1, but it balances quantities instead of values. R5 is mathematically equivalent to R1 as long as the prices associated with all transactions in the row are the same, which is the case in MF and AC rows without specification CET, CETQ, or CETINF. During normal model solution, where QSOL and QCSOL are eliminated, R5 is actually replaced by R1. If the model turns out to be numerically singular, however, then R5 is implemented as shown above, and an attempt is made to find where the structural singularity comes from. This is explained in more detail in chapter 9.

#### C1-EQUATION:

$$YSOL(j) - \text{SUM}(i, TSOL(i, j) )$$

COMMENTS: The equation simply states that the total expenditure of the account is the sum of the individual payment from the accounts. The payments have no special structure, and the equation is used in most columns without a special pattern of specifications.

#### C2-EQUATION:

$$PSOL(j) - (1+THETA(j)) / (1+E(j)) * \text{SUM}(i \text{ \$ SPECS}(i, j) \text{ ne } (ITAX, FTAX, MARKUP), A(i, j) * PSOL(i)/(1+EC(i, j)))$$

COMMENTS: The equation defines the price of the output of an IO production function as a weighted sum of the prices of the inputs, adjusted for tax. The weights are the input intensities, A(i,j), adjusted for changes in the efficiency of their use, EC(i,j) and E(j). The equation is mathematically equivalent to C1 but it describes the price relationship better. The equation is used in all columns that contain specification IO without having a RENT specification. It is also used in columns with the combination FEXCHD and FTAX, in which case A(i,j) is derived from the base SAM as TBASE(i,j)/YBASE(j) and EC and E are zero. Finally, it defines a consumer price

index for a QSHR consumption system. In this last case EC, E, and THETA are all zero and the equation takes the simpler form below.

If there are no efficiency changes, i.e.  $EC(i,j) = E(j) = 0$ , and no tax, i.e.  $THE- TA(j) = 0$ , then the equations takes on the simpler form

$$PSOL(j) - SUM(1, A(1,j) * PSOL(1))$$

C3-EQUATION:

$$PSOL(j) - (1+THETA(j))/(1+THETA0(j)) / (1+E(j)) * PROD(1 \$ SPECS(1,j) ne (ITAX,MARKUP), (PSOL(1)/(1+EC(1,j)))) ** A(1,j))$$

COMMENTS: The equation is similar to C2 except that it applies to a CD production function. This equation is not equivalent to C1; C1 is automatically satisfied with a CD production function, and C3 is the cost function associated with CD. The equation is used in all columns that have CD specifications without having a RENT specification. It also defines a consumer price index for a VSHR consumption system. In this case EC, E, and THETA are all zero and the equation takes the simpler form below.

If there are no efficiency changes, i.e.  $EC(i,j) = E(j) = 0$ , and no tax, i.e.  $THE- TA(j) = THETA0(j) = 0$ , then the equation takes on the simpler form

$$PSOL(j) - PROD(1, PSOL(1) ** A(1,j) )$$

C4-EQUATION:

$$PSOL(j) - (1+THETA(j)) / (1+E(j)) * SUM(1 \$ SPECS(1,j) ne (ITAX,MARKUP), A(1,j)*(PSOL(1)/(1+EC(1,j))))** (1-SIGMA(j)) ) ** (1/(1-SIGMA(j)))$$

COMMENTS: The equation is similar to C2 and C3; it applies to the general CES production function, and C2 and C3 are in fact special cases of it. It is used in all columns with CES specifications as long as SIGMA does not have one of its special values (0, 1, or INF), and as long as the column has no RENT specification.

If there are no efficiency changes, i.e.  $EC(i,j) = 0$  and  $E(j) = 0$ , and no tax, i.e.  $THETA(j) = 0$ , then the equation takes on the simpler form

$$PSOL(j) - SUM(1, A(1,j) * PSOL(1) ** (1-SIGMA(j)) ) ** (1/(1-SIGMA(j)))$$

## C5-EQUATION:

$$PSOL(j) - SUM(1, TSOL(1,j)) / SUM(1, TSOL(1,j)) / (PBAS(1)*PSOL(1)) / (SUM(1, TBASE(1,j)) / SUM(1, TBASE(1,j)) / PBAS(1))$$

COMMENTS: This equation defines the price of the output in a column with AVRG specifications. The nominator sum in the first line is the value of all inputs, and the denominator is the 'absolute' quantity of the inputs, derived by dividing the payments by the base period 'absolute' price and the price index, so the price is equal to value divided by quantity. The second line contains the same formula as the first line but derived for the base case. It is included to scale PSOL(j) so it becomes an index with value 1 in the base case.

C5 does not guarantee that the account is balanced, so it is always combined with C1 in the same column, i.e., we have two column equations in AVRG columns.

## C6-EQUATION:

$$PSOL(j) - (1 + THETA(j)) * A0(1,j) * WP(1,j) * PSOL(1),$$

where i is the row with specification IMPORT.

COMMENTS: The equation defines the price of an imported commodity after taxes and in domestic currency. When more than one tax is levied, each tax is based on pretax values. PSOL(i) is the price of the Rest-Of-World account, i.e. the exchange rate, and A0(i,j) is a constant calibrated from the base SAM as explained under specification IMPORT in Appendix A. C6 is mathematically equivalent to C1, but it shows the price relationship much more clearly. The equation is used in all columns with an IMPORT specification and without a RENT specification.

## C7-EQUATION:

$$PSOL(j) - YSOL(j) / SUM(1, FQ(1,j))$$

COMMENTS: The equation defines a consumer price index for the QEXO consumption system, defined as the total value of the account divided by the sum of the quantities. It is used in all columns with specification QEXO. The price index guarantees that QSO(j) is equal to SUM(i, QCSOL(i,j)).

**C8-EQUATION:**

$$PSOL(j) = YSOL(j) / \text{SUM}(i, FV(i,j)/PSOL(i))$$

**COMMENTS:** The equation defines a consumer price index for the VEXO consumption system, defined as the total value of the column divided by the sum of the quantities. It is used in all columns with specification VEXO. The price index guarantees that QSOL(j) is equal to SUM(i, QCSOL(i,j)).

**C9-EQUATION:**

$$PSOL(j) = \text{UNDEFined}$$

**COMMENTS:** The equation defines the price index in a column with DQEXO or DVEXO specifications, i.e. a column that models changes in stocks. Since the quantities in such a column can be negative, there is no real price index; for the purpose of counting variables and equations, however, C9 and the corresponding price variable are still included in the model.

C9 is used during the intermediate solution of the model. If the quantities in the column in the final solution are significantly different from zero, then we still compute a price index based on C7 for DQEXO consumption systems and C8 for DVEXO consumption systems. Significantly different from zero is defined as

$$\text{abs}(\text{SUM}(i, \text{QCSOL}(i,j))) > 0.1 * \text{SUM}(i, \text{abs}(\text{QCSOL}(i,j)))$$

**CA-EQUATION:**

$$PSOL(j) = PSOL(1LES) * YSOL(j) / (\text{YSOL}(j) - \text{SUM}(i \text{ \&SPECS}(i,j) \text{ eq DQEXO, } (PSOL(1LES) - PSOL(i)) * FV(i,j)))$$

**COMMENTS:** This is a price index in a column with DQEXO and one 1LES specification, i.e. an expanded LES column. (See Appendix A, specification LES for details of this expansion.) The resulting price index is the price index of the overall LES column.

Q1-EQUATION:

$$PSOL(j) * QSOL(j) - YSOL(j)$$

COMMENTS: The equation applies to all accounts that have a defined quantity, i.e. all accounts with TYPE defined as MF, INSTC, and AC.

## APPENDIX C

### MODEL PARAMETERS

This appendix describes the parameters used in a GAMS-HERCULES model, explaining their use and any restrictions that they must satisfy (e.g., some of them cannot be negative, and others must add to one in a row or column), defining their default values if any, and explaining how calibrated values are returned to the modeler.

We use the following notation for parameters. The account table is referred to as AT and the cell table as CT. Account parameter ABC refers to column ABC in the account table, i.e., AT(\*,"ABC") where \* indicates some index, and ABC represents some legal column label, and cell parameter DEF refers to plane DEF in the cell table, i.e., CT(\*,\*, "DEF") where DEF represents some legal plane label. Similarly, ABC(i) refers to the element in the account table in account i and column ABC, i.e. AT(-i,"ABC"), and DEF(i,j) refers to the element in the cell table in row i, column j, and plane ABC, i.e., CT(i,j,"DEF").

For all parameters that need not be supplied explicitly, HERCULES returns the value that was used to produce the solution in an additional column in the account table as a plane in the cell table. The name of this column or plane is defined under "Actual Value" below. If the model is solved successfully, the actual value will be the target value, i.e. the value supplied by the modeler, or if none was supplied, the default value. If the solution process is not successful but stops somewhere between the base solution and the final solution, then the actual value will be a weighted average of the target value and the default value.

Note that GAMS can handle special values like +INF and -INF (plus and minus infinity), EPS (epsilon or effectively zero), UNDF (undefined) and NA (not available). The HERCULES system will interpret EPS as an explicit zero which is different from the zero GAMS default value. EPS can be used everywhere. Other special values are not allowed unless explicitly noted.

Also note that automatic calibration of shares can only take place in columns or rows that contain significant column or row totals, i.e. totals that are numerically greater than approximately 0.01 percent of the largest account total. The only exception is columns and rows with only one element, where the share is naturally always one. Shares that cannot be calibrated automatically **MUST** be defined explicitly.

**ACCOUNT SPECIFIC PARAMETERS:****Parameter E.**

**USE:** The parameter E is an efficiency parameter. It defines general changes in the efficiency of production functions. The definition is as follows: If a set of inputs creates one unit of output before the efficiency change, then the same set of inputs will create  $(1+E)$  units after the efficiency change. E is used with all production function specifications, i.e., CD, CES, CES2, IO, and SUBST.

**RESTRICTIONS:** The parameter must be greater than -1.

**DEFAULT:** The default value for E is the same as the default GAMS value, i.e., zero corresponding to no efficiency change.

**ACTUAL VALUE:** The value of E actually used in the model is returned in column E-USED of the account table.

**Parameter PFIX.**

**USE:** The parameter PFIX defines the exogenous price associated with a FIX of P, PQ, NP, and NPQ.

**RESTRICTIONS:** The parameter must be greater than zero.

**DEFAULT:** If the value of PFIX in an account is undefined, i.e., has the default GAMS value of zero, then it is assigned the value 1.

**ACTUAL VALUE:** The value of PFIX actually used in the model is the value of PSOL returned by the solution algorithm.

**Parameter PBAS.**

**USE:** The parameter PBAS defines an absolute price level or an absolute exchange rate in the base case. It is used in rows with specification AVRG or columns with specification FALOC0.

**RESTRICTIONS:** The parameter must be greater than zero.

**DEFAULT:** There is no default value, i.e. a value must be defined by the modeler.

**ACTUAL VALUE:** The value of PBAS actually used in the model is always the value defined in GAMS, since there is no default for PBAS.

**Parameter QFIX.**

**USE:** The parameter QFIX defines the exogenous quantity associated with a FIX of Q, PQ, or NPQ.

**RESTRICTIONS:** The parameter must be greater than or equal to zero.

**DEFAULT:** If the value of QFIX in an account is undefined, i.e., if it has the default GAMS value of zero, then it is assigned the value YBASE for the account. If YBASE is nonzero and QFIX should be zero, then define QFIX to be EPS.

**ACTUAL VALUE:** The value of QFIX actually used in the model is the value of QSOL returned by the solution algorithm.

**Parameter SIGMA.**

**USE:** The parameter SIGMA is an elasticity of substitution between the inputs in a CES production function. It is only used with specification CES and CES2.

**RESTRICTIONS:** SIGMA must be greater than or equal to zero. It can be +INF (plus infinity) in which case the inputs are perfect substitutes, corresponding to a SUBST specification.

**DEFAULT:** There is no default value, i.e., a value must be defined by the modeler. If SIGMA should be zero then define it to be EPS.

**ACTUAL VALUE:** The value of SIGMA actually used in the model is always the value defined in GAMS, since there is no default for SIGMA.

**Parameter SIGMAR.**

**USE:** The parameter SIGMAR is an elasticity of substitution between the outputs in a CET transformation function. It is only used with specification CET.

**RESTRICTIONS:** SIGMAR must be less than or equal to zero. It can be -INF (minus infinity) in which case the outputs are perfect substitutes.

**DEFAULT:** There is no default value, i.e., a value must be defined by the modeler. If SIGMA should be zero then define it to be EPS.

**ACTUAL VALUE:** The value of SIGMAR actually used in the model is always the value defined in GAMS, since there is no default for SIGMAR.

**Parameter YFIX.**

**USE:** The parameter YFIX defines the exogenous value associated with a FIX of Y or NY.

**RESTRICTIONS:** None.

**DEFAULT:** If the value of YFIX in an account is undefined, i.e. has the default GAMS value of zero, then it is assigned the value YBASE. If YBASE is nonzero and YFIX should be zero, then define YFIX to be EPS.

**ACTUAL VALUE:** The value of YFIX actually used in the model is the value of YSOL returned by the solution algorithm.

**CELL SPECIFIC PARAMETERS:****Parameter A.**

**USE:** The A parameter is a share parameter. It defines how total income is divided among the cells in the column. It is used explicitly with specification CD, CES, CES2, IO, QSHR, and VSHR, and implicitly with IMPORT, ITAX, MARKUP, and RENT. The distinction between explicit and implicit use is that A can only be defined by the modeler if it is used explicitly.

**RESTRICTIONS:** The parameter must be greater than or equal to zero, and the values must add to one in each column.

**DEFAULT:** If all explicitly used values of A in one column are undefined, i.e., have the default GAMS value of zero, then the shares in that column are computed from the base SAM,  $A(i,j) = TBASE(i,j)/YBASE(j)$ . Implicitly used shares are always computed from this formula. If one value in a column is defined, then all explicitly used values in the column must be defined, possibly with the default GAMS value of zero, and they must together with implicitly used A-values satisfy the restrictions above.

**ACTUAL VALUE:** The explicitly used values of A, either computed from the base SAM or taken from the input values, are returned in plane A-USED of the cell table.

**Parameter ALPHA.**

**USE:** The ALPHA parameter defines committed expenditure in a linear expenditure system. It is only used with specification LES.

**RESTRICTIONS:** None.

**DEFAULT:** The default value for ALPHA is the same as the default GAMS value, i.e., zero or no committed expenditure.

**ACTUAL VALUE:** The value of ALPHA actually used in the model is returned in plane ALPHA-USED of the cell table.

#### Parameter B.

**USE:** The parameter B is a share parameter. It defines how overall output is divided among the cells in the row. It is only used with specification CET.

**RESTRICTIONS:** The parameter must be greater than or equal to zero, and the values must add to one in each row.

**DEFAULT:** If all values of B in one row are undefined, i.e., have the default GAMS value of zero, then the shares in that row are computed from the base SAM,  $B(i,j) = TBASE(i,j)/YBASE(i)$ . If one value in a row is defined, then all values in the row must be defined, possibly with the default GAMS value of zero, and they must satisfy the restrictions above.

**ACTUAL VALUE:** The value of B actually used in the model, either computed from the base SAM or taken from the input values, is returned in plane B-USED of the cell table.

#### Parameter BETA.

**USE:** The BETA parameter defines the proportion of discretionary income spent on discretionary expenditure in a linear expenditure system. It is only used with specification LES.

**RESTRICTIONS:** The values of BETA in a column must add to one.

**DEFAULT:** If all values of BETA in one column are undefined, i.e., have the default GAMS value of zero, then the shares in that column are computed from the base SAM after the committed expenditures (ALPHA) have been subtracted,  $BETA(i,j) = (TBASE(i,j) - ALPHA(i,j))/(YBASE(j) - SUM(k,ALPHA(k,j)))$ . If one value in a column is defined, then all values in the column must be defined, possibly with the default GAMS value zero, and they must satisfy the restrictions above.

**ACTUAL VALUE:** The value of BETA actually used in the model, either computed from the base SAM or taken from the input values, is returned in the plane BETA-USED of the cell table.

#### Parameter C.

**USE:** The parameter C is a share parameter. It defines the shares in which the overall amount of foreign exchange is divided among the cells in the row, measured in foreign exchange units. It is only used with specification FALOC0.

**RESTRICTIONS:** The parameter must be greater than or equal to zero, and the values must add to one in each row.

**DEFAULT:** If all values of C in one row are undefined, i.e., have the default GAMS value of zero, then the shares in that row are computed from the base SAM,  $C(i,j) = \text{TBASE}(i,j) / \text{PBAS}(j) / \text{sum}(k, \text{TBASE}(i,k) / \text{PBAS}(k))$ . If one value in a row is defined, then all values in the row must be defined, possibly with the default GAMS value zero, and they must satisfy the restrictions above.

**ACTUAL VALUE:** The value of C actually used in the model, either computed from the base SAM or taken from the input values, is returned in the plane C-USED of the cell table.

#### Parameter EC.

**USE:** The parameter EC is an efficiency parameter. It defines input specific changes in efficiency in production functions. The definition is as follows: One unit of input after the efficiency change is as effective as  $(1+EC)$  units before the efficiency change. EC is used with all production function specifications, i.e., CD, CES, CES2, IO, and SUBST.

**RESTRICTIONS:** The parameter must be greater than -1.

**DEFAULT:** The default value for EC is the same as the default GAMS value, i.e., zero or no efficiency change.

**ACTUAL VALUE:** The value of EC actually used in the model is returned in plane EC-USED of the cell table.

#### Parameter ETA.

**USE:** The parameter ETA is an export demand elasticity. ETA is only used with specification EXPORT.

**RESTRICTIONS:** The parameter must be greater than or equal to zero. It can be +INF (plus infinity), in which case the price of the exported good will be equal to the world price (parameter WP) multiplied by the exchange rate, and the quantity of exports will be a residual.

**DEFAULT:** There is no default value, i.e., a value must be defined by the modeler. If ETA should be zero then define it to be EPS.

**ACTUAL VALUE:** The value of ETA actually used in the model is always the value defined in GAMS, since there is no default for ETA.

**Parameter FQ.**

**USE:** The parameter FQ is a base quantity flow parameter that, possibly together with relative prices and/or exchange rates, defines the payment in the cell. It is used with specification EXPORT, QEXO, and VEXO.

**RESTRICTIONS:** The parameter must be greater than or equal to zero.

**DEFAULT:** If the value of FQ in a cell is undefined, i.e., has the default GAMS value of zero, then it is assigned the value of the flow from the base SAM,  $FQ(i,j) = TBASE(i,j)$ .

**ACTUAL VALUE:** The value of FQ actually used in the model, either computed from the base SAM or taken from the input values, is returned in plane FQ-USED of the cell table.

**Parameter FV.**

**USE:** The parameter FV is a base value flow parameter that, possibly together with a price or an exchange rate, defines the payment in the cell. It is used with specification PQEXO, DVEXO, FEXO and TEXO.

**RESTRICTIONS:** None.

**DEFAULT:** If the value of FV in a cell is undefined, i.e., has the default GAMS value of zero, then it is assigned the value of the flow from the base SAM,  $FV(i,j) = TBASE(i,j)$ .

**ACTUAL VALUE:** The value of FV actually used in the model, either computed from the base SAM or taken from the input values, is returned in plane FV-USED of the cell table.

**NOTE:** The key difference between FQ and FV is that cells that use FQ always describe quantities and therefore must be positive, while cells with FV describe pure monetary payments or changes in quantities, and therefore can be negative.

**Parameter SIGMAC.**

**USE:** The parameter SIGMAC is an elasticity of substitution between the inputs at the lower level of a two level CES production function. It is only used with specification CES2.

**RESTRICTIONS:** SIGMAC must be greater than or equal to zero. It can be +INF (plus infinity), in which case the inputs are perfect substitutes. (See specification SUBST).

**DEFAULT:** There is no default value, i.e., a value must be defined by the modeler. If SIGMAC should be zero then define it to be EPS.

**ACTUAL VALUE:** The value of SIGMAC actually used in the model is always the value defined in GAMS, since there is no default for SIGMAC.

#### Parameter THETA.

**USE:** The parameter THETA is a tax or markup rate. There can be more than one tax or markup in each column and THETA is always based on all non-tax and non-markup payments in the column. THETA is used with specification FTAX, ITAX, and MARKUP.

**RESTRICTIONS:** Each individual THETA must be greater than minus one, and the sum of all THETAs in a column must also be greater than minus one.

**DEFAULT:** If the value of THETA in a cell is undefined, i.e., has the default GAMS value of zero, then it is calibrated from the base SAM,  $THETA(i,j) = TBASE(i,j) / (YBASE(j) - SUM(cells with THETA, TBASE(*,j)))$ . If there is more than one ITAX and/or MARKUP specification in a column, then some of the THETA values may be calibrated automatically and others may be defined explicitly.

**ACTUAL VALUE:** The value of THETA actually used in the model, either computed from the base SAM or taken from the input values, is returned in plane THETA-USED of the cell table.

#### Parameter WP.

**USE:** The parameter WP is a world price index. WP is used with specifications EXPORT and IMPORT.

**RESTRICTIONS:** WP must be greater than zero.

**DEFAULT:** If the value of WP in a cell is undefined, i.e., has the default GAMS value of zero, then it is assigned the value 1.

**ACTUAL VALUE:** The value of WP actually used in the model is returned in the WP-USED plane of the cell table.

## APPENDIX D

### HERCULES KEYWORDS

#### D.1 Introduction

The keyword used by HERCULES fall into two main groups called Acronyms, and Parameters.

Acronyms are real keywords. They can be used in account table in the TYPE and FIX columns, and in the cell table in the SPECS plane. The list below lists them as 'Account Type Acronym', 'Account Fix Acronym', and 'Specification Acronym', respectively. All acronyms must be declared as ACRONYMS in GAMS before they are used. The Specification Acronyms are explained in detail in Appendix A while the remaining acronyms are explained in the text.

The word parameter is used to denote names of columns in the account table and planes in the cell table, i.e. Labels in GAMS. They fall into four categories: 'User Definable Account Specific Parameter', 'User Definable Cell Specific Parameter', 'Account Specific Parameter defined by HERCULES', and 'Cell Specific Parameter defined by HERCULES', depending on whether they describe columns in the account table or planes in the cell table, and whether they are inputs to or output from HERCULES. It is not necessary to declare the labels used to represent the parameters, and we recommend that you do not create a set with these labels and use it for domain checking. The interpretation of the parameters is given in Appendix C.

#### D.2 Keywords

|            |   |
|------------|---|
| A          | - User Definable Cell Specific Parameter      |
| A-USED     | - Cell Specific Parameter defined by HERCULES |
| AC         | - Account Type Acronym                        |
| ALPHA      | - User Definable Cell Specific Parameter      |
| ALPHA-USED | - Cell Specific Parameter defined by HERCULES |
| AVRG       | - Specification Acronym                       |
| B          | - User Definable Cell Specific Parameter      |
| B-USED     | - Cell Specific Parameter defined by HERCULES |
| BETA       | - User Definable Cell Specific Parameter      |
| BETA-USED  | - Cell Specific Parameter defined by HERCULES |
| C          | - User Definable Cell Specific Parameter      |

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|         |  |
|---------|--|
| C-USED  | - Cell Specific Parameter defined by HERCULES    |
| CD      | - Specification Acronym                          |
| CES     | - Specification Acronym                          |
| CES2    | - Specification Acronym                          |
| CET     | - Specification Acronym                          |
| CETINF  | - Specification Acronym                          |
| CET0    | - Specification Acronym                          |
| DPSOL   | - Account Specific Parameter defined by HERCULES |
| DQCSOL  | - Cell Specific Parameter defined by HERCULES    |
| DQEXO   | - Specification Acronym                          |
| DQSOL   | - Account Specific Parameter defined by HERCULES |
| DTAX    | - Specification Acronym                          |
| DTSOL   | - Cell Specific Parameter defined by HERCULES    |
| DVEXO   | - Specification Acronym                          |
| DYSOL   | - Account Specific Parameter defined by HERCULES |
| E       | - User Definable Account Specific Parameter      |
| E-USED  | - Account Specific Parameter defined by HERCULES |
| EC      | - User Definable Cell Specific Parameter         |
| EC-USED | - Cell Specific Parameter defined by HERCULES    |
| ETA     | - User Definable Cell Specific Parameter         |
| EXPORT  | - Specification Acronym                          |
| EXPINF  | - Specification Acronym                          |
| EXP1    | - Specification Acronym                          |
| EXP0    | - Specification Acronym                          |
| FALOC   | - Specification Acronym                          |
| FALOCI  | - Specification Acronym                          |
| FALOC0  | - Specification Acronym                          |
| FAVRG   | - Specification Acronym                          |
| FDIST   | - Specification Acronym                          |
| FEXCHD  | - Specification Acronym                          |
| FEXCHE  | - Specification Acronym                          |
| FEXO    | - Specification Acronym                          |
| FEXOF   | - Specification Acronym                          |
| FEXOT   | - Specification Acronym                          |
| FQ      | - User Definable Cell Specific Parameter         |
| FQ-USED | - Cell Specific Parameter defined by HERCULES    |
| FRENT   | - Specification Acronym                          |
| FTAX    | - Specification Acronym                          |
| FV      | - User Definable Cell Specific Parameter         |
| FV-USED | - Cell Specific Parameter defined by HERCULES    |
| IDIST   | - Specification Acronym                          |
| IMPORT  | - Specification Acronym                          |

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|            |  |
|------------|--|
| INST       | - Account Type Acronym                           |
| INSTC      | - Account Type Acronym                           |
| IO         | - Specification Acronym                          |
| ITAX       | - Specification Acronym                          |
| MF         | - Account Type Acronym                           |
| MARKUP     | - Specification Acronym                          |
| NEQ-NVAR   | - Account Specific Parameter defined by HERCULES |
| NEQ-NVAR   | - Cell Specific Parameter defined by HERCULES    |
| NMF        | - Account Type Acronym                           |
| NP         | - Account Fix Acronym                            |
| NPQ        | - Account Fix Acronym                            |
| NY         | - Account Fix Acronym                            |
| P          | - Account Fix Acronym                            |
| PBAS       | - User Definable Account Specific Parameter      |
| PFIX       | - User Definable Account Specific Parameter      |
| PSOL       | - Account Specific Parameter defined by HERCULES |
| Q          | - Account Fix Acronym                            |
| QCSOL      | - Cell Specific Parameter defined by HERCULES    |
| QEXO       | - Specification Acronym                          |
| QFIX       | - User Definable Account Specific Parameter      |
| QSHR       | - Specification Acronym                          |
| QSOL       | - Account Specific Parameter defined by HERCULES |
| ROW        | - Account Type Acronym                           |
| SIGMA      | - User Definable Account Specific Parameter      |
| SIGMAC     | - User Definable Cell Specific Parameter         |
| SIGMAR     | - User Definable Account Specific Parameter      |
| SPECS      | - User Definable Cell Specific Parameter         |
| SUBST      | - Specification Acronym                          |
| TAX        | - Account Type Acronym                           |
| TBASE      | - User Definable Cell Specific Parameter         |
| TEXO       | - Specification Acronym                          |
| THETA      | - User Definable Cell Specific Parameter         |
| THETA-USED | - Cell Specific Parameter defined by HERCULES    |
| TNMF       | - Account Type Acronym                           |
| TRANSF     | - Specification Acronym                          |
| TSOL       | - Cell Specific Parameter defined by HERCULES    |
| UNSPEC     | - Specification Acronym                          |
| VE XO      | - Specification Acronym                          |
| VSHR       | - Specification Acronym                          |
| WP         | - User Definable Cell Specific Parameter         |
| WP-USED    | - Cell Specific Parameter defined by HERCULES    |
| Y          | - Account Fix Acronym                            |

- YFIX - User Definable Account Specific Parameter
- YSOL - Account Specific Parameter defined by HERCULES

\$TITLE MODELA: INITIAL DEMONSTRATION MODEL  
 \* THE FOLLOWING MODEL IS THE INITIAL MODEL IN DRUD AND KENDRICK:  
 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".  
 \* IT DESCRIBES A SIMPLE MODEL WITH TWO PRODUCTION SECTORS, TWO FACTORS  
 \* OF PRODUCTION, AND TWO HOUSEHOLDS.

```
SET ACC ACCOUNTS /
      LABOR
      CAPITAL
      HHLR-RURAL
      HHLR-URBAN
      FOOD
      CLOTHING /;
```

```
ALIAS (ACC,ACCP);
```

```
ACRONYMS MF MARKET FACTOR ACCOUNT
          INSTC INSTITUTIONS CONSUMPTION ACCOUNT
          AC ACTIVITY OR COMMODITY ACCOUNT

          Q QUANTITY FIXED
          NP PRICE FIXED AS A NUMERAIRE

          CD COBB DOUGLAS PRODUCTION FUNCTION SPECIFICATION
          IDIST INCOME DISTRIBUTION SPECIFICATION
          VSHR FIXED VALUE SHARE CONSUMPTION SYSTEM;
```

```
TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX
```

|            | LABOR | CAPITAL | HHLR-RURAL | HHLR-URBAN | FOOD | CLOTHING |
|------------|-------|---------|------------|------------|------|----------|
| LABOR      |       |         |            |            | 75   | 85       |
| CAPITAL    |       |         |            |            | 50   | 60       |
| HHLR-RURAL | 90    | 30      |            |            |      |          |
| HHLR-URBAN | 70    | 80      |            |            |      |          |
| FOOD       |       |         | 60         | 65         |      |          |
| CLOTHING   |       |         | 60         | 85         |      |          |

```
TABLE SPEC(ACC,ACC) SPECIFICATIONS TABLE
```

|            | LABOR | CAPITAL | HHLR-RURAL | HHLR-URBAN | FOOD | CLOTHING |
|------------|-------|---------|------------|------------|------|----------|
| LABOR      |       |         |            |            | CD   | CD       |
| CAPITAL    |       |         |            |            | CD   | CD       |
| HHLR-RURAL | IDIST | IDIST   |            |            |      |          |
| HHLR-URBAN | IDIST | IDIST   |            |            |      |          |
| FOOD       |       |         | VSHR       | VSHR       |      |          |
| CLOTHING   |       |         | VSHR       | VSHR       |      |          |

```
* DEFINE CELL ARRAY
```

```
PARAMETER CT(ACC,ACC,*) CELL TABLE;
          CT(ACC,ACCP,"TBASE") - SAM(ACC,ACCP);
          CT(ACC,ACCP,"SPECS") - SPEC(ACC,ACCP);
```

TABLE AT(ACC,\*) ACCOUNT TABLE

|            | TYPE  | FIX |
|------------|-------|-----|
| LABOR      | MF    | Q   |
| CAPITAL    | MF    | Q   |
| HHLR-RURAL | INSTC |     |
| HHLR-URBAN | INSTC | NP  |
| FOOD       | AC    |     |
| CLOTHING   | AC    |     |

MODEL MODELA INITIAL DEMONSTRATION MODEL  
/ ACC, AT, CT /;

DISPLAY "ACCOUNT AND CELL TABLES BEFORE SOLVE:",  
AT,CT;

SOLVE MODELA USING HERCULES;

DISPLAY "ACCOUNT AND CELL TABLES AFTER FIRST SOLVE:",  
AT,CT;

- \* EXPERIMENT INFORMATION:
- \* CHANGE THE QUANTITY OF CAPITAL BY A FACTOR 1.1 FROM THE BASE VALUE.

AT("CAPITAL","QFIX")- 1.1\*AT("CAPITAL","QSOL");

SOLVE MODELA USING HERCULES;

DISPLAY "ACCOUNT AND CELL TABLES AFTER SECOND SOLVE:",  
AT,CT;

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1

MODELB: G O V E R N M E N T   A N D   T A X E S

2 \* THE FOLLOWING MODEL IS DESCRIBED IN THE CHAPTER ON  
 3 \* GOVERNMENT AND TAXES IN DRUD AND KENDRICK:  
 4 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".

5  
 6 SET ACC ACCOUNTS /  
 7       LABOR  
 8       CAPITAL  
 9       HHL- INCM  
 10       HHL- CONS  
 11       GOVERNMENT  
 12       INDR- TAX  
 13       ACT- AGRCLT  
 14       ACT- INDSTR  
 15       COM- AGRCLT  
 16       COM- INDSTR /

17  
 18 ALIAS (ACC,ACCP);

19  
 20 ACRONYMS MF       MARKET FACTOR ACCOUNT  
 21           INST     INSTITUTIONS INCOME ACCOUNT  
 22           INSTC    INSTITUTIONS COMSUMPTION ACCOUNT  
 23           TAX      INDIRECT TAX ACCOUNT  
 24           AC       ACTIVITY OR COMMODITY ACCOUNT  
 25  
 26           NP       PRICE FIXED - NUMARIARE  
 27           Q        QUANTITY FIXED  
 28  
 29           CD       COBB DOUGLAS PRODUCTION FUNCTION SPECIFICATION  
 30           IDIST    INCOME DISTRIBUTION SPECIFICATION  
 31           IO       INPUT OUTPUT SPECIFICATION  
 32           ITAX     INDIRECT TAX SPECIFICATION  
 33           VSHR     FIXED VALUE SHARE SPECIFICATION

34  
 35 TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX

|                | LABOR     | CAPITAL     | HHL- INCM   | HHL- CONS   | GOVERNMENT  |
|----------------|-----------|-------------|-------------|-------------|-------------|
| 39 LABOR       |           |             |             |             |             |
| 40 CAPITAL     |           |             |             |             |             |
| 41 HHL- INCM   | 160       | 110         |             |             |             |
| 42 HHL- CONS   |           |             | 240         |             |             |
| 43 GOVERNMENT  |           |             | 30          |             |             |
| 44 INDR- TAX   |           |             |             |             |             |
| 45 ACT- AGRCLT |           |             |             |             |             |
| 46 ACT- INDSTR |           |             |             |             |             |
| 47 COM- AGRCLT |           |             |             | 140         | 15          |
| 48 COM- INDSTR |           |             |             | 100         | 55          |
| 49             |           |             |             |             |             |
| 50 +           | INDR- TAX | ACT- AGRCLT | ACT- INDSTR | COM- AGRCLT | COM- INDSTR |

51

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 MODELB: G O V E R N M E N T A N D T A X E S

|    |            |    |    |  |     |     |
|----|------------|----|----|--|-----|-----|
| 52 | LABOR      |    | 95 |  | 65  |     |
| 53 | CAPITAL    |    | 30 |  | 80  |     |
| 54 | HHLD-INCM  |    |    |  |     |     |
| 55 | HHLD-CONS  |    |    |  |     |     |
| 56 | GOVERNMENT | 40 |    |  |     |     |
| 57 | INDR-TAX   |    |    |  | 30  | 10  |
| 58 | ACT-AGRCLT |    |    |  | 125 |     |
| 59 | ACT-INDSTR |    |    |  |     | 145 |
| 60 | COM-AGRCLT |    |    |  |     |     |
| 61 | COM-INDSTR |    |    |  |     |     |

62

63 TABLE SPEC(ACC,ACC) SPECIFICATIONS TABLE

64

65 LABOR CAPITAL HHLD-INCM HHLD-CONS GOVERNMENT

66

67 LABOR

68 CAPITAL

69 HHLD-INCM IDIST IDIST

70 HHLD-CONS IDIST

71 GOVERNMENT IDIST

72 INDR-TAX

73 ACT-AGRCLT

74 ACT-INDSTR

75 COM-AGRCLT VSHR VSHR

76 COM-INDSTR VSHR VSHR

77

78 + INDR-TAX ACT-AGRCLT ACT-INDSTR COM-AGRCLT COM-INDSTR

79

80 LABOR CD CD

81 CAPITAL CD CD

82 HHLD-INCM

83 HHLD-CONS

84 GOVERNMENT IDIST

85 INDR-TAX ITAX ITAX

86 ACT-AGRCLT IO

87 ACT-INDSTR IO

88 COM-AGRCLT

89 COM-INDSTR

90

91 PARAMETER CT(ACC,ACC,\*) CELL TABLE;

92

93 CT(ACC,ACCP,"TBASE") - SAM(ACC,ACCP);

94 CT(ACC,ACCP,"SPECS") - SPEC(ACC,ACCP);

95

96 TABLE AT(ACC,\*) ACCOUNT TABLE

97

98 TYPE FIX

99

100 LABOR MF Q

101 CAPITAL MF Q

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MODELB: G O V E R N M E N T   A N D   T A X E S

```

102          HHL-D-INCM      INST
103          HHL-D-CONS      INSTC      NP
104          GOVERNMENT      INSTC
105          INDR-TAX        TAX
106          ACT-AGRCLT      AC
107          ACT-INDSTR      AC
108          COM-AGRCLT      AC
109          COM-INDSTR      AC
110
111  MODEL  MODELB  EXAMPLE WITH GOVERNMENT AND TAXES
112          / ACC, AT, CT /;
113
114  DISPLAY "ACCOUNT AND CELL TABLES BEFORE SOLVE",
115          AT,CT;
116
117  SOLVE  MODELB  USING  HERCULES;
118
119  DISPLAY "ACCOUNT AND CELL TABLES AFTER BASE SOLVE",
120          AT,CT;
121
122  *  EXPERIMENT INFORMATION:
123  *  NEW INDIRECT TAX RATES: ONE THIRD THE BASE RATE FOR AGRICULTURE AND
124  *  THREE TIMES THE BASE RATE FOR INDUSTRY.
125
126  CT("INDR-TAX", "COM-AGRCLT", "THETA") -
127          CT("INDR-TAX", "COM-AGRCLT", "THETA-USED")/3;
128  CT("INDR-TAX", "COM-INDSTR", "THETA") -
129          CT("INDR-TAX", "COM-INDSTR", "THETA-USED")*3;
130
131  SOLVE  MODELB  USING  HERCULES;
132
133  DISPLAY "ACCOUNT AND CELL TABLES AFTER EXPERIMENT SOLVE",
134          AT,CT;
135
136  *  DEFINE, EXTRACT, AND DISPLAY THE CURRENT PRICE AND CONSTANT PRICE
137  *  SOLUTION SAMS.
138
139  PARAMETER CURSAM(ACC,ACC) CURRENT PRICE SOLUTION SAM
140          CONSAM(ACC,ACC) CONSTANT PRICE SOLUTION SAM;
141
142  CURSAM(ACC,ACCP) - CT(ACC,ACCP,"TSOL");
143  CONSAM(ACC,ACCP) - CT(ACC,ACCP,"QCSOL");
144
145  DISPLAY CURSAM,CONSAM;

```

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 MODEL C: I N T E R M E D I A T E I N P U T S

2 \* THE FOLLOWING MODEL IS DESCRIBED IN THE CHAPTER ON  
 3 \* INTERMEDIATE INPUTS IN DRUD AND KENDRICK:  
 4 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".

5  
 6 SET ACC ACCOUNTS /  
 7 LABOR LABOR  
 8 CAPITAL CAPITAL  
 9 HOUSEHLDI HOUSEHOLD INCOME ACCOUNT  
 10 HOUSEHLDC HOUSEHOLD CONSUMPTION ACCOUNT  
 11 GOVERNMENT GOVERNMENT EXPENDITURE ACCOUNT  
 12 INDR-TAX INDIRECT TAX ACCOUNT  
 13 VA-AGRCLT VALUE ADDED IN AGRICULTURE  
 14 VA-INDSTR VALUE ADDED IN INDUSTRY  
 15 ACT-AGRCLT PRODUCTION ACTIVITY FOR AGRICULTURE  
 16 ACT-INDSTR PRODUCTION ACTIVITY FOR INDUSTRY  
 17 COM-AGRCLT COMMODITY MARKETING FOR AGRICULTURE  
 18 COM-INDSTR COMMODITY MARKETING FOR INDUSTRY /

19  
 20 ALIAS (ACC,ACCP)  
 21

22 ACRONYMS MF MARKET FACTOR ACCOUNT  
 23 INST INSTITUTIONS INCOME ACCOUNT  
 24 INSTC INSTITUTIONS CONSUMPTION ACCOUNT  
 25 TAX INDIRECT TAX ACCOUNT  
 26 AC ACTIVITY-COMMODITY ACCOUNT  
 27  
 28 NP PRICE EXOGENOUS - NUMERAIRE  
 29 Q QUANTITY EXOGENOUS  
 30  
 31 CD COBB-DOUGLAS PRODUCTION FUNCTION  
 32 IDIST INCOME DISTRIBUTION SPECIFICATION  
 33 IO INPUT-OUTPUT SPECIFICATION  
 34 ITAX INDIRECT TAX SPECIFICATION  
 35 VSHR VALUE SHARE CONSUMPTION SYSTEM  
 36

37 TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX WITH INTERMEDIATE INPUTS

38  
 39 LABOR CAPITAL HOUSEHLDI HOUSEHLDC GOVERNMENT INDR-TAX  
 40 HOUSEHLDI 160 110  
 41 HOUSEHLDC 240  
 42 GOVERNMENT 30 40  
 43 COM-AGRCLT 145 15  
 44 COM-INDSTR 95 55  
 45  
 46 + VA-AGRCLT VA-INDSTR ACT-AGRCLT ACT-INDSTR COM-AGRCLT  
 COM-INDSTR  
 47 LABOR 95 65  
 48 CAPITAL 30 80  
 49 INDR-TAX 30 10  
 50 VA-AGRCLT 125

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 MODEL C: I N T E R M E D I A T E I N P U T S

```

51 VA-INDSTR                145
52 ACT-AGRCLT                200
53 ACT-INDSTR                235
54 COM-AGRCLT                40    30
55 COM-INDSTR                35    60
56
57 TABLE SPEC(ACC,ACC) SPECIFICATIONS TABLE
58
59           LABOR    CAPITAL    HOUSEHLDI    HOUSEHLC    GOVERNMENT    INDR-TAX
60 HOUSEHLDI    IDIST        IDIST
61 HOUSEHLC        IDIST
62 GOVERNMENT        IDIST
63 COM-AGRCLT        VSHR        VSHR
64 COM-INDSTR        VSHR        VSHR
65
66 +           VA-AGRCLT    VA-INDSTR    ACT-AGRCLT    ACT-INDSTR    COM-AGRCLT
                                           COM-INDSTR
67 LABOR            CD        CD
68 CAPITAL          CD        CD
69 INDR-TAX                ITAX        ITAX
70 VA-AGRCLT                IO
71 VA-INDSTR                IO
72 ACT-AGRCLT                IO
73 ACT-INDSTR                IO
74 COM-AGRCLT                IO    IO
75 COM-INDSTR                IO    IO
76
77 * DEFINE CELL ARRAY
78
79 PARAMETER CT(ACC,ACC,*) CELL ARRAY;
80           CT(ACC,ACCP,"TBASE") - SAM(ACC,ACCP);
81           CT(ACC,ACCP,"SPECS") - SPEC(ACC,ACCP);
82
83 TABLE AT(ACC,*) ACCOUNT TABLE
84
85           TYPE    FIX
86 LABOR          MF    Q
87 CAPITAL        MF    Q
88 HOUSEHLDI      INST
89 HOUSEHLC       INSTC    NP
90 GOVERNMENT     INSTC
91 INDR-TAX       TAX
92 VA-AGRCLT      AC
93 VA-INDSTR      AC
94 ACT-AGRCLT     AC
95 ACT-INDSTR     AC
96 COM-AGRCLT     AC
97 COM-INDSTR     AC
98
99 MODEL MODEL C EXAMPLE MODEL WITH INTERMEDIATE INPUTS

```

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3

MODEL C: I N T E R M E D I A T E I N P U T S

```
100          / ACC, AT, CT /;
101
102 SOLVE MODEL C USING HERCULES;
103
104 * THE FOLLOWING EXPERIMENTS EVALUATES THE INFLUENCE OF CHANGES IN TAX
105 * RATES ON FINAL CONSUMPTION. FIRST DEFINE SETS AND PARAMETERS, AND SAVE
106 * THE BASE VALUES.
107
108 SET CA(ACC)  COMMODITY ACCOUNTS / COM-AGRCLT, COM-INDSTR /
109     CASES    EXPERIMENT CASES / BASE-CASE, "+10%AG-TAX", "+10%IN-TAX" / .
110 PARAMETER  COMM(CA,CASES) SUMMARY TABLE OF COMMODITY BALANCE
111     ELAST(CA,CASES) ELASTICITY OF COMMODITY SUPPLY W.R.T.
                                CHANGES IN TAX RATES;
112
113 COMM(CA,"BASE-CASE") - AT(CA,"QSOL");
114
115 * EXPERIMENT 1:
116 * INCREASE AGRICULTURAL TAX BY 10 PERCENT.
117
118 CT("INDR-TAX","COM-AGRCLT","THETA") -
119     CT("INDR-TAX","COM-AGRCLT","THETA-USED")*1.10;
120
121 SOLVE MODEL C USING HERCULES;
122
123 COMM(CA,"+10%AG-TAX") - AT(CA,"QSOL");
124
125 * EXPERIMENT 2:
126 * INCREASE INDUSTRIAL TAX BY 10 PERCENT.
127
128 CT("INDR-TAX","COM-INDSTR","THETA") -
129     CT("INDR-TAX","COM-INDSTR","THETA-USED")*1.10;
130 CT("INDR-TAX","COM-AGRCLT","THETA") - 0;
131
132 SOLVE MODEL C USING HERCULES;
133
134 COMM(CA,"+10%IN-TAX") - AT(CA,"QSOL");
135
136 * COMPUTE ELASTICITIES AND DISPLAY RESULTS
137
138 ELAST(CA,CASES) - (COMM(CA,CASES) / COMM(CA,"BASE-CASE") - 1) / 0.1;
139 DISPLAY COMM,ELAST;
```

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1

MODEL: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
 DEFINITION OF ACCOUNT SET AND ACRONYMS

3 \* THE FOLLOWING MODEL IS DESCRIBED IN THE CHAPTER ON FOREIGN TRADE,  
 4 \* INVESTMENT, AND SAVINGS IN DRUD AND KENDRICK:  
 5 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".

6  
 7 SET ACC ACCOUNTS /  
 8 LABOR LABOR  
 9 CAPITAL CAPITAL  
 10 HOUSEHLD-I HOUSEHOLD INCOME ACCOUNT  
 11 HOUSEHLD-C HOUSEHOLD CONSUMPTION ACCOUNT  
 12 GOVERNMT-I GOVERNMENT INCOME ACCOUNT  
 13 GOVERNMT-C GOVERNMENT EXPENDITURE ACCOUNT  
 14 SAVING-INV SAVINGS AND INVESTMENTS  
 15 INDR-TAX INDIRECT TAX ACCOUNT  
 16 VAL-ADD-AG VALUE ADDED IN AGRICULTURE  
 17 VAL-ADD-IN VALUE ADDED IN INDUSTRY  
 18 ACT-AGRCLT PRODUCTION ACTIVITY FOR AGRICULTURE  
 19 ACT-INDSTR PRODUCTION ACTIVITY FOR INDUSTRY  
 20 COM-DOM-AG DOMESTIC COMMODITIES IN AGRICULTURE  
 21 COM-DOM-IN DOMESTIC COMMODITIES IN INDUSTRY  
 22 COM-IMP-AG IMPORTED COMMODITIES IN AGRICULTURE  
 23 COM-IMP-IN IMPORTED COMMODITIES IN INDUSTRY  
 24 COM-CMP-AG COMPOSITE COMMODITIES IN AGRICULTURE  
 25 COM-CMP-IN COMPOSITE COMMODITIES IN INDUSTRY  
 26 COM-EXP-AG EXPORTED COMMODITIES IN AGRICULTURE  
 27 COM-EXP-IN EXPORTED COMMODITIES IN INDUSTRY  
 28 REST-WORLD REST OF THE WORLD ACCOUNT /

29  
 30 ALIAS (ACC,ACCP);

31  
 32 ACRONYMS MF MARKET FACTOR ACCOUNT  
 33 INST INSTITUTIONS INCOME ACCOUNT  
 34 INSTC INSTITUTIONS CONSUMPTION ACCOUNT  
 35 TAX INDIRECT TAX ACCOUNT  
 36 AC ACTIVITY-COMMODITY ACCOUNT  
 37 ROW REST OF THE WORLD ACCOUNT  
 38  
 39 NP PRICE EXOGENOUS - NUMERAIRE  
 40 Q QUANTITY EXOGENOUS  
 41  
 42 CD COBB-DOUGLAS PRODUCTION FUNCTION  
 43 CES CES PRODUCTION FUNCTION  
 44 EXPORT EXPORT DEMAND FROM THE REST OF THE WORLD  
 45 IDIST INCOME DISTRIBUTION SPECIFICATION  
 46 IMPORT PAYMENTS FOR IMPORTS  
 47 IO INPUT-OUTPUT SPECIFICATION  
 48 ITAX INDIRECT TAX SPECIFICATION  
 49 - QEXO FIXED QUANTITY CONSUMPTION SYSTEM  
 50 QSHR FIXED QUANTITY SHARE CONSUMPTION SYSTEM  
 51 UNSPEC UNSPECIFIED OR RESIDUAL  
 52 - VEXO SPECIFICATION FOR EXOGENOUS VALUE  
 53 VSHR VALUE SHARE CONSUMPTION SYSTEM

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 MODEL: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
 DEFINITION OF SOCIAL ACCOUNTING MATRIX

```

55 TABLE SAM(ACC,ACC) "SOCIAL ACCOUNTING MATRIX WITH FOREIGN TRADE,
                           INVESTMENT, AND SAVINGS"
56
57           LABOR      CAPITAL  HOUSEHLD-I  HOUSEHLD-C
58 HOUSEHLD-I      160         110
59 HOUSEHLD-C                                210
60 GOVERNMT-I                                20
61 SAVING-INV                                40
62 COM-CMP-AG                                130
63 COM-CMP-IN                                80
64
65 +           GOVERNMT-I  GOVERNMT-C  SAVING-INV  INDR-TAX
66 GOVERNMT-I                                90
67 GOVERNMT-C      70
68 SAVING-INV      40
69 COM-CMP-AG                                15
70 COM-CMP-IN                                55
71
72 +           VAL-ADD-AG  VAL-ADD-IN  ACT-AGRCLT  ACT-INDSTR
73 LABOR           95         65
74 CAPITAL         30         80
75 VAL-ADD-AG                                125
76 VAL-ADD-IN                                145
77 COM-CMP-AG                                40
78 COM-CMP-IN                                35
79
80 +           COM-DOM-AG  COM-DOM-IN  COM-IMP-AG  COM-IMP-IN
81 INDR-TAX        20         10
82 ACT-AGRCLT     140
83 ACT-INDSTR                                185
84 REST-WORLD                                50
85
86 +           COM-CMP-AG  COM-CMP-IN  COM-EXP-AG  COM-EXP-IN  REST-WORLD
87 SAVING-INV                                20
88 INDR-TAX                                15
89 ACT-AGRCLT                                60
90 ACT-INDSTR                                50
91 COM-DOM-AG     160
92 COM-DOM-IN                                195
93 COM-IMP-AG      70
94 COM-IMP-IN                                120
95 COM-EXP-AG                                75
96 COM-EXP-IN                                55

```

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 MODEL: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
 DEFINITION OF SPECIFICATION AND CELL TABLES

```

98 TABLE SPEC(ACC,ACC) SPECIFICATION TABLE
99
100          LABOR      CAPITAL  HOUSEHLD-I  HOUSEHLD-C
101 HOUSEHLD-I      IDIST      IDIST
102 HOUSEHLD-C                      IDIST
103 GOVERNMT-I                      IDIST
104 SAVING-INV                      IDIST
105 COM-CMP-AG                      VSHR
106 COM-CMP-IN                      VSHR
107
108 +          GOVERNMT-I  GOVERNMT-C  SAVING-INV  INDR-TAX
109 GOVERNMT-I                      IDIST
110 GOVERNMT-C      UNSPEC
111 SAVING-INV      UNSPEC
112 COM-CMP-AG                      QEXO      QSHR
113 COM-CMP-IN                      QEXO      QSHR
114
115 +          VAL-ADD-AG  VAL-ADD-IN  ACT-AGRCLT  ACT-INDSTR
116 LABOR          CD          CD
117 CAPITAL        CD          CD
118 VAL-ADD-AG                      IO
119 VAL-ADD-IN                      IO
120 COM-CMP-AG                      IO
121 COM-CMP-IN                      IO
122
123 +          COM-DOM-AG  COM-DOM-IN  COM-IMP-AG  COM-IMP-IN
124 INDR-TAX      ITAX      ITAX      ITAX      ITAX
125 ACT-AGRCLT    IO
126 ACT-INDSTR    IO
127 REST-WORLD    IMPORT      IMPORT
128
129 +          COM-CMP-AG  COM-CMP-IN  COM-EXP-AG  COM-EXP-IN  REST-WORLD
130 SAVING-INV    UNSPEC
131 INDR-TAX      ITAX      ITAX
132 ACT-AGRCLT    IO
133 ACT-INDSTR    IO
134 COM-DOM-AG    CES
135 COM-DOM-IN    CES
136 COM-IMP-AG    CES
137 COM-IMP-IN    CES
138 COM-EXP-AG    EXPORT
139 COM-EXP-IN    EXPORT
140
141 SET ACCEX(ACC) EXPORTED COMMODITIES /COM-EXP-AG,COM-EXP-IN/
142
143 PARAMETER ETAS(ACCEX) ELASTICITY OF DEMAND FOR EXPORTS /
144          COM-EXP-AG - 3.0, COM-EXP-IN - 1.5 /
145
146 * DEFINE AND FILL THE CELL TABLE:
147

```

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4

MODEL: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
DEFINITION OF SPECIFICATION AND CELL TABLES

```
148  PARAMETER  CT(ACC,ACC,*)  CELL TABLE;  
149  
150      CT(ACC,ACCP,"TBASE")      - SAM(ACC,ACCP);  
151      CT(ACC,ACCP,"SPECS")      - SPEC(ACC,ACCP);  
152      CT(ACCEX,"REST-WORLD","ETA") - ETAS(ACCEX);
```

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 MODEL: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
 ACCOUNT TABLE AND ACCOUNT TOTALS

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```

154 TABLE AT(ACC,*) ACCOUNT TABLE
155
156           TYPE    FIX    SIGMA
157     LABOR      MF      Q
158     CAPITAL    MF      Q
159     HOUSEHLD-I INST
160     HOUSEHLD-C INSTC
161     GOVERNMT-I INST
162     GOVERNMT-C INSTC
163     SAVING-INV INSTC  Q
164     INDR-TAX   TAX
165     VAL-ADD-AG AC
166     VAL-ADD-IN AC
167     ACT-AGRCLT AC
168     ACT-INDSTR AC
169     COM-DOM-AG AC
170     COM-DOM-IN AC
171     COM-IMP-AG AC
172     COM-IMP-IN AC
173     COM-CMP-AG AC          3.0
174     COM-CMP-IN AC          0.5
175     COM-EXP-AG AC
176     COM-EXP-IN AC
177     REST-WORLD ROW      NP
178
179 PARAMETER TOTALS(ACC,*) ACCOUNT TOTALS AND IMBALANCES FOR THE SAM;
180
181     TOTALS(ACC, "ROW-TOTAL") - SUM(ACCP, SAM(ACC, ACCP));
182     TOTALS(ACCP, "COL-TOTAL") - SUM(ACC, SAM(ACC, ACCP));
183     TOTALS(ACC, "DIFFERENCE") - TOTALS(ACC, "ROW-TOTAL") -
184                               TOTALS(ACC, "COL-TOTAL");
185
186 DISPLAY "CHECK FOR BALANCE OF BASE SAM:", TOTALS;

```

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 MODEL D: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
 MODEL DEFINITION, EXPERIMENTS SOLUTION AND REPORT

```

188  MODEL  MODEL D  "MODEL WITH FOREIGN TRADE, INVESTMENT, AND SAVINGS"
189          / ACC, AT, CT / ;
190
191  * DEFINE SETS AND PARAMETERS FOR REPORT TABLES THAT SUMMARIZE
192  * ALL EXPERIMENTS AND DEFINE THE BASE CASE.
193
194  SET  COM(ACC)  "DOMESTIC, IMPORTED, COMPOSITE, AND EXPORTED COMMODITIES"
195          / COM-DOM-AG, COM-DOM-IN, COM-IMP-AG, COM-IMP-IN,
196          COM-CMP-AG, COM-CMP-IN, COM-EXP-AG, COM-EXP-IN /;
197
198  PARAMETER  REPORTQ(COM,*)  QUANTITIES OF ALL COMMODITIES
199          REPORTP(COM,*)  PRICES OF ALL COMMODITIES;
200
201  * REPORT -BASE CASE-
202
203          REPORTQ(COM,"BASE-CASE") - TOTALS(COM,"COL-TOTAL");
204          REPORTP(COM,"BASE-CASE") - 1;
205
206  * EXPERIMENT DATA: CHANGES IN WORLD PRICES
207  * WORLD AGRICULTURAL PRICES ARE INCREASED BY 10% AND
208  * WORLD INDUSTRIAL PRICES ARE DECREASED BY 10%.
209
210          CT("REST-WORLD","COM-IMP-AG","WP") - 1.1;
211          CT("REST-WORLD","COM-IMP-IN","WP") - 0.9;
212          CT("COM-EXP-AG","REST-WORLD","WP") - 1.1;
213          CT("COM-EXP-IN","REST-WORLD","WP") - 0.9;
214
215  SOLVE  MODEL D  USING  HERCULES;
216
217  DISPLAY "ACCOUNT AND CELL INFORMATION TABLES AFTER SOLVING BASE MODEL:",
218          AT,CT;
219
220  * REPORT - SOLUTION BASE MODEL:
221
222          REPORTQ(COM,"BASE-SOLUT") - AT(COM,"QSOL");
223          REPORTP(COM,"BASE-SOLUT") - AT(COM,"PSOL");
224
225  * DEFINE AND SOLVE AN ALTERNATIVE MODEL WITH AGRICULTURAL
226  * EXPORT ELASTICITY INFINITY
227  * NOTE THAT WORLD PRICES ARE STILL 10% HIGHER FOR AGRICULTURAL GOODS
228  * AND 10% LOWER FOR INDUSTRIAL GOODS.
229
230          CT("COM-EXP-AG","REST-WORLD","ETA") - INF;
231
232  SOLVE  MODEL D  USING  HERCULES;
233
234  * REPORT - SOLUTION TO MODEL 2:
235
236          REPORTQ(COM,"ETA-AG-INF") - AT(COM,"QSOL");
237          REPORTP(COM,"ETA-AG-INF") - AT(COM,"PSOL");

```

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MODEL D: FOREIGN TRADE, INVESTMENT, AND SAVINGS  
MODEL DEFINITION, EXPERIMENTS SOLUTION AND REPORT

```
238
239 * BEFORE SOLVING THE NEXT MODEL, RESTORE THE INPUT PART OF THE CELL
240 * TABLE TO ITS ORIGINAL CONTENT AGAIN, I.E. RESET THE EXPORT
241 * ELASTICITIES TO THEIR BASE VALUES:
242
243     CT(ACCEX,"REST-WORLD","ETA") - ETAS(ACCEX);
244
245 * DEFINE AND SOLVE AN ALTERNATIVE MODEL WITH AGRICULTURAL
246 * IMPORT ELASTICITY INFINITY.
247
248     AT("COM-CMP-AG","SIGMA") - INF;
249
250 SOLVE MODEL D USING HERCULES;
251
252 * REPORT - SOLUTION TO MODEL 3:
253
254     REPORTQ(COM,"SGM-AG-INF") - AT(COM,"QSOL");
255     REPORTP(COM,"SGM-AG-INF") - AT(COM,"PSOL");
256
257 * DISPLAY SUMMARY TABLES
258
259     DISPLAY REPORTQ,REPORTP;
```

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1

MODELE: MODEL WITH INCOME DISTRIBUTION  
DEFINITION OF ACCOUNT SET AND ACRONYMS

3 \* THE FOLLOWING MODEL IS DESCRIBED IN THE CHAPTER ON  
 4 \* INSTITUTIONAL DISAGGREGATION AND INCOME DISTRIBUTION  
 5 \* IN DRUD AND KENDRICK:  
 6 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".

7  
 8 SET ACC ACCOUNTS /  
 9 LABOR LABOR  
 10 CAPITAL-AG CAPITAL IN AGRICULTURE  
 11 CAPITAL-IN CAPITAL IN INDUSTRY  
 12 HOU-RU-IN INCOME OF RURAL HOUSEHOLDS  
 13 HOU-RU-CO CONSUMPTION OF RURAL HOUSEHOLDS  
 14 HOU-UR-IN INCOME OF URBAN HOUSEHOLDS  
 15 HOU-UR-CO CONSUMPTION OF URBAN HOUSEHOLDS  
 16 COMPANIES COMPANIES (PRIMARYLY THE FINANCIAL PART)  
 17 GOVERN-IN GOVERNMENT INCOME  
 18 GOVERN-CO GOVERNMENT CONSUMPTION  
 19 INDR-TAX INDIRECT TAX  
 20 SAVING-INV SAVINGS AND INVESTMENT  
 21 VAL-ADD-AG VALUE-ADDED IN AGRICULTURE  
 22 VAL-ADD-IN VALUE-ADDED IN INDUSTRY  
 23 PROD-AGR PRODUCTION ACTIVITY FOR AGRICULTURE  
 24 PROD-IND PRODUCTION ACTIVITY FOR INDUSTRY  
 25 COM-AGRIC COMMODITY MARKETING FOR AGRICULTURE  
 26 COM-INDUS COMMODITY MARKETING FOR INDUSTRY /

27  
 28 ALIAS (ACC,ACCP);  
 29

30 ACRONYMS MF MARKET FACTOR ACCOUNT  
 31 INST INSTITUTIONS ACCOUNT  
 32 INSTC INSTITUTIONS CONSUMPTION ACCOUNT  
 33 TAX INDIRECT TAX ACCOUNT  
 34 AC ACTIVITY OR COMMODITY ACCOUNT  
 35  
 36 NP PRICE FIXED - NUMERAIRE  
 37 Q QUANTITY FIXED  
 38  
 39 CD COBB-DOUGLAS PRODUCTION FUNCTION  
 40 IO INPUT-OUTPUT SPECIFICATION  
 41 ITAX INDIRECT TAX SPECIFICATION  
 42 VSHR VALUE SHARE SPECIFICATION  
 43 TEXO EXOGENOUS VALUE (IN DOMESTIC CURRENCY)  
 44 QSHR QUANTITY SHARE SPECIFICATION  
 45 IDIST INCOME DISTRIBUTION SPECIFICATION  
 46 UNSPEC UNSPECIFIED OR RESIDUAL

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MODELE: MODEL WITH INCOME DISTRIBUTION

DEFINITION OF SOCIAL ACCOUNTING MATRIX

```

48 TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX
49
50          LABOR  CAPITAL-AG  CAPITAL-IN  HOU-RU-IN  HOU-RU-CO
51
52 HOU-RU-IN          80          20
53 HOU-RU-CO          100
54 HOU-UR-IN          80          10          25
55 COMPANIES          45
56 GOVERN-IN          10          5
57 SAVING-INV          10
58 COM-AGRIC          80
59 COM-INDUS          20
60
61          +  HOU-UR-IN  HOU-UR-CO  COMPANIES  GOVERN-IN  GOVERN-CO
62
63 HOU-RU-IN          5          10
64 HOU-UR-IN          15          5
65 HOU-UR-CO          100
66 GOVERN-IN          15          10
67 GOVERN-CO          60
68 SAVING-INV          15          20          15
69 COM-AGRIC          60          15
70 COM-INDUS          40          45
71
72          +  INDR-TAX  SAVING-INV  VAL-ADD-AG  VAL-ADD-IN  PROD-AGR
73
74 LABOR          95          65
75 CAPITAL-AG          30
76 CAPITAL-IN          80
77 GOVERN-IN          50
78 VAL-ADD-AG          125
79 COM-AGRIC          10          40
80 COM-INDUS          50          35
81
82          +  PROD-IND  COM-AGRIC  COM-INDUS
83
84 INDR-TAX          35          15
85 VAL-ADD-IN          145
86 PROD-AGR          200
87 PROD-IND          235
88 COM-AGRIC          30
89 COM-INDUS          60

```

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3

MODELE: MODEL WITH INCOME DISTRIBUTION

DEFINITION OF SPECIFICATION AND CELL TABLES

```

91 TABLE SPEC(ACC,ACC) SPECIFICATIONS TABLE
92
93          LABOR  CAPITAL-AG  CAPITAL-IN  HOU-RU-IN  HOU-RU-CO
94
95 HOU-RU-IN          IDIST          IDIST
96 HOU-RU-CO          IDIST          IDIST          IDIST
97 HOU-UR-IN          IDIST          IDIST          IDIST
98 COMPANIES          IDIST          IDIST
99 GOVERN-IN          IDIST          IDIST
100 SAVING-INV          IDIST          IDIST
101 COM-AGRIC          VSHR
102 COM-INDUS          VSHR
103
104          +  HOU-UR-IN  HOU-UR-CO  COMPANIES  GOVERN-IN  GOVERN-CO
105
106 HOU-RU-IN          IDIST          IDIST          TEXO
107 HOU-UR-IN          IDIST          IDIST          TEXO
108 HOU-UR-CO          IDIST          IDIST
109 GOVERN-IN          IDIST          IDIST
110 GOVERN-CO          IDIST          UNSPEC
111 SAVING-INV          IDIST          IDIST          UNSPEC
112 COM-AGRIC          VSHR          VSHR          QSHR
113 COM-INDUS          VSHR          VSHR          QSHR
114
115          +  INDR-TAX  SAVING-INV  VAL-ADD-AG  VAL-ADD-IN  PROD-AGR
116
117 LABOR          CD          CD
118 CAPITAL-AG          CD          CD
119 CAPITAL-IN          CD          CD
120 GOVERN-IN          IDIST
121 VAL-ADD-AG          IDIST          IO
122 COM-AGRIC          QSHR          IO
123 COM-INDUS          QSHR          IO
124
125          +  PROD-IND  COM-AGRIC  COM-INDUS
126
127 INDR-TAX          ITAX          ITAX
128 VAL-ADD-IN          IO          IO
129 PROD-AGR          IO          IO
130 PROD-IND          IO          IO
131 COM-AGRIC          IO          IO
132 COM-INDUS          IO          IO
133
134 * DEFINE CELL ARRAY
135
136 PARAMETER CT(ACC,ACC,*) CELL TABLE;
137
138          CT(ACC,ACCP,"TBASE") - SAM(ACC,ACCP);
139          CT(ACC,ACCP,"SPECS") - SPEC(ACC,ACCP);

```

GAMS 2.05 PC AT/XT  
 MODELE: MODEL WITH INCOME DISTRIBUTION  
 ACCOUNT TABLE AND ACCOUNT TOTALS

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```

141 TABLE AT(ACC,*) ACCOUNT TABLE
142
143           TYPE      FIX
144     LABOR          MF      Q
145     CAPITAL-AG     MF      Q
146     CAPITAL-IN     MF      Q
147     HOU-RU-IN     INST
148     HOU-RU-CO     INSTC
149     HOU-UR-IN     INST
150     HOU-UR-CO     INSTC   NP
151     COMPANIES     INST
152     GOVERN-IN     INST
153     GOVERN-CO     INSTC   Q
154     INDR-TAX      TAX
155     SAVING-INV    INSTC
156     VAL-ADD-AG    AC
157     VAL-ADD-IN    AC
158     PROD-AGR      AC
159     PROD-IND      AC
160     COM-AGRIC     AC
161     COM-INDUS     AC
162
163 PARAMETER TOTALS(ACC,*) ACCOUNT TOTALS AND IMBALANCES FOR THE SAM;
164
165 TOTALS(ACC,"ROW-TOTAL") - SUM(ACCP,SAM(ACC,ACCP));
166 TOTALS(ACCP,"COL-TOTAL") - SUM(ACC,SAM(ACC,ACCP));
167 TOTALS(ACC,"DIFFERENCE") - TOTALS(ACC,"ROW-TOTAL") -
168                               TOTALS(ACC,"COL-TOTAL");
169
170 DISPLAY "CHECK FOR BALANCE OF BASE SAM:",TOTALS;

```

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 MODELE: MODEL WITH INCOME DISTRIBUTION  
 DEFINITION AND SOLUTION OF MODEL, EXPERIMENT AND REPORT

```

172 MODEL MODELE EXAMPLE MODEL WITH INCOME DISTRIBUTION
173           / ACC, AT, CT /;
174
175 * DEFINE SETS AND PARAMETERS FOR REPORT STATEMENTS AND DEFINE BASE CASE
176
177 SET FINUSE(ACC) ACCOUNTS FOR FINAL USE
178           / HOU-RU-CO, HOU-UR-CO , GOVERN-CO, SAVING-INV /
179 CASES SET OF CASES TO BE MODELED
180           / BASE-CASE, "GOVCON+10%", "AG-TAX+10%", "IN-TAX+10%" /
181 PQY SET WITH PRICE QUANTITY AND VALUE
182           / PRICE, QUANTITY, VALUE /
183 T(CASES) COLUMN LABELS FOR REPORTS
184
185 PARAMETER REPORT(PQY,FINUSE,CASES) CONSUMPTION FOR EACH HOUSEHOLD
186           ELAST(PQY,FINUSE,CASES) ELASTICITY OF CONSUMPTION
187 SCALAR DELTA CHANGE IN POLICY VARIABLES IN
188                                     EXPERIMENTS /0.10/ ;
189
189 * REPORT STATEMENTS FOR BASE CASE
190
191 T("BASE-CASE") - YES;
192 REPORT("VALUE" ,FINUSE,T) - TOTALS(FINUSE,"COL-TOTAL");
193 REPORT("QUANTITY",FINUSE,T) - TOTALS(FINUSE,"COL-TOTAL");
194 REPORT("PRICE" ,FINUSE,T) - 1;
195
196 * EXPERIMENT 1:
197 * GROWTH OF 10% IN GOVERNMENT CONSUMPTION:
198
199 AT("GOVERN-CO", "QFIX") - (1+DELTA)*TOTALS("GOVERN-CO", "COL-TOTAL");
200
201 SOLVE MODELE USING HERCULES;
202
203 * REPORT STATEMENTS FOR EXPERIMENT 1:
204
205 T(T) - NO; T("GOVCON+10%") - YES;
206 REPORT("VALUE" ,FINUSE,T) - AT(FINUSE,"YSOL");
207 REPORT("QUANTITY",FINUSE,T) - AT(FINUSE,"QSOL");
208 REPORT("PRICE" ,FINUSE,T) - AT(FINUSE,"PSOL");
209
210 * EXPERIMENT 2:
211 * ADD 10% TO THE AGRICULTURE TAX:
212
213 * FIRST, UNDO EXPERIMENT 1. IT IS SUFFICIENT TO RESET QFIX TO THE
214 * DEFAULT VALUE 0. THEN, COMPUTE THE NEW INDIRECT TAX RATE FOR
215 * AGRICULTURAL COMMODITIES.
216
217 AT("GOVERN-CO", "QFIX") - 0;
218 CT("INDR-TAX", "COM-AGRIC", "THETA") -
219 CT("INDR-TAX", "COM-AGRIC", "THETA-USED") + DELTA;
220

```

GAMS 2.05 PC AT/XT 90/04/11 08:48:38 PAGE 6  
MODELE: MODEL WITH INCOME DISTRIBUTION  
DEFINITION AND SOLUTION OF MODEL, EXPERIMENT AND REPORT

```
221 SOLVE MODELE USING HERCULES;
222
223 * REPORT STATEMENTS FOR EXPERIMENT 2
224
225 T(T) - NO; T("AG-TAX+10%") - YES;
226 REPORT("VALUE" ,FINUSE,T) - AT(FINUSE,"YSOL");
227 REPORT("QUANTITY",FINUSE,T) - AT(FINUSE,"QSOL");
228 REPORT("PRICE" ,FINUSE,T) - AT(FINUSE,"PSOL");
229
230 * EXPERIMENT 3:
231 * ADD 10% TO THE INDUSTRIAL TAX:
232
233 * FIRST UNDO THE AGRICULTURAL TAX INCREASE BY SETTING THETA TO THE
234 * DEFAULT 0. THEN, COMPUTE THE NEW TAX RATE FOR INDUSTRIAL
235 * COMMODITIES.
236
237 CT("INDR-TAX","COM-AGRIC","THETA") - 0;
238 CT("INDR-TAX","COM-INDUS","THETA") -
239 CT("INDR-TAX","COM-INDUS","THETA-USED") + DELTA;
240
241 SOLVE MODELE USING HERCULES;
242
243 * REPORT STATEMENTS FOR EXPERIMENT 3:
244
245 T(T) - NO; T("IN-TAX+10%") - YES;
246 REPORT("VALUE" ,FINUSE,T) - AT(FINUSE,"YSOL");
247 REPORT("QUANTITY",FINUSE,T) - AT(FINUSE,"QSOL");
248 REPORT("PRICE" ,FINUSE,T) - AT(FINUSE,"PSOL");
249
250 * COMPUTE ELASTICITIES FOR ALL EXPERIMENTS AND DISPLAY REPORT TABLES:
251
252 ELAST(PQY,FINUSE,CASES) -
253 (REPORT(PQY,FINUSE,CASES)/REPORT(PQY,FINUSE,"BASE-CASE")-1)/DELTA;
254
255 DISPLAY REPORT,ELAST;
```

GAMS 2.05 PC AT/XT 90/04/11 08:51:10 PAGE 1  
 MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
 DEFINITION OF ACCOUNT SET AND ACRONYMS

3 \* THE FOLLOWING MODEL IS DESCRIBED IN THE CHAPTER ON NATIONAL ACCOUNTS  
 4 \* IN DRUD AND KENDRICK:  
 5 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".

6  
 7 SET ACC ACCOUNTS /  
 8 LABOR LABOR  
 9 CAPITAL-AG CAPITAL IN AGRICULTURE  
 10 CAPITAL-IN CAPITAL IN INDUSTRY  
 11 FAC-ABROAD FACTORS EMPLOYED ABROAD  
 12 HOU-RU-IN INCOME OF RURAL HOUSEHOLDS  
 13 HOU-RU-CO CONSUMPTION OF RURAL HOUSEHOLDS  
 14 HOU-UR-IN INCOME OF URBAN HOUSEHOLDS  
 15 HOU-UR-CO CONSUMPTION OF URBAN HOUSEHOLDS  
 16 COMPANIES COMPANIES  
 17 GOVERN-IN GOVERNMENT INCOME  
 18 GOVERN-CO GOVERNMENT CONSUMPTION  
 19 INDR-TAX INDIRECT TAX  
 20 SAVING-INV SAVINGS AND INVESTMENT  
 21 VAL-ADD-AG VALUE-ADDED IN AGRICULTURE  
 22 VAL-ADD-IN VALUE-ADDED IN INDUSTRY  
 23 PROD-AGR PRODUCTION ACTIVITY FOR AGRICULTURE  
 24 PROD-IND PRODUCTION ACTIVITY FOR INDUSTRY  
 25 COM-DOM-AG DOMESTIC AGRICULTURAL COMMODITIES  
 26 COM-DOM-IN DOMESTIC INDUSTRIAL COMMODITIES  
 27 COM-IMP-AG IMPORTED AGRICULTURAL COMMODITIES  
 28 COM-IMP-IN IMPORTED INDUSTRIAL COMMODITIES  
 29 COM-CMP-AG COMPOSITE AGRICULTURAL COMMODITIES  
 30 COM-CMP-IN COMPOSITE INDUSTRIAL COMMODITIES  
 31 COM-EXP-AG AGRICULTURAL EXPORT COMMODITIES  
 32 COM-EXP-IN INDUSTRIAL EXPORT COMMODITIES  
 33 REST-WORLD REST OF THE WORLD ACCOUNT/  
 34 ALIAS (ACC,ACCP);  
 35  
 36 ACRONYMS AC ACTIVITY OR COMMODITY ACCOUNT  
 37 INST INSTITUTIONS ACCOUNT  
 38 INSTC INSTITUTIONS CONSUMPTION ACCOUNT  
 39 MF MARKET FACTOR ACCOUNT  
 40 NMF NON-MARKET FACTOR ACCOUNT  
 41 ROW REST OF THE WORLD ACCOUNT  
 42 TAX INDIRECT TAX ACCOUNT  
 43 NP PRICE FIXED - NUMERAIRE  
 44 Q QUANTITY FIXED  
 45 CD COBB-DOUGLAS PRODUCTION FUNCTION  
 46 CES CES PRODUCTION FUNCTION  
 47 EXPORT DEMAND FROM THE REST OF THE WORLD FOR EXPORTS  
 48 FEXO EXOGENOUS VALUE (IN FOREIGN CURRENCY)  
 49 IDIST INCOME DISTRIBUTION SPECIFICATION  
 50 IMPORT PAYMENT TO THE REST OF THE WORLD FOR IMPORTED GOODS  
 51 IO INPUT-OUTPUT SPECIFICATION  
 52 ITAX INDIRECT TAX SPECIFICATION

GAMS 2.05 PC AT/XT 90/04/11 08:51:10 PAGE 2  
MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
DEFINITION OF ACCOUNT SET AND ACRONYMS

|    |        |   |
|----|--------|---|
| 53 | LES    | LINEAR EXPENDITURE SYSTEM SPECIFICATION |
| 54 | QSHR   | QUANTITY SHARE SPECIFICATION            |
| 55 | TEXO   | EXOGENOUS VALUE (IN DOMESTIC CURRENCY)  |
| 56 | UNSPEC | UNSPECIFIED OR RESIDUAL                 |
| 57 | VSHR   | VALUE SHARE SPECIFICATION               |

GAMS 2.05 PC AT/XT 90/04/11 08:51:10 PAGE 3  
 MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
 DEFINITION OF SOCIAL ACCOUNTING MATRIX

```

59 TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX
60
61          LABOR  CAPITAL-AG  CAPITAL-IN  FAC-ABROAD  HOU-RU-IN
62 HOU-RU-IN      80          20
63 HOU-RU-CO
64 HOU-UR-IN      80          10          25          5
65 COMPANIES
66 GOVERN-IN
67 SAVING-INV
68 REST-WORLD
69
70          +  HOU-RU-CO  HOU-UR-IN  HOU-UR-CO  COMPANIES  GOVERN-IN
71 HOU-RU-IN
72 HOU-UR-IN
73 HOU-UR-CO
74 COMPANIES
75 GOVERN-IN
76 GOVERN-CO
77 SAVING-INV
78 COM-CMP-AG      76          54
79 COM-CMP-IN      26          54
80 REST-WORLD
81
82          +  GOVERN-CO  INDR-TAX  SAVING-INV  VAL-ADD-AG  VAL-ADD-IN
83 LABOR
84 CAPITAL-AG
85 CAPITAL-IN
86 GOVERN-IN
87 COM-CMP-AG      20          15
88 COM-CMP-IN      60          85
89
90          +  PROD-AGR  PROD-IND  COM-DOM-AG  COM-DOM-IN  COM-IMP-AG
91 INDR-TAX
92 VAL-ADD-AG      125
93 VAL-ADD-IN
94 PROD-AGR
95 PROD-IND
96 COM-CMP-AG      40          30
97 COM-CMP-IN      35          60
98 REST-WORLD
99
100         +  COM-IMP-IN  COM-CMP-AG  COM-CMP-IN  COM-EXP-AG  COM-EXP-IN
101 INDR-TAX
102 PROD-AGR
103 PROD-IND
104 COM-DOM-AG
105 COM-DOM-IN
106 COM-IMP-AG
107 COM-IMP-IN
108 REST-WORLD

```

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MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
DEFINITION OF SOCIAL ACCOUNTING MATRIX

|     |              |    |
|-----|--------------|----|
| 109 |              |    |
| 110 | + REST-WORLD |    |
| 111 | FAC-ABROAD   | 5  |
| 112 | HOU-RU-IN    | 2  |
| 113 | HOU-UR-IN    | 6  |
| 114 | GOVERN-IN    | 5  |
| 115 | SAVING-INV   | 20 |
| 116 | COM-EXP-AG   | 70 |
| 117 | COM-EXP-IN   | 50 |

GAMS 2.05 PC AT/XT 90/04/11 08:51:10 PAGE 5  
 MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
 DEFINITION OF SPECIFICATIONS AND CELL TABLE

|     |            |               |                      |            |            |            |            |
|-----|------------|---------------|----------------------|------------|------------|------------|------------|
| 119 | TABLE      | SPEC(ACC,ACC) | SPECIFICATIONS TABLE |            |            |            |            |
| 120 |            |               |                      |            |            |            |            |
| 121 |            |               | LABOR                | CAPITAL-AG | CAPITAL-IN | FAC-ABROAD | HOU-RU-IN  |
| 122 | HOU-RU-IN  |               | IDIST                | IDIST      |            |            |            |
| 123 | HOU-RU-CO  |               |                      |            |            |            | IDIST      |
| 124 | HOU-UR-IN  |               | IDIST                | IDIST      | IDIST      | IDIST      |            |
| 125 | COMPANIES  |               |                      |            | IDIST      |            |            |
| 126 | GOVERN-IN  |               |                      |            | IDIST      |            | IDIST      |
| 127 | SAVING-INV |               |                      |            |            |            | IDIST      |
| 128 | REST-WORLD |               |                      |            | IDIST      |            |            |
| 129 |            |               |                      |            |            |            |            |
| 130 |            | +             | HOU-RU-CO            | HOU-UR-IN  | HOU-UR-CO  | COMPANIES  | GOVERN-IN  |
| 131 | HOU-RU-IN  |               |                      | IDIST      |            |            | TEXO       |
| 132 | HOU-UR-IN  |               |                      |            |            | IDIST      | TEXO       |
| 133 | HOU-UR-CO  |               |                      | IDIST      |            |            |            |
| 134 | COMPANIES  |               |                      | IDIST      |            |            | TEXO       |
| 135 | GOVERN-IN  |               |                      | IDIST      |            | IDIST      |            |
| 136 | GOVERN-CO  |               |                      |            |            |            | UNSPEC     |
| 137 | SAVING-INV |               |                      | IDIST      |            | IDIST      | UNSPEC     |
| 138 | COM-CMP-AG |               | LES                  |            | LES        |            |            |
| 139 | COM-CMP-IN |               | LES                  |            | LES        |            |            |
| 140 | REST-WORLD |               |                      | IDIST      |            |            |            |
| 141 |            |               |                      |            |            |            |            |
| 142 |            | +             | GOVERN-CO            | INDR-TAX   | SAVING-INV | VAL-ADD-AG | VAL-ADD-IN |
| 143 | LABOR      |               |                      |            |            | CD         | CD         |
| 144 | CAPITAL-AG |               |                      |            |            | CD         |            |
| 145 | CAPITAL-IN |               |                      |            |            |            | CD         |
| 146 | GOVERN-IN  |               |                      | IDIST      |            |            |            |
| 147 | COM-CMP-AG |               | QSHR                 |            | QSHR       |            |            |
| 148 | COM-CMP-IN |               | QSHR                 |            | QSHR       |            |            |
| 149 |            |               |                      |            |            |            |            |
| 150 |            | +             | PROD-AGR             | PROD-IND   | COM-DOM-AG | COM-DOM-IN | COM-IMP-AG |
| 151 | INDR-TAX   |               |                      |            | ITAX       | ITAX       | ITAX       |
| 152 | VAL-ADD-AG |               | IO                   |            |            |            |            |
| 153 | VAL-ADD-IN |               |                      | IO         |            |            |            |
| 154 | PROD-AGR   |               |                      |            | IO         |            |            |
| 155 | PROD-IND   |               |                      |            |            | IO         |            |
| 156 | COM-CMP-AG |               | IO                   | IO         |            |            |            |
| 157 | COM-CMP-IN |               | IO                   | IO         |            |            |            |
| 158 | REST-WORLD |               |                      |            |            |            | IMPORT     |
| 159 |            |               |                      |            |            |            |            |
| 160 |            | +             | COM-IMP-IN           | COM-CMP-AG | COM-CMP-IN | COM-EXP-AG | COM-EXP-IN |
| 161 | INDR-TAX   |               | ITAX                 |            |            | ITAX       | ITAX       |
| 162 | PROD-AGR   |               |                      |            |            | IO         |            |
| 163 | PROD-IND   |               |                      |            |            |            | IO         |
| 164 | COM-DOM-AG |               |                      | CES        |            |            |            |
| 165 | COM-DOM-IN |               |                      |            | CES        |            |            |
| 166 | COM-IMP-AG |               |                      | CES        |            |            |            |
| 167 | COM-IMP-IN |               |                      |            | CES        |            |            |
| 168 | REST-WORLD |               | IMPORT               |            |            |            |            |

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MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
DEFINITION OF SPECIFICATIONS AND CELL TABLE

```
169
170          + REST-WORLD
171 FAC-ABROAD          FEXO
172 HOU-RU-IN          FEXO
173 HOU-UR-IN          FEXO
174 GOVERN-IN          FEXO
175 SAVING-INV          UNSPEC
176 COM-EXP-AG          EXPORT
177 COM-EXP-IN          EXPORT
178
179 SET ACCEX(ACC) EXPORTED COMMODITIES
180     / COM-EXP-AG , COM-EXP-IN /
181
182 PARAMETER ETAS(ACCEX) ELASTICITY OF DEMAND FOR EXPORTS /
183           COM-EXP-AG - 3.0 , COM-EXP-IN - 1.5 /
184
185 PARAMETER CT(ACC,ACC,*) CELL TABLE;
186
187     CT(ACC,ACCP,"TBASE")          - SAM(ACC,ACCP);
188     CT(ACC,ACCP,"SPECS")          - SPEC(ACC,ACCP);
189     CT(ACCEX,"REST-WORLD","ETA") - ETAS(ACCEX);
```

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MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM

ACCOUNT TABLE

| 191 | TABLE | AT(ACC,*)  | ACCOUNT TABLE |     |       |
|-----|-------|------------|---------------|-----|-------|
| 192 |       |            |               |     |       |
| 193 |       |            | TYPE          | FIX | SIGMA |
| 194 |       | LABOR      | MF            | Q   |       |
| 195 |       | CAPITAL-AG | MF            | Q   |       |
| 196 |       | CAPITAL-IN | MF            | Q   |       |
| 197 |       | FAC-ABROAD | NMF           |     |       |
| 198 |       | HOU-RU-IN  | INST          |     |       |
| 199 |       | HOU-RU-CO  | INSTC         |     |       |
| 200 |       | HOU-UR-IN  | INST          |     |       |
| 201 |       | HOU-UR-CO  | INSTC         |     |       |
| 202 |       | COMPANIES  | INST          |     |       |
| 203 |       | GOVERN-IN  | INST          |     |       |
| 204 |       | INDR-TAX   | TAX           |     |       |
| 205 |       | GOVERN-CO  | INSTC         | Q   |       |
| 206 |       | SAVING-INV | INSTC         | Q   |       |
| 207 |       | VAL-ADD-AG | AC            |     |       |
| 208 |       | VAL-ADD-IN | AC            |     |       |
| 209 |       | PROD-AGR   | AC            |     |       |
| 210 |       | PROD-IND   | AC            |     |       |
| 211 |       | COM-DOM-AG | AC            |     |       |
| 212 |       | COM-DOM-IN | AC            |     |       |
| 213 |       | COM-IMP-AG | AC            |     |       |
| 214 |       | COM-IMP-IN | AC            |     |       |
| 215 |       | COM-CMP-AG | AC            | 3.0 |       |
| 216 |       | COM-CMP-IN | AC            | 0.5 |       |
| 217 |       | COM-EXP-AG | AC            |     |       |
| 218 |       | COM-EXP-IN | AC            |     |       |
| 219 |       | REST-WORLD | ROW           | NP  |       |

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 MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
 MODEL AND EXPERIMENT DEFINITION AND SOLUTION REPORT

```

221  MODEL  MODEL F  EXAMPLE MODEL WITH NATIONAL ACCOUNTS SUMMARIES
222          / ACC, AT, CT /;
223
224  * EXPERIMENT:  WORLD AGRICULTURAL PRICES ARE INCREASED BY 10% AND
225  *              WORLD INDUSTRIAL PRICES ARE DECREASED BY 10%.
226
227          CT("REST-WORLD", "COM-IMP-AG", "WP") - 1.1;
228          CT("REST-WORLD", "COM-IMP-IN", "WP") - 0.9;
229          CT("COM-EXP-AG", "REST-WORLD", "WP") - 1.1;
230          CT("COM-EXP-IN", "REST-WORLD", "WP") - 0.9;
231
232  SOLVE  MODEL F  USING  HERCULES;
233
234  *  SETS AND MAPPING DEFINING THE SUMMARY REPORT.
235
236  SET  HEAD  TABLE HEADERS /
237          BASE-CASE, CURRENT, CONSTANT, PRICE-INDX /
238  LINES TABLE LINES /
239          GDP-FC      GROSS DOMESTIC PRODUCT AT FACTOR COST
240          NET-I-TAX   NET INDIRECT TAXES
241          INCM-EFF    INCOME EFFECT
242          FINAL-USE   TOTAL DOMESTIC FINAL USE
243          EXPORT      TOTAL EXPORTS F.O.B.
244          IMPORT      TOTAL IMPORTS C.I.F.
245          GDP-MP      GROSS DOMESTIC PRODUCT AT MARKET PRICES
246          T-TRADE     TERMS OF TRADE
247          G-D-INCOME  GROSS DOMESTIC INCOME
248          RES-GAP     RESOURCE GAP /
249  HD(HEAD)  TABLE HEADERS DIRECTLY COMPUTABLE FROM CELL TABLE /
250          BASE-CASE, CURRENT, CONSTANT /
251  AL(LINES) AGGREGATES COMPUTED DIRECTLY FROM THE SOLUTION /
252          GDP-FC, NET-I-TAX, FINAL-USE, IMPORT, EXPORT /
253  PL(LINES) SUBSET OF TABLES LINES WITH PRICES /
254          GDP-FC, FINAL-USE, EXPORT, IMPORT, GDP-MP /
255  CLAB LABELS IN CELL TABLE FOR SOLUTION EXTRACTION /
256          TBASE,      TSOL,      QCSOL /
257
258  HMAP(HD,CLAB) MAPPING FROM TABLE HEADERS TO LABELS IN CELL TABLE /
259          BASE-CASE.TBASE
260          CURRENT .TSOL
261          CONSTANT .QCSOL /
262
263  LMAP(AL,ACC,ACC) MAPPING FROM TABLE LINES TO CELLS IN THE SAM /
264          GDP-FC      .((LABOR,CAPITAL-AG).VAL-ADD-AG,
265                    (LABOR,CAPITAL-IN).VAL-ADD-IN),
266          NET-I-TAX  .INDR-TAX.(COM-DOM-AG,COM-DOM-IN,
267                    COM-IMP-AG,COM-IMP-IN,
268                    COM-EXP-AG,COM-EXP-IN),
269          FINAL-USE .(COM-CMP-AG,COM-CMP-IN).(HOU-RU-CO,HOU-UR-CO,
270                    GOVERN-CO,SAVING-INV),

```

GAMS 2.05 PC AT/XT 90/04/11 08:51:10 PAGE 9  
 MODEL F: NATIONAL ACCOUNTS AGGREGATES DEFINED FROM THE SAM  
 MODEL AND EXPERIMENT DEFINITION AND SOLUTION REPORT

```

271          EXPORT      .(COM-EXP-AG,COM-EXP-IN).REST-WORLD,
272          IMPORT      .REST-WORLD.(COM-IMP-AG,COM-IMP-IN) /
273
274  PARAMETER GDPTAB(LINES,HEAD) GDP SUMMARY TABLE;
275
276  GDPTAB(AL,HD) = SUM((ACC,ACCP) $ LMAP(AL,ACC,ACCP),
277                    SUM(CLAB $ HMAP(HD,CLAB), CT(ACC,ACCP,CLAB) ));
278  GDPTAB("GDP-MP",HD)      - GDPTAB("FINAL-USE",HD)
279                          + GDPTAB("EXPORT",HD)
280                          - GDPTAB("IMPORT",HD);
281  GDPTAB("INCM-EFF",HD)    - GDPTAB("GDP-MP",HD)
282                          - GDPTAB("GDP-FC",HD)
283                          - GDPTAB("NET-I-TAX",HD);
284  GDPTAB(PL,"PRICE-INDX") - GDPTAB(PL,"CURRENT")
285                          / GDPTAB(PL,"CONSTANT");
286  GDPTAB("T-TRADE","CONSTANT") - GDPTAB("EXPORT","CURRENT")
287                          / GDPTAB("IMPORT","PRICE-INDX")
288                          - GDPTAB("EXPORT","CONSTANT");
289  GDPTAB("G-D-INCOME",HD)  - GDPTAB("GDP-MP",HD)
290                          + GDPTAB("T-TRADE",HD);
291  GDPTAB("RES-GAP",HD)     - GDPTAB("IMPORT",HD)
292                          - GDPTAB("EXPORT",HD)
293                          - GDPTAB("T-TRADE",HD);
294  DISPLAY GDPTAB;

```

GAMS 2.05 PC AT/XT 90/04/11 08:51:55 PAGE 1  
 MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
 DEFINITION OF ACCOUNT SET AND ACRONYMS

3 \* THE FOLLOWING MODEL IS DESCRIBED IN THE CHAPTER ON CLOSURE RULES  
 4 \* IN DRUD AND KENDRICK:  
 5 \* "HERCULES - A MODELING SYSTEM FOR LARGE ECONOMYWIDE MODELS".

6  
 7 SET ACC ACCOUNTS /  
 8 LABOR LABOR  
 9 CAPITAL-AG CAPITAL IN AGRICULTURE  
 10 CAPITAL-IN CAPITAL IN INDUSTRY  
 11 HOUSEHLD-I HOUSEHOLD INCOME ACCOUNT  
 12 HOUSEHLD-C HOUSEHOLD CONSUMPTION ACCOUNT  
 13 COMPANIES ACCOUNT FOR COMPANIES  
 14 GOVERNMT-I GOVERNMENT INCOME ACCOUNT  
 15 GOVERNMT-C GOVERNMENT EXPENDITURE ACCOUNT  
 16 INDR-TAX INDIRECT TAX ACCOUNT  
 17 SAVING-HH HOUSEHOLD SAVINGS  
 18 SAVING-COM COMPANY SAVINGS  
 19 SAVING-GOV GOVERNMENT SAVINGS  
 20 INVEST-HH HOUSEHOLD INVESTMENTS  
 21 INVEST-COM COMPANY INVESTMENTS  
 22 INVEST-GOV GOVERNMENT INVESTMENTS  
 23 ACT-AGRCLT PRODUCTION ACTIVITY FOR AGRICULTURE  
 24 ACT-INDSTR PRODUCTION ACTIVITY FOR INDUSTRY  
 25 COM-DOM-AG DOMESTIC COMMODITIES IN AGRICULTURE  
 26 COM-DOM-IN DOMESTIC COMMODITIES IN INDUSTRY  
 27 COM-IMP-AG IMPORTED COMMODITIES IN AGRICULTURE  
 28 COM-IMP-IN IMPORTED COMMODITIES IN INDUSTRY  
 29 COM-CMP-AG COMPOSITE COMMODITIES IN AGRICULTURE  
 30 COM-CMP-IN COMPOSITE COMMODITIES IN INDUSTRY  
 31 COM-EXP-AG EXPORTED COMMODITIES IN AGRICULTURE  
 32 COM-EXP-IN EXPORTED COMMODITIES IN INDUSTRY  
 33 REST-WORLD REST OF THE WORLD ACCOUNT /

34  
 35 ALIAS (ACC,ACCP);  
 36

37 ACRONYMS MF MARKET FACTOR ACCOUNT  
 38 NMF NON MARKET FACTOR  
 39 INST INSTITUTIONS INCOME ACCOUNT  
 40 INSTC INSTITUTIONS CONSUMPTION ACCOUNT  
 41 TAX INDIRECT TAX ACCOUNT  
 42 AC ACTIVITY-COMMODITY ACCOUNT  
 43 ROW REST OF THE WORLD ACCOUNT  
 44  
 45 NP PRICE EXOGENOUS - NUMERAIRE  
 46 P PRICE EXOGENOUS  
 47 Q QUANTITY EXOGENOUS  
 48  
 49 CD COBB-DOUGLAS PRODUCTION FUNCTION  
 50 CES CES PRODUCTION FUNCTION  
 51 CES2 TWO LEVEL CES PRODUCTION FUNCTION  
 52 EXPORT EXPORT DEMAND FROM THE REST OF THE WORLD

GAMS 2.05 PC AT/XT 90/04/11 08:51:55 PAGE 2  
MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
DEFINITION OF ACCOUNT SET AND ACRONYMS

|    |        |   |
|----|--------|---|
| 53 | FEXO   | EXOGENOUS IN FOREIGN EXCHANGE           |
| 54 | IDIST  | INCOME DISTRIBUTION SPECIFICATION       |
| 55 | IMPORT | PAYMENTS FOR IMPORTS                    |
| 56 | IO     | INPUT-OUTPUT SPECIFICATION              |
| 57 | ITAX   | INDIRECT TAX SPECIFICATION              |
| 58 | MARKUP | MARKUP OVER AND ABOVE COSTS             |
| 59 | QEXO   | FIXED QUANTITY CONSUMPTION SYSTEM       |
| 60 | QSHR   | FIXED QUANTITY SHARE CONSUMPTION SYSTEM |
| 61 | UNSPEC | UNSPECIFIED OR RESIDUAL                 |
| 62 | VEXO   | SPECIFICATION FOR EXOGENOUS VALUE       |
| 63 | VSHR   | VALUE SHARE CONSUMPTION SYSTEM          |

GAMS 2.05 PC AT/XT 90/04/11 08:51:55 PAGE 3  
 MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
 DEFINITION OF SOCIAL ACCOUNTING MATRIX

```

65 TABLE SAM(ACC,ACC) SOCIAL ACCOUNTING MATRIX WITH FLOW OF FUNDS
66
67          LABOR  CAPITAL-AG  CAPITAL-IN  HOUSEHLD-I  HOUSEHLD-C
68 HOUSEHLD-I      160          30          40
69 HOUSEHLD-C
70 COMPANIES
71 GOVERNMT-I
72 SAVING-HH
73 COM-CMP-AG
74 COM-CMP-IN
75
76          +  COMPANIES  GOVERNMT-I  GOVERNMT-C  INDR-TAX  SAVING-HH
77 HOUSEHLD-I      5
78 GOVERNMT-I      5
79 GOVERNMT-C
80 SAVING-COM      30
81 SAVING-GOV
82 INVEST-HH
83 INVEST-GOV
84 COM-CMP-AG
85 COM-CMP-IN
86
87          +  SAVING-COM  SAVING-GOV  INVEST-HH  INVEST-COM  INVEST-GOV
88 INVEST-COM      25
89 INVEST-GOV      5          35
90 COM-CMP-AG
91 COM-CMP-IN
92
93          +  ACT-AGRCLT  ACT-INDSTR  COM-DOM-AG  COM-DOM-IN  COM-IMP-AG
94 LABOR          95          65
95 CAPITAL-AG      30
96 CAPITAL-IN
97 INDR-TAX
98 ACT-AGRCLT
99 ACT-INDSTR
100 COM-CMP-AG      40          30
101 COM-CMP-IN      35          60
102 REST-WORLD
103
104          +  COM-IMP-IN  COM-CMP-AG  COM-CMP-IN  COM-EXP-AG  COM-EXP-IN
105 INDR-TAX        20
106 ACT-AGRCLT
107 ACT-INDSTR
108 COM-DOM-AG
109 COM-DOM-IN
110 COM-IMP-AG
111 COM-IMP-IN
112 REST-WORLD      100
113
114          +  REST-WORLD

```

GAMS 2.05 PC AT/XT  
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MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
DEFINITION OF SOCIAL ACCOUNTING MATRIX

|     |            |    |
|-----|------------|----|
| 115 | INVEST-GOV | 20 |
| 116 | COM-EXP-AG | 75 |
| 117 | COM-EXP-IN | 55 |

GAMS 2.05 PC AT/XT 90/04/11 08:51:55 PAGE 5  
 MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
 DEFINITION OF SPECIFICATION TABLE

|     |            |               |                     |            |            |            |            |
|-----|------------|---------------|---------------------|------------|------------|------------|------------|
| 119 | TABLE      | SPEC(ACC,ACC) | SPECIFICATION TABLE |            |            |            |            |
| 120 |            |               |                     |            |            |            |            |
| 121 |            |               | LABOR               | CAPITAL-AG | CAPITAL-IN | HOUSEHLD-I | HOUSEHLD-C |
| 122 | HOUSEHLD-I |               | IDIST               | IDIST      | IDIST      |            |            |
| 123 | HOUSEHLD-C |               |                     |            |            | IDIST      |            |
| 124 | COMPANIES  |               |                     |            | IDIST      |            |            |
| 125 | GOVERNMT-I |               |                     |            |            | IDIST      |            |
| 126 | SAVING-HH  |               |                     |            |            | IDIST      |            |
| 127 | COM-CMP-AG |               |                     |            |            |            | VSHR       |
| 128 | COM-CMP-IN |               |                     |            |            |            | VSHR       |
| 129 |            |               |                     |            |            |            |            |
| 130 | +          | COMPANIES     | GOVERNMT-I          | GOVERNMT-C |            | INDR-TAX   | SAVING-HH  |
| 131 | HOUSEHLD-I |               | IDIST               |            |            |            |            |
| 132 | GOVERNMT-I |               | IDIST               |            |            | IDIST      |            |
| 133 | GOVERNMT-C |               |                     | UNSPEC     |            |            |            |
| 134 | SAVING-COM |               | IDIST               |            |            |            |            |
| 135 | SAVING-GOV |               |                     | UNSPEC     |            |            |            |
| 136 | INVEST-HH  |               |                     |            |            |            | IDIST      |
| 137 | INVEST-GOV |               |                     |            |            |            | IDIST      |
| 138 | COM-CMP-AG |               |                     |            | QEXO       |            |            |
| 139 | COM-CMP-IN |               |                     |            | QEXO       |            |            |
| 140 |            |               |                     |            |            |            |            |
| 141 | +          | SAVING-COM    | SAVING-GOV          | INVEST-HH  | INVEST-COM | INVEST-GOV |            |
| 142 | INVEST-COM |               | IDIST               |            |            |            |            |
| 143 | INVEST-GOV |               | IDIST               | IDIST      |            |            |            |
| 144 | COM-CMP-AG |               |                     |            | QSHR       | QSHR       |            |
| 145 | COM-CMP-IN |               |                     |            | QSHR       | QSHR       | QSHR       |
| 146 |            |               |                     |            |            |            |            |
| 147 | +          | ACT-AGRCLT    | ACT-INDSTR          | COM-DOM-AG | COM-DOM-IN | COM-IMP-AG |            |
| 148 | LABOR      | CES2          | IO                  |            |            |            |            |
| 149 | CAPITAL-AG | CES2          |                     |            |            |            |            |
| 150 | CAPITAL-IN |               | MARKUP              |            |            |            |            |
| 151 | INDR-TAX   |               |                     | ITAX       | ITAX       | ITAX       |            |
| 152 | ACT-AGRCLT |               |                     | IO         |            |            |            |
| 153 | ACT-INDSTR |               |                     |            | IO         |            |            |
| 154 | COM-CMP-AG | CES2          | IO                  |            |            |            |            |
| 155 | COM-CMP-IN | CES2          | IO                  |            |            |            |            |
| 156 | REST-WORLD |               |                     |            |            |            | IMPORT     |
| 157 |            |               |                     |            |            |            |            |
| 158 | +          | COM-IMP-IN    | COM-CMP-AG          | COM-CMP-IN | COM-EXP-AG | COM-EXP-IN |            |
| 159 | INDR-TAX   | ITAX          |                     |            | ITAX       | ITAX       |            |
| 160 | ACT-AGRCLT |               |                     |            | IO         |            |            |
| 161 | ACT-INDSTR |               |                     |            |            |            | IO         |
| 162 | COM-DOM-AG |               | CES                 |            |            |            |            |
| 163 | COM-DOM-IN |               |                     | CES        |            |            |            |
| 164 | COM-IMP-AG |               | CES                 |            |            |            |            |
| 165 | COM-IMP-IN |               |                     | CES        |            |            |            |
| 166 | REST-WORLD | IMPORT        |                     |            |            |            |            |
| 167 |            |               |                     |            |            |            |            |
| 168 | +          | REST-WORLD    |                     |            |            |            |            |

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MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
DEFINITION OF SPECIFICATION TABLE

|     |            |        |
|-----|------------|--------|
| 169 | INVEST-GOV | UNSPEC |
| 170 | COM-EXP-AG | EXPORT |
| 171 | COM-EXP-IN | EXPORT |

GAMS 2.05 PC AT/XT 90/04/11 08:51:55 PAGE 7  
 MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
 ACCOUNT TABLE AND CELL TABLE

```

173 TABLE AT(ACC,*) ACCOUNT TABLE
174
175           TYPE    FIX    SIGMA
176     LABOR      MF      Q
177     CAPITAL-AG MF      Q
178     CAPITAL-IN  NMF
179     HOUSEHLD-I INST
180     HOUSEHLD-C INSTC
181     COMPANIES  INST
182     GOVERNMT-I INST
183     GOVERNMT-C INSTC
184     SAVING-HH  INST
185     SAVING-COM INST
186     SAVING-GOV INST
187     INVEST-HH  INSTC
188     INVEST-COM INSTC
189     INVEST-GOV INSTC  Q
190     INDR-TAX   TAX
191     ACT-AGRCLT AC           EPS
192     ACT-INDSTR AC
193     COM-DOM-AG AC
194     COM-DOM-IN AC
195     COM-IMP-AG AC
196     COM-IMP-IN AC
197     COM-CMP-AG AC           3.0
198     COM-CMP-IN AC           0.5
199     COM-EXP-AG AC
200     COM-EXP-IN AC
201     REST-WORLD ROW      NP
202
203 SET          ACCEX(ACC)  EXPORTED COMMODITIES /COM-EXP-AG,COM-EXP-IN/
204              INP(ACC)   INPUTS INTO AGRICULTURE /
205                      LABOR, CAPITAL-AG, COM-CMP-AG, COM-CMP-IN /
206 PARAMETER    ETAS(ACCEX) ELASTICITY OF DEMAND FOR EXPORTS /
207                      COM-EXP-AG - 3.0, COM-EXP-IN - 1.5 /
208              SIGMAC(INP) ELASTICITIES IN CES2 COLUMN IN AGRICULTURE /
209                      LABOR      - 1, CAPITAL-AG - 1,
210                      COM-CMP-AG - EPS, COM-CMP-IN - EPS /
211
212 * DEFINE AND FILL THE CELL TABLE;
213
214 PARAMETER    CT(ACC,ACC,*) CELL TABLE;
215
216 CT(ACC,ACCP,"TBASE")      - SAM(ACC,ACCP);
217 CT(ACC,ACCP,"SPECS")     - SPEC(ACC,ACCP);
218 CT(ACCEX,"REST-WORLD","ETA") - ETAS(ACCEX);
219 CT(INP,"ACT-AGRCLT","SIGMAC") - SIGMAC(INP);

```

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MODELG: FLOW OF FUNDS MATRIX AND CHANGING CLOSURE RULES.  
MODEL DEFINITION, EXPERIMENTS SOLUTION, AND REPORT

```
221 MODEL      MODELG      MODEL WITH FLOW OF FUNDS MATRIX / ACC, AT, CT /
222 SET        CON(ACC)    CONSUMING INSTITUTIONS
223           CASES      EXPERIMENT CASES /
224                   BASE-CASE, BASE-SOLUT, FIXED-WAGE, FIX-WG+RNT /
225 PARAMETER  RQ(*,CASES) CONSUMPTION LEVELS FOR ALL CONSUMING INSTITUTIONS;
226 CON(ACC) $ (AT(ACC,"TYPE") EQ INSTC) - YES;
227
228 * BASE MODEL AND BASE CASE
229
230 SOLVE MODELG USING HERCULES;
231 RQ(CON,"BASE-CASE") - AT(CON,"QSOL");
232
233 * EXPERIMENT: INCREASE GOVERNMENT INVESTMENT BY 20%
234
235 AT("INVEST-GOV","QFIX") - 1.2 * AT("INVEST-GOV","YBASE");
236
237 SOLVE MODELG USING HERCULES;
238 RQ(CON,"BASE-SOLUT") - AT(CON,"QSOL");
239
240 * DEFINE AND SOLVE A SECOND MODEL WITH FIXED WAGE RATE AND
241 * UNEMPLOYED LABOR. THE EXPERIMENT IS UNCHANGED.
242
243 AT("LABOR","FIX") - P;
244 SOLVE MODELG USING HERCULES;
245 RQ(CON,"FIXED-WAGE") - AT(CON,"QSOL");
246
247 * DEFINE AND SOLVE A THIRD MODEL WITH FIXED RENTAL RATE FOR CAPITAL
248 * IN AGRICULTURE. LABOR STILL HAS FIXED WAGE AND THE EXPERIMENT IS THE
249 * SAME.
250
251 AT("CAPITAL-AG","FIX") - P;
252 SOLVE MODELG USING HERCULES;
253 RQ(CON,"FIX-WG+RNT") - AT(CON,"QSOL");
254
255 RQ("TOTAL",CASES) - SUM(CON,RQ(CON,CASES));
256 DISPLAY RQ;
```

GAMS 2.05 PC AT/XT  
MODEL WITH MORE EQUATIONS THAN VARIABLES

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```

3 SET ACC ACCOUNTS /
4     LABOR
5     CAPITAL
6     HOUSEHOLD
7     ACTIVITY /
8
9 ACRONYMS MF     MARKET FACTOR ACCOUNT
10         INSTC  INSTITUTIONS CONSUMPTION ACCOUNT
11         AC     ACTIVITY COMMODITY ACCOUNT
12         NPQ   FIXED QUANTITY AND PRICE - NUMERAIRE
13         P     FIXED PRICE
14         Q     FIXED QUANTITY
15         IO    INPUT OUTPUT SPECIFICATION
16         VSHR  FIXED VALUE SHARE CONSUMPTION SYSTEM
17         IDIST INCOME DISTRIBUTION
18
19 TABLE CT(ACC,ACC,*) CELL TABLE
20         TBASE  SPECS
21 LABOR .ACTIVITY 100 IO
22 CAPITAL .ACTIVITY 150 IO
23 HOUSEHOLD.LABOR 100 IDIST
24 HOUSEHOLD.CAPITAL 150 IDIST
25 ACTIVITY .HOUSEHOLD 250 VSHR
26
27 TABLE AT(ACC,*) ACCOUNT TABLE
28         TYPE  FIX
29 LABOR MF NPQ
30 CAPITAL MF Q
31 HOUSEHOLD INSTC
32 ACTIVITY AC P
33
34 MODEL ERR MODEL WITH MORE EQUATIONS THAN VARIABLES / ACC,AT,CT /
35 SOLVE ERR USING HERCULES;
36
37 ALIAS(ACC,ACCP);
38 PARAMETER NENVA(ACC) EQUATIONS MINUS VARIABLES PER ACCOUNT
39           NENVC(ACC,ACC) EQUATIONS MINUS VARIABLES PER CELL;
40 NENVA(ACC) - AT(ACC,"NEQ-NVAR");
41 NENVC(ACC,ACCP) - CT(ACC,ACCP,"NEQ-NVAR");
42 DISPLAY NENVA,NENVC;

```

COMPILATION TIME - 0.043 MINUTES

GAMS 2.05 PC AT/XT 90/05/09 16:41:23 PAGE 3  
 MODEL WITH MORE EQUATIONS THAT VARIABLES  
 SOLUTION REPORT SOLVE ERR USING HERCULES FROM LINE 35

H E R C U L E S --- VERSION 1.14 FROM 90/02/28

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 DK-2880 Bagsvaerd  
 Denmark

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 Bagsvaerd  
 Denmark

| SAM STATISTICS:  | ACCOUNTS | CELLS |
|------------------|----------|-------|
| BEFORE EXPANSION | 4        | 5     |
| AFTER EXPANSION  | 4        | 5     |

| MODEL STATISTICS: | TOTAL | EXPLICIT | IMPLICIT | EXOGENOUS |
|-------------------|-------|----------|----------|-----------|
| VARIABLES         |       |          |          |           |
| P-VARIABLES       | 4     | 2        | 2        | 2         |
| Q-VARIABLES       | 4     |          | 4        | 2         |
| Y-VARIABLES       | 4     | 4        | 0        | 0         |
| T-VARIABLES       | 5     | 5        |          |           |
| C-VARIABLES       | 3     |          | 3        |           |
| RESIDUAL          | 1     | 1        |          |           |
| TOTAL             | 21    | 12       | 9        | 4         |

| EQUATIONS        | TOTAL | EXPLICIT | IMPLICIT |
|------------------|-------|----------|----------|
| ROW EQUATIONS    | 4     | 4        |          |
| COLUMN EQUATIONS | 2     | 2        |          |
| P*Q-Y EQUATIONS  | 4     | 2        | 2        |
| T(I,J) EQUATIONS | 5     | 5        |          |
| C(I,J) EQUATIONS | 3     |          | 3        |
| FIXED VARIABLES  | 3     |          | 3        |
| NUMERAIRE        | 1     |          | 1        |
| TOTAL            | 22    | 13       | 9        |

VARIABLE AND EQUATION BALANCE BY MAJOR ACCOUNT TYPE:  
 (CELLS ARE COUNTED WITH THEIR COLUMN, EXCEPT IN  
 REST OF WORLD ACCOUNTS WHERE CELLS IN INSTITUTIONS  
 ROWS ARE COUNTED WITH INSTITUTIONS)

|  | VARIABLES | EQUATIONS | IMBALANCE |
|--|-----------|-----------|-----------|
| FACTORS                                  | 8         | 8         | 0         |
| INSTITUTIONS                             | 5         | 5         | 0         |
| ACTIVITIES/COMMODITIES/<br>REST-OF-WORLD | 7         | 8         | -1        |
| INDIRECT TAXES                           | 0         | 0         | 0         |

GAMS 2.05 PC AT/XT 90/05/09 16:41:23 PAGE 4  
 MODEL WITH MORE EQUATIONS THAN VARIABLES  
 SOLUTION REPORT SOLVE ERR USING HERCULES FROM LINE 35

|                    |    |    |    |
|--------------------|----|----|----|
| NUMERAIRE/RESIDUAL | 1  | 1  |    |
| TOTALS             | 21 | 22 | -1 |

\*\*\*\* ERROR. MODEL DOES NOT HAVE THE SAME NUMBER OF VARIABLES AS EQUATIONS.  
 THERE ARE 1 EQUATIONS TOO MANY.

DISPLAY THE "NEQ-NVAR" COLUMN OF THE ACCOUNT TABLE  
 AND THE "NEQ-NVAR" PLANE OF THE CELL TABLE.

POSITIVE NUMBERS INDICATE UNMATCHED EQUATIONS AND  
 NEGATIVE NUMBERS INDICATE UNMATCHED VARIABLES  
 IN THE CORRESPONDING ACCOUNT OR CELL.

TRY TO MATCH UNMATCHED VARIABLES WITH UNMATCHED EQUATIONS.

\*\*\*\* THE JACOBIAN OF THE MODEL IS STRUCTURALLY SINGULAR.  
 A SUBSYSTEM IS OVERDETERMINED.  
 THE FOLLOWING 6 VARIABLES ARE THE ONLY ONES APPEARING IN  
 THE FOLLOWING 7 EQUATIONS:

VARIABLES:

PARAMETER CT(LABOR,ACTIVITY,TSOL)  
 PARAMETER CT(CAPITAL,ACTIVITY,TSOL)  
 PARAMETER AT(CAPITAL,PSOL)  
 PARAMETER AT(LABOR,YSOL)  
 PARAMETER AT(CAPITAL,YSOL)  
 PARAMETER AT(ACTIVITY,YSOL)

EQUATIONS:

ACCOUNT EQUATION R5 IN  
 SET ACC(LABOR)  
 ACCOUNT EQUATION R5 IN  
 SET ACC(CAPITAL)  
 ACCOUNT EQUATION C2 IN  
 SET ACC(ACTIVITY)  
 ACCOUNT EQUATION Q1 IN  
 SET ACC(LABOR)  
 ACCOUNT EQUATION Q1 IN  
 SET ACC(CAPITAL)  
 CELL EQUATION QSHR IN  
 PARAMETER CT(LABOR,ACTIVITY,SPECS)  
 CELL EQUATION QSHR IN  
 PARAMETER CT(CAPITAL,ACTIVITY,SPECS)

SOLUTION TIME .021 MINUTES

WORK SPACE USED -- 179 WORDS.  
 WORK SPACE AVAILABLE -- 27170 WORDS.

GAMS 2.05 PC AT/XT

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5

MODEL WITH MORE EQUATIONS THAN VARIABLES

SOLUTION REPORT SOLVE ERR USING HERCULES FROM LINE 35

## S O L U T I O N   S U M M A R Y

NEQ-NVAR

ACTIVITY      -1.000

GAMS 2.05 PC AT/XT  
MODEL WITH MORE EQUATIONS THAN VARIABLES  
E X E C U T I N G

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6

---- 42 PARAMETER NENVA EQUATIONS MINUS VARIABLES PER ACCOUNT  
ACTIVITY -1.000

---- 42 PARAMETER NENVC EQUATIONS MINUS VARIABLES PER CELL  
(ALL ZERO)

\*\*\*\* FILE SUMMARY

INPUT D:\HERCLIB\HERCER9.GMS  
OUTPUT D:\HERCLIB\HERCER9.LST

\*\*\*\* USER ERROR(S) ENCOUNTERED

EXECUTION TIME - 0.073 MINUTES

GAMS 2.05 PC AT/XT 90/05/09 16:41:34 PAGE 1  
 MODEL WITH OVERDETERMINED AND UNDERDETERMINED SUBMODELS

```

3 SET ACC ACCOUNTS /
4     LABOR
5     CAPITAL
6     HOUSEHOLD
7     ACTIVITY /
8
9 ACRONYMS MF MARKET FACTOR ACCOUNT
10 INSTC INSTITUTIONS CONSUMPTION ACCOUNT
11 AC ACTIVITY COMMODITY ACCOUNT
12 NPQ FIXED QUANTITY AND PRICE - NUMERAIRE
13 Q FIXED QUANTITY
14 IO INPUT OUTPUT SPECIFICATION
15 VSHR FIXED VALUE SHARE CONSUMPTION SYSTEM
16 IDIST INCOME DISTRIBUTION
17
18 TABLE CT(ACC,ACC,*) CELL TABLE
19           TBASE  SPECS
20 LABOR .ACTIVITY 100 IO
21 CAPITAL .ACTIVITY 150 IO
22 HOUSEHOLD.LABOR 100 IDIST
23 HOUSEHOLD.CAPITAL 150 IDIST
24 ACTIVITY .HOUSEHOLD 250 VSHR
25
26 TABLE AT(ACC,*) ACCOUNT TABLE
27           TYPE  FIX
28 LABOR MF NPQ
29 CAPITAL MF Q
30 HOUSEHOLD INSTC
31 ACTIVITY AC
32
33 MODEL SUB MODEL WITH OVER- AND UNDERDETERMINED SUBBLOCKS / ACC,AT,CT /
34 SOLVE SUB USING HERCULES;
```

COMPILATION TIME - 0.037 MINUTES

GAMS 2.05 PC AT/XT 90/05/09 16:41:46 PAGE 3  
 MODEL WITH OVERDETERMINED AND UNDERDETERMINED SUBMODELS  
 SOLUTION REPORT SOLVE SUB USING HERCULES FROM LINE 34

H E R C U L E S --- VERSION 1.14 FROM 90/02/28

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 Denmark

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 Denmark

| SAM STATISTICS:  | ACCOUNTS | CELLS |
|------------------|----------|-------|
| BEFORE EXPANSION | 4        | 5     |
| AFTER EXPANSION  | 4        | 5     |

| MODEL STATISTICS: | TOTAL | EXPLICIT | IMPLICIT | EXOGENOUS |
|-------------------|-------|----------|----------|-----------|
| VARIABLES         |       |          |          |           |
| P-VARIABLES       | 4     | 3        | 1        | 1         |
| Q-VARIABLES       | 4     |          | 4        | 2         |
| Y-VARIABLES       | 4     | 4        | 0        | 0         |
| T-VARIABLES       | 5     | 5        |          |           |
| C-VARIABLES       | 3     |          | 3        |           |
| RESIDUAL          | 1     | 1        |          |           |
| TOTAL             | 21    | 13       | 8        | 3         |

| EQUATIONS        | TOTAL | EXPLICIT | IMPLICIT |
|------------------|-------|----------|----------|
| ROW EQUATIONS    | 4     | 4        |          |
| COLUMN EQUATIONS | 2     | 2        |          |
| P*Q-Y EQUATIONS  | 4     | 2        | 2        |
| T(I,J) EQUATIONS | 5     | 5        |          |
| C(I,J) EQUATIONS | 3     |          | 3        |
| FIXED VARIABLES  | 2     |          | 2        |
| NUMERAIRE        | 1     |          | 1        |
| TOTAL            | 21    | 13       | 8        |

VARIABLE AND EQUATION BALANCE BY MAJOR ACCOUNT TYPE:  
 (CELLS ARE COUNTED WITH THEIR COLUMN, EXCEPT IN  
 REST OF WORLD ACCOUNTS WHERE CELLS IN INSTITUTIONS  
 ROWS ARE COUNTED WITH INSTITUTIONS)

|  | VARIABLES | EQUATIONS | IMBALANCE |
|--|-----------|-----------|-----------|
| FACTORS                                  | 8         | 8         | 0         |
| INSTITUTIONS                             | 5         | 5         | 0         |
| ACTIVITIES/COMMODITIES/<br>REST-OF-WORLD | 7         | 7         | 0         |
| INDIRECT TAXES                           | 0         | 0         | 0         |

GAMS 2.05 PC AT/XT 90/05/09 16:41:46 PAGE 4  
 MODEL WITH OVERDETERMINED AND UNDERDETERMINED SUBMODELS  
 SOLUTION REPORT SOLVE SUB USING HERCULES FROM LINE 34

|                    |    |    |   |
|--------------------|----|----|---|
| NUMERAIRE/RESIDUAL | 1  | 1  |   |
| TOTALS             | 21 | 21 | 0 |

SIZE OF LARGEST SIMULTANEOUS BLOCK: 5  
 TOTAL NUMBER OF SPIKES: 1

\*\*\*\* THE JACOBIAN OF THE MODEL IS STRUCTURALLY SINGULAR.  
 A SUBSYSTEM IS OVERDETERMINED.  
 THE FOLLOWING 3 VARIABLES ARE THE ONLY ONES APPEARING IN  
 THE FOLLOWING 4 EQUATIONS:

VARIABLES:

PARAMETER AT(ACTIVITY,QSOL)  
 PARAMETER CT(LABOR,ACTIVITY,QCSOL)  
 PARAMETER CT(CAPITAL,ACTIVITY,QCSOL)

EQUATIONS:

CELL EQUATION QSHR IN  
 PARAMETER CT(CAPITAL,ACTIVITY,SPECS)  
 ACCOUNT EQUATION R5 IN  
 SET ACC(LABOR)  
 ACCOUNT EQUATION R5 IN  
 SET ACC(CAPITAL)  
 CELL EQUATION QSHR IN  
 PARAMETER CT(LABOR,ACTIVITY,SPECS)

SOLUTION TIME .028 MINUTES

WORK SPACE USED -- 207 WORDS.  
 WORK SPACE AVAILABLE -- 27170 WORDS.

#### S O L U T I O N S U M M A R Y

( NO VALUES RETURNED )

\*\*\*\* FILE SUMMARY

INPUT D:\HERCLIB\HERCER10.GMS  
 OUTPUT D:\HERCLIB\HERCER10.LST

\*\*\*\* USER ERROR(S) ENCOUNTERED

EXECUTION TIME - 0.066 MINUTES

GAMS 2.05 PC AT/XT  
MODEL WITH NUMERICALLY SINGULAR JACOBIAN

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```

3  SET ACC ACCOUNTS /
4      LABOR
5      CAPITAL
6      HOUSEHOLD
7      ACTIV-1
8      ACTIV-2 /
9
10 ACRONYMS MF      MARKET FACTOR ACCOUNT
11          INSTC  INSTITUTIONS CONSUMPTION ACCOUNT
12          AC     ACTIVITY COMMODITY ACCOUNT
13          NP     FIXED PRICE - NUMERAIRE
14          P      FIXED PRICE
15          Q      FIXED QUANTITY
16          CD     COBB DOUGLAS PRODUCTION FUNCTION
17          VSHR   FIXED VALUE SHARE CONSUMPTION SYSTEM
18          IDIST  INCOME DISTRIBUTION
19
20 TABLE CT(ACC,ACC,*) GELL TABLE
21          TBASE   SPECS
22 LABOR      .ACTIV-1    50    CD
23 LABOR      .ACTIV-2    50    CD
24 CAPITAL    .ACTIV-1    75    CD
25 CAPITAL    .ACTIV-2    75    CD
26 HOUSEHOLD .LABOR      100   IDIST
27 HOUSEHOLD .CAPITAL    150   IDIST
28 ACTIV-1    .HOUSEHOLD  125   VSHR
29 ACTIV-2    .HOUSEHOLD  125   VSHR
30
31 TABLE AT(ACC,*) ACCOUNT TABLE
32          TYPE   FIX
33 LABOR      MF
34 CAPITAL    MF     Q
35 HOUSEHOLD  INSTC  NP
36 ACTIV-1    AC
37 ACTIV-2    AC     P
38
39 MODEL SING MODEL WITH NUMERICALLY SINGULAR JACOBIAN / ACC,AT,CT /
40 SOLVE SING USING HERCULES;

```

COMPILATION TIME - 0.039 MINUTES

GAMS 2.05 PC AT/XT 90/05/09 16:42:09 PAGE 3  
 MODEL WITH NUMERICALLY SINGULAR JACOBIAN  
 SOLUTION REPORT SOLVE SING USING HERCULES FROM LINE 40

HERCULES --- VERSION 1.14 FROM 90/02/28

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| SAM STATISTICS:  | ACCOUNTS | CELLS |
|------------------|----------|-------|
| BEFORE EXPANSION | 5        | 8     |
| AFTER EXPANSION  | 5        | 8     |

| MODEL STATISTICS: | TOTAL | EXPLICIT | IMPLICIT | EXOGENOUS |
|-------------------|-------|----------|----------|-----------|
| VARIABLES         |       |          |          |           |
| P-VARIABLES       | 5     | 3        | 2        | 2         |
| Q-VARIABLES       | 5     |          | 5        | 1         |
| Y-VARIABLES       | 5     | 5        | 0        | 0         |
| T-VARIABLES       | 8     | 8        |          |           |
| C-VARIABLES       | 6     |          | 6        |           |
| RESIDUAL          | 1     | 1        |          |           |
| TOTAL             | 30    | 17       | 13       | 3         |

| EQUATIONS        | TOTAL | EXPLICIT | IMPLICIT |
|------------------|-------|----------|----------|
| ROW EQUATIONS    | 5     | 5        |          |
| COLUMN EQUATIONS | 3     | 3        |          |
| P*Q-Y EQUATIONS  | 5     | 1        | 4        |
| T(I,J) EQUATIONS | 8     | 8        |          |
| C(I,J) EQUATIONS | 6     |          | 6        |
| FIXED VARIABLES  | 2     |          | 2        |
| NUMERAIRE        | 1     |          | 1        |
| TOTAL            | 30    | 17       | 13       |

VARIABLE AND EQUATION BALANCE BY MAJOR ACCOUNT TYPE:  
 (CELLS ARE COUNTED WITH THEIR COLUMN, EXCEPT IN  
 REST OF WORLD ACCOUNTS WHERE CELLS IN INSTITUTIONS  
 ROWS ARE COUNTED WITH INSTITUTIONS)

|  | VARIABLES | EQUATIONS | IMBALANCE |
|--|-----------|-----------|-----------|
| FACTORS                                  | 8         | 7         | 1         |
| INSTITUTIONS                             | 7         | 7         | 0         |
| ACTIVITIES/COMMODITIES/<br>REST-OF-WORLD | 14        | 15        | -1        |
| INDIRECT TAXES                           | 0         | 0         | 0         |

GAMS 2.05 PC AT/XT 90/05/09 16:42:09 PAGE 4  
MODEL WITH NUMERICALLY SINGULAR JACOBIAN  
SOLUTION REPORT SOLVE SING USING HERCULES FROM LINE 40

|                    |    |    |   |
|--------------------|----|----|---|
| NUMERAIRE/RESIDUAL | 1  | 1  |   |
| TOTALS             | 30 | 30 | 0 |

SIZE OF LARGEST SIMULTANEOUS BLOCK: 7  
TOTAL NUMBER OF SPIKES: 2

\*\*\*\* MODEL ERROR \*\* THE JACOBIAN IS SINGULAR  
THE JACOBIAN COLUMN FOR THE FOLLOWING VARIABLE IS  
LINEARLY DEPENDENT ON OTHER COLUMNS:  
PARAMETER AT(LABOR,PSOL)

SOLUTION TIME .027 MINUTES

WORK SPACE USED -- 276 WORDS.  
WORK SPACE AVAILABLE -- 27170 WORDS.

#### SOLUTION SUMMARY

NEQ-NVAR

LABOR 1.000  
ACTIV-2 -1.000

#### \*\*\*\* FILE SUMMARY

INPUT D:\HERCLIB\HERCER11.GMS  
OUTPUT D:\HERCLIB\HERCER11.LST

#### \*\*\*\* USER ERROR(S) ENCOUNTERED

EXECUTION TIME - 0.066 MINUTES

GAMS 2.05 PC AT/XT  
 MODEL IN WHICH A PRICE APPROACHES INFINITY

90/05/09 16:44:22 PAGE 1

```

3  SET ACC ACCOUNTS /
4      LABOR
5      CAPITAL
6      HOUSEHOLD
7      ACTIVITY /
8
9  ACRONYMS MF      MARKET FACTOR ACCOUNT
10         INSTC   INSTITUTIONS CONSUMPTION ACCOUNT
11         AC      ACTIVITY COMMODITY ACCOUNT
12         NPQ     FIXED QUANTITY AND PRICE - NUMERAIRE
13         Q       FIXED QUANTITY
14         CD      COBB DOUGLAS PRODUCTION FUNCTION
15         VSHR    FIXED VALUE SHARE CONSUMPTION SYSTEM
16         IDIST   INCOME DISTRIBUTION
17
18  TABLE CT(ACC,ACC,*) CELL TABLE
19              TBASE   SPECS
20  LABOR      .ACTIVITY  100    CD
21  CAPITAL    .ACTIVITY  150    CD
22  HOUSEHOLD .LABOR      100    IDIST
23  HOUSEHOLD .CAPITAL    150    IDIST
24  ACTIVITY  .HOUSEHOLD  250    VSHR
25
26  TABLE AT(ACC,*) ACCOUNT TABLE
27              TYPE    FIX    QFIX
28  LABOR        MF      NPQ
29  CAPITAL      MF      Q      EPS
30  HOUSEHOLD    INSTC
31  ACTIVITY     AC
32
33  MODEL IN MODEL IN WHICH A VARIABLES APPROACHES INFINITY / ACC,AT,CT /
34  SOLVE IN USING HERCULES;

```

COMPILATION TIME - 0.035 MINUTES

GAMS 2.05 PC AT/XT 90/05/09 16:44:34 PAGE 3  
 MODEL IN WHICH A PRICE APPROACHES INFINITY  
 SOLUTION REPORT SOLVE IN USING HERCULES FROM LINE 34

HERCULES --- VERSION 1.14 FROM 89/11/06

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 BAGSVAERD - DENMARK

| SAM STATISTICS:  | ACCOUNTS | CELLS |
|------------------|----------|-------|
| BEFORE EXPANSION | 4        | 5     |
| AFTER EXPANSION  | 4        | 5     |

| MODEL STATISTICS: | TOTAL | EXPLICIT | IMPLICIT | EXOGENOUS |
|-------------------|-------|----------|----------|-----------|
| VARIABLES         |       |          |          |           |
| P-VARIABLES       | 4     | 3        | 1        | 1         |
| Q-VARIABLES       | 4     |          | 4        | 2         |
| Y-VARIABLES       | 4     | 4        | 0        | 0         |
| T-VARIABLES       | 5     | 5        |          |           |
| C-VARIABLES       | 3     |          | 3        |           |
| RESIDUAL          | 1     | 1        |          |           |
| TOTAL             | 21    | 13       | 8        | 3         |

| EQUATIONS        | TOTAL | EXPLICIT | IMPLICIT |
|------------------|-------|----------|----------|
| ROW EQUATIONS    | 4     | 4        |          |
| COLUMN EQUATIONS | 2     | 2        |          |
| P*Q-Y EQUATIONS  | 4     | 2        | 2        |
| T(I,J) EQUATIONS | 5     | 5        |          |
| C(I,J) EQUATIONS | 3     |          | 3        |
| FIXED VARIABLES  | 2     |          | 2        |
| NUMERAIRE        | 1     |          | 1        |
| TOTAL            | 21    | 13       | 8        |

VARIABLE AND EQUATION BALANCE BY MAJOR ACCOUNT TYPE:  
 (CELLS ARE COUNTED WITH THEIR COLUMN, EXCEPT IN  
 REST OF WORLD ACCOUNTS WHERE CELLS IN INSTITUTIONS  
 ROWS ARE COUNTED WITH INSTITUTIONS)

|  | VARIABLES | EQUATIONS | IMBALANCE |
|--|-----------|-----------|-----------|
| FACTORS                                  | 8         | 8         | 0         |
| INSTITUTIONS                             | 5         | 5         | 0         |
| ACTIVITIES/COMMODITIES/<br>REST-OF-WORLD | 7         | 7         | 0         |
| INDIRECT TAXES                           | 0         | 0         | 0         |

GAMS 2.05 PC AT/XT 90/05/09 16:44:34 PAGE 4  
 MODEL IN WHICH A PRICE APPROACHES INFINITY  
 SOLUTION REPORT SOLVE IN USING HERCULES FROM LINE 34

|                    |    |    |   |
|--------------------|----|----|---|
| NUMERAIRE/RESIDUAL | 1  | 1  |   |
| TOTALS             | 21 | 21 | 0 |

SIZE OF LARGEST SIMULTANEOUS BLOCK: 0  
 TOTAL NUMBER OF SPIKES: 0

\*\*\*\* RATE OF CHANGE GREATER THAN 100 IN  
 PARAMETER AT(CAPITAL, PSOL)

THE COLUMNS "DPSOL", "DQSOL", AND "DYSOL" OF THE  
 ACCOUNT TABLE CONTAIN THE RATES OF CHANGE OF PSOL,  
 QSOL, AND YSOL, RESPECTIVELY.

THE PLANES "DTSOL" AND "DQCSOL" OF THE CELL TABLE  
 CONTAIN THE RATES OF CHANGE OF TSOL AND QCSOL,  
 RESPECTIVELY.

LOOK FOR LARGE VALUES. THEY INDICATE COMPONENTS OF  
 THE MODEL THAT ARE RUNNING WILD.

\*\*\*\* SOLUTION PROCESS STOPS AT TIME PERIOD .953

#### G D P SUMMARY

|                      | BASE    | SOLUTION          |                    | PRICE<br>INDEX |
|----------------------|---------|-------------------|--------------------|----------------|
|                      |         | CURRENT<br>PRICES | CONSTANT<br>PRICES |                |
| GDP AT FACTOR COST   | 250.000 | 250.000           | 107.031            | 2.336          |
| NET INDIRECT TAXES   | .000    | .000              | .000               |                |
| INCOME EFFECT        |         |                   | -67.174            |                |
| FINAL USE            | 250.000 | 250.000           | 39.857             | 6.272          |
| EXPORTS              | .000    | .000              | .000               |                |
| IMPORTS              | .000    | .000              | .000               |                |
| GDP AT MARKET PRICES | 250.000 | 250.000           | 39.857             | 6.272          |

EXIT -- INTERMEDIATE SOLUTION  
 TIME STEPS 9  
 NEWTON ITERATIONS 0

SOLUTION TIME .034 MINUTES

WORK SPACE USED -- 204 WORDS.  
 WORK SPACE AVAILABLE -- 27664 WORDS.

GAMS 2.05 PC AT/XT 90/05/09 16:44:34 PAGE 5  
 MODEL IN WHICH A PRICE APPROACHES INFINITY  
 SOLUTION REPORT SOLVE IN USING HERCULES FROM LINE 34

## S O L U T I O N S U M M A R Y

|           | PSOL   | QSOL    | YSOL    | YBASE   | RESIDUAL |
|-----------|--------|---------|---------|---------|----------|
| LABOR     | 1.000  | 100.000 | 100.000 | 100.000 |          |
| CAPITAL   | 21.333 | 7.031   | 150.000 | 150.000 |          |
| HOUSEHOLD | 6.272  | 39.857  | 250.000 | 250.000 |          |
| ACTIVITY  | 6.272  | 39.857  | 250.000 | 250.000 | -67.174  |

## S O L U T I O N S U M M A R Y

|           | + DPSOL | DQSOL    |
|-----------|---------|----------|
| CAPITAL   | 227.556 | -150.000 |
| HOUSEHOLD | 45.529  | -438.505 |
| ACTIVITY  | 45.529  | -438.505 |

## \*\*\*\* FILE SUMMARY

INPUT D:\HERCLIB\HERCER12.GMS  
 OUTPUT D:\HERCLIB\HERCER12.LST

## \*\*\*\* USER ERROR(S) ENCOUNTERED

EXECUTION TIME - 0.072 MINUTES